MATH 128A Numerical Analysis Discussion Section

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Aitken's Δ^2 Method

$$\widehat{p_n} = p_n - \frac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n}$$

- Accelerate the convergence of a sequence that is linearly convergent.
- Forward difference := $\Delta p_n = p_{n+1} - p_n$
- Aitken's method can be written as

$$\widehat{p_n} = p_n - \frac{(\Delta p_n)^2}{\Delta^2 p_n}$$

Steffensen's Method

$$p_0^{(i)} = \{\Delta^2\}\left(p_0^{(i-1)}\right)$$
$$p_1^{(i)} = g\left(p_0^{(i)}\right)$$
$$p_2^{(i)} = g\left(p_1^{(i)}\right)$$
$$p_0^{(i+1)} = \{\Delta^2\}\left(\boldsymbol{p_0^{(i)}}\right)$$

- Accelerate the order of convergence of the given fixed point method using Aitken's method

Horner's Method

$$P(x) = (x - x_0)Q(x) + b_0$$

- Make given polynomial be nested form
- If we get well approximated root x_*, we can reduce the given polynomial

$$P(x) \approx (x - x_*)Q(x)$$

Muller's Method

$$P(x) = a(x - p_2)^2 + b(x - p_2) + c$$

where P(x) goes through 3 points (p0, f(p0)), (p1,f(p1)), (p2,f(p2))

- One of the root of P(x) will be p3, namely

$$p_3 = p_2 - \frac{2c}{b + \text{sgn}(b)\sqrt{b^2 - 4ac}},$$

Muller's Method

$$c = f(p_2),$$

$$b = \frac{(p_0 - p_2)^2[f(p_1) - f(p_2)] - (p_1 - p_2)^2[f(p_0) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_0 - p_1)},$$

$$a = \frac{(p_1 - p_2)[f(p_0) - f(p_2)] - (p_0 - p_2)[f(p_1) - f(p_2)]}{(p_0 - p_2)(p_1 - p_2)(p_0 - p_1)}.$$

- Used Lagrange polynomials (Sec 3.1)