MATH 128A Numerical Analysis Discussion Section

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Outline

- Multiple Integral
  - Simpson's Double Integral
- IVP for ODEs
  - Well-posedness
- (Forward) Euler's Method
  - $x_{n+1} = x_n + hf(x_n, t_n)$
- High-order Taylor Method
  - $x_{n+1} = x_n + hT(x_n, t_n)$

Multiple Integral

- In the textbook
  - Apply the quadrature rule dimension by dimension
  - Find a huge formula

- In practical
  - Define the quadrature points and weights at the reference domain [-1,1]^N
  - Using the change of variables and the for loops

Multiple Integral

- Regular way (Composite Simpson's rule)

$$\iint_R f(x,y)\,dA = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx,$$

$$\int_a^b \int_c^d f(x,y)\,dy\,dx = \frac{k}{3}\Bigg[\int_a^b f(x,y_0)\,dx + 2\sum_{j=1}^{(m/2)-1} \int_a^b f(x,y_{2j})\,dx + 4\sum_{j=1}^{m/2} \int_a^b f(x,y_{2j-1})\,dx + \int_a^b f(x,y_m)\,dx \Bigg] - \frac{(d-c)k^4}{180} \int_a^b \frac{\partial^4 f}{\partial y^4}(x,\mu)\,dx.$$

IVP for ODEs

- Well-posed
  - Have a unique solution
    - Checking Lipschitz condition on the Convex set D
  - Residence against small perturbations
    - Truncation error could be a perturbation
    - Example : $y' = 10^{100}y$

Euler's Method

- Forward Euler's method
  - $x_{n+1} = x_n + hf(x_n, t_n)$

- Error analysis(w/ truncation error)

$$|y(t_{j+1}) - u_{j+1}| \leq \frac{1}{L}\left(\frac{hM}{2} + \frac{\delta}{h}\right)\left(e^{L(t_{j+1}-a)} - 1\right) + \delta\, e^{L(t_{j+1}-a)}.$$

  - As t grows, the error increases exponentially.
  - Requires really huge N for adequate error bounds.
  - We can not take arbitrary small h.

High-order Taylor Method

- High-order Taylor Method
  - $x_{n+1} = x_n + hT(x_n, t_n)$
  - $T(x_n, t_n) = f(x_n, t_n) + \frac{h}{2}\frac{df}{dt}(x_n, t_n) + \cdots + \frac{h^{k-1}}{k!}\frac{d^{k-1}f}{dx^{k-1}}(x_n, t_n)$
- LTE

$$\tau_{i+1}(h) = \frac{y_{i+1} - y_i}{h} - T^{(n)}(t_i, y_i) = \frac{h^n}{(n+1)!}f^{(n)}(\xi_i, y(\xi_i)),$$

  - $O(h^n)$