

Mathematics 219, Fall 2024

Fraydoun Rezakhanlou

Lectures: TT 12:30-2:00 pm, Evans 891

Office Hours: TT 5:00-6:30 pm, Evans 803

Prerequisites : Math 202 AB, or familiarity with measure theory

The main goal of the theory of dynamical system is the study of the global orbit structure of maps and flows. This course reviews some fundamental concepts and results in the theory of dynamical systems with an emphasis on differentiable dynamics.

Several important notions in the theory of dynamical systems have their roots in the work of Maxwell, Boltzmann and Gibbs who tried to explain the macroscopic behavior of fluids and gases on the basic of the classical dynamics of many particle systems. The notion of *ergodicity* was introduced by Boltzmann as a property satisfied by a Hamiltonian flow on its constant energy surfaces. Boltzmann also initiated a mathematical expression for the *entropy* and the *entropy production* to derive Maxwell's description for the equilibrium states. Gibbs introduced the notion of *mixing systems* to explain how reversible mechanical systems could approach equilibrium states. The ergodicity and mixing are only two possible properties in the hierarchy of stochastic behavior of a dynamical system. Hopf invented a versatile method for proving the ergodicity of geodesic flows. The key role in Hopf's approach is played by the hyperbolicity. Lyapunov exponents and Kolmogorov–Sinai entropy are used to measure the hyperbolicity of a system.

Here is an outline of the course:

1. Examples: Linear systems. Translations on Tori. Arnold can map.
Baker's transformation. Geodesic flows. Sinai's billiard. Lorentz gas.
2. Invariant measures. Ergodic theory. Kolmogorov-Sinai Entropy. Lyapunov exponents.
Hyperbolic systems. Smale horseshoe.
3. Perron-Frobenius operator. Bowen-Ruelle-Sinai measures.
4. Pesin's theorem. Ruelle's inequality.
5. Billiards.

Grading: Your grade will be based on your performance on weekly assigned homework assignments, and your participation in the class. The answer to homework assignments must be typed (and preferrably written in LaTeX).