MATH 128A Numerical Analysis Discussion Section

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Announcement

- Homework submission
  - ASSIGN the pages to the questions, please.
- Homework correction and comment
  - 1.1.26 : n -> n+1
  - 1.2.22 : applying chopping to each term first
- Quiz #1
  - From 10:40 to 10:50

Brief Review

- Iterative method
  - Bisection method
  - Fixed point theorem
- Newton's method
  - Requires $f'(x)$
  - The most effective method
- Secant method
  - Approximate $f'(p_{n-1}) \approx \frac{f(p_{n-1})-f(p_{n-2})}{p_{n-1}-p_{n-2}}$
  - Slower than Newton's method

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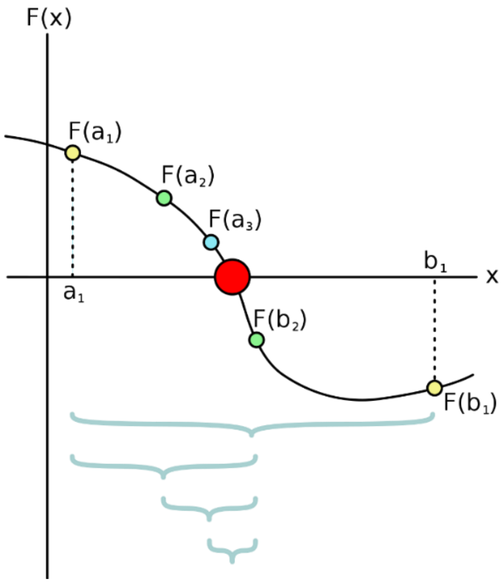
- Bisection method
  - Based on the Intermediate value theorem
  - half -> half -> …
  - Very slow, but reliable(or stable).



Fixed point method

- Find the root of

$$f(x) = x^2 - x - 1$$

- How?
  - $x = x^2 - 1$
  - $x = 1 + \frac{1}{x}$
  - $x = \sqrt{x + 1}$

Fixed point theorem

(Fixed-Point Theorem)

Let $g \in C[a,b]$ be such that $g(x) \in [a,b]$, for all x in $[a,b]$. Suppose, in addition, that g' exists on (a,b) and that a constant $0 < k < 1$ exists with

$$|g'(x)| \leq k, \quad \text{for all } x \in (a,b).$$

Then for any number p_0 in $[a,b]$, the sequence defined by

$$p_n = g(p_{n-1}), \quad n \geq 1,$$

converges to the unique fixed point p in $[a,b]$. ■

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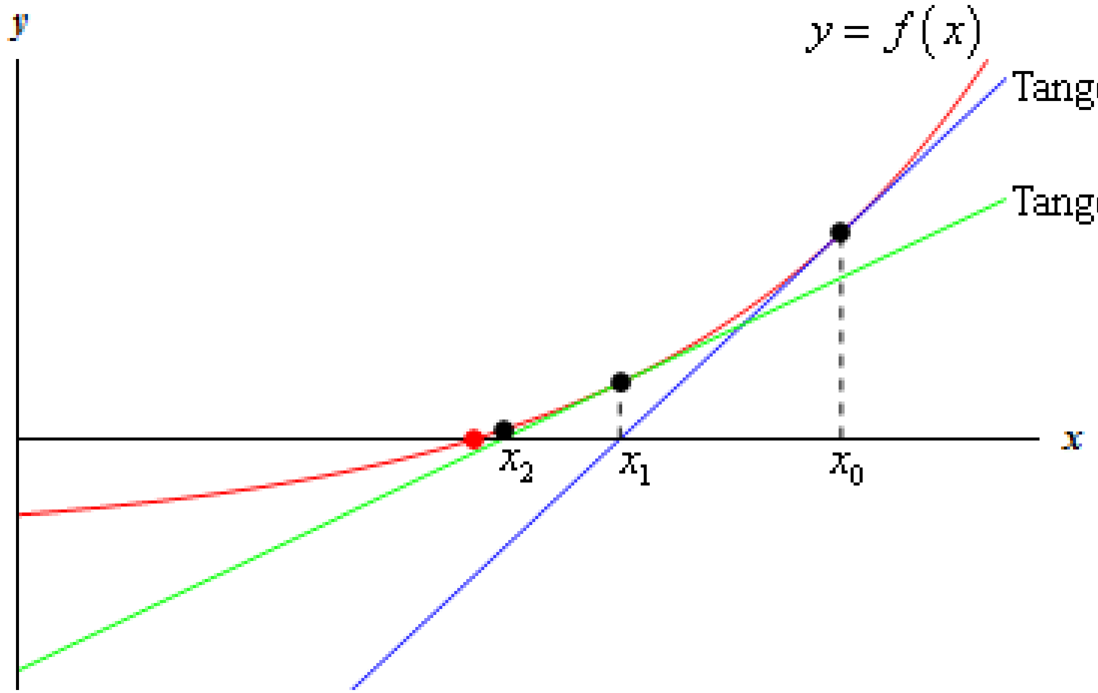
Newton's method



- f(x) is continuous and differentiable near the root
- $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Converging very fast.

Newton's method - cautions

- f(x) is continuous and differentiable
- Stationary points
- Converging to a different root
- Diverging
  - $f(x) = x^3 - 5x$

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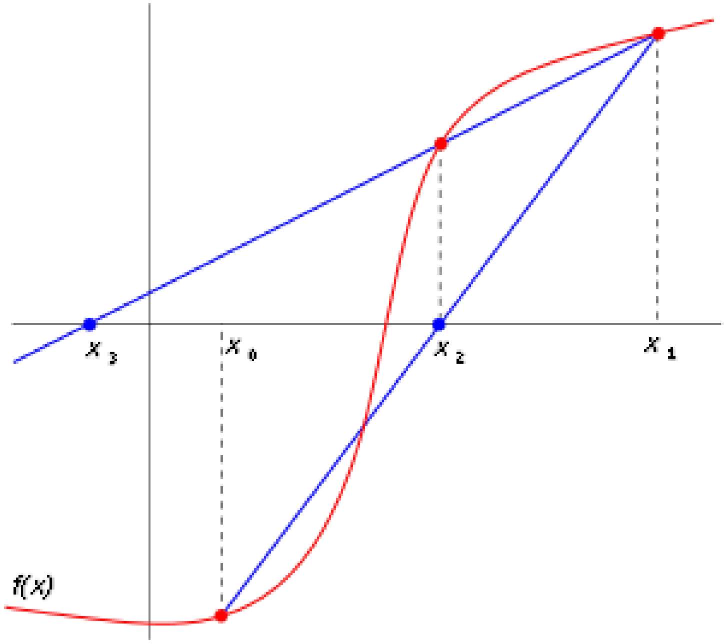
Secant method



- Start with two distinct points
- $x_n = x_{n-1} - \frac{(x_{n-1} - x_{n-2})}{(f(x_{n-1}) - f(x_{n-2}))} f(x_n)$
- Don't need to calculate derivatives

Quiz