MATH 128A Numerical Analysis Discussion Section

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Announcement

- First Quiz : Sep. 7 in DIS section. Covers Sec 1.1, 1.2, 1.3, 2.1 (HW #1).

- Each quiz will cover contents from 2 previous homework

Announcement

- Office : 743 Evans Hall
- Office Hours : Mon 13:00-15:00pm

Announcement

- Account Information
  - Username : !cmfmath128a
  - Password : c@1b3arssubtraction

Brief Review

- Floating point numbers
  - Inevitable errors
  - Structure
  - Finite-digit arithmetic
- Error analysis
  - Absolute error $|p - p^*|$
  - Relative error $\frac{|p-p^*|}{|p|}$
- Rate of convergence
  - Measure how fast the sequence converges

Floating point numbers

- Floating point numbers
  - Inevitable errors
    - Impossible to store the TRUE value in the format for computing machine (e.g. π)
  - Structure
    - How many bits it uses -> Single/Double/Quad
    - Sign(s) / exponent(d) / mantissa(f)

Floating point numbers

- Floating point numbers
  - Finite-digit arithmetic
    - Rounding / Chopping
    - The order DOES matter

      $$(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$$

    - Smallest to largest / nested form

Error analysis

- Error analysis
  - Error formula
    - Absolute error $|p - p^*|$
    - Relative error $\frac{|p-p^*|}{|p|}$
  - abs/rel error
    - Round 1024 to 1000
    - abs : 24
    - rel : 24/1024 = 0.0234375 = 2.34375e-2

Rate of convergence

- Rate of convergence
  - $\{\alpha_n\}_{n\in\mathbb{N}}$, $|\alpha_n - \alpha| \leq K|\beta_n|$. Then this seq. converges with rate of convergence $O(\beta_n)$.
  - Basically, the rate of convergence measures the convergence speed of a (well-known) seq.
  - In general, we are interested in the fastest β_n
    - One quick way; Find β_n s.t. $\lim_{n\to\infty} \frac{|\alpha_n - \alpha|}{\beta_n} = \lambda \neq 0$

    OR

$$\lim_{h\to 0} \frac{|f(h) - F|}{|g(h)|} = \lambda \neq 0$$

Rate of convergence

- Rate of convergence
  - Ex $\lim_{h \to 0}(1 - \cos h) = 0$. How fast?
    - There are two ways.

      i) Trial & error

      ii) Use Taylor expansion

Rate of convergence

- $\lim_{h \to 0} |1 - \cos h| = 0$
  - convergence is faster than $O(h^0)$
- $\lim_{h \to 0} \frac{|1-\cos h|}{|h|} = 0$
  - convergence is faster than $O(h^1)$
- $\lim_{h \to 0} \frac{|1-\cos h|}{|h^2|} = \frac{1}{2}$
  - Finally we have the rate of convergence $O(h^2)$

Rate of convergence

- Since $\cos h = 1 - \frac{1}{2}h^2 + O(h^4)$,
$$|\cos h - 1| = \frac{1}{2}h^2 + O(h^4) \le h^2$$
for small h
- So, we have the rate of convergence $\mathrm{O}(h^2)$