Solution to Homework 10

1. (4.2.6) $y = C_1e^{2t} + C_2e^{3t}$.
2. (4.2.10)
$$y = C_1e^{\frac{1}{2}t} + C_2te^{\frac{1}{2}t}.$$
3. (4.2.16)
$$y = \frac{4}{3}e^t - \frac{1}{3}e^{3t}.$$
4. (4.2.28) Independent.
5. (4.2.34)
   (a) By the definition of 2×2 determinant.
   (b) True by Lemma 1.
   (c) In this case $ay_1 = by_2$ so the matrix has rank 1 and its determinant is always 0.
6. (4.3.8) The roots to the auxiliary equation is $\frac{-1 \pm \sqrt{5}i}{2}$, so the general solutions are
$$y = C_1e^{-t/2}\cos\frac{\sqrt{5}}{2}t + C_2e^{-t/2}\sin\frac{\sqrt{5}}{2}t.$$
7. (4.3.16)
$$y = C_1e^{\frac{\sqrt{53}-3}{2}t} + C_2e^{\frac{\sqrt{53}+3}{2}t}.$$
8. (4.3.24)
$$y = \frac{1}{3}\sin 3t + \cos 3t.$$
9. (4.3.29)
   (a) $y = C_1e^{-t} + C_2e^t\sin(\sqrt{2}t) + C_3e^t\cos(\sqrt{2}t)$.
   (b) $y = C_1e^{2t} + C_2e^{-2t}\sin(3t) + C_3e^{-2t}\cos(3t)$.
   (c) $y = C_1\sin 3t + C_2\sin 2t + C_3\cos 3t + C_4\cos 2t$.
10. (4.4.4) Yes.
11. (4.4.6) Yes.
12. (4.4.18) $y_p = -2t\cos 2t$.
13. (4.4.22) $x_p = 2t^4e^t$.
14. (4.4.30) $y = C_1e^t\sin t + C_2e^t\cos t$.
15. (4.4.34) $y_p = -\frac{1}{4}e^{-t}$.