1. Let \( u = \ln x \), \( dv = x \, dx \Rightarrow du = dx / x \), \( v = \frac{1}{2} x^2 \). Then by Equation 2, \( \int u \, dv = uv - \int v \, du \),

\[
\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{2} x^2 - \frac{1}{2} \int x^2 \ln x - \frac{1}{2} \frac{1}{2} x^2 + C
= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C
\]

5. Let \( u = r \), \( dv = e^{r/2} \, dr \Rightarrow du = dr \), \( v = 2e^{r/2} \). Then \( \int r e^{r/2} \, dr = 2e^{r/2} - \int 2e^{r/2} \, dr = 2e^{r/2} - 4e^{r/2} + C \).

7. Let \( u = x^2 \), \( dv = \sin \pi x \, dx \Rightarrow du = 2x \, dx \) and \( v = -\frac{1}{\pi} \cos \pi x \). Then

\[
I = \int x^2 \sin \pi x \, dx = -\frac{1}{\pi} x^2 \cos \pi x + \frac{2}{\pi} \int x \cos \pi x \, dx (\ast).
\]

Next let \( U = x \), \( dV = \cos \pi x \, dx \Rightarrow dU = dx \), \( V = \frac{1}{\pi} \sin \pi x \), so

\[
\int x \cos \pi x \, dx = \frac{1}{\pi} \sin \pi x - \frac{1}{\pi} \int \sin \pi x \, dx = \frac{1}{\pi} \sin \pi x + \frac{1}{\pi} \cos \pi x + C_1.
\]

Substituting for \( \int x \cos \pi x \, dx \) in (\ast), we get

\[
I = -\frac{1}{\pi} x^2 \cos \pi x + \frac{2}{\pi} \left( \frac{1}{\pi} \sin \pi x + \frac{1}{\pi} \cos \pi x + C_1 \right) = -\frac{1}{\pi} x^2 \cos \pi x + \frac{2}{\pi^2} \sin \pi x + \frac{2}{\pi^3} \cos \pi x + C,
\]

where \( C = \frac{2}{\pi} C_1 \).

13. First let \( u = (\ln x)^2 \), \( dv = dx \Rightarrow du = 2 \ln x \cdot \frac{1}{x} \, dx \), \( v = x \). Then by Equation 2,

\[
I = \int (\ln x)^2 \, dx = x(\ln x)^2 - 2 \int x \ln x \cdot \frac{1}{x} \, dx = x(\ln x)^2 - 2 \int \ln x \, dx.
\]

Next let \( U = \ln x \), \( dV = dx \Rightarrow dU = 1/x \, dx \), \( V = x \) to get \( \int \ln x \, dx = -\int x \cdot (1/x) \, dx = x \ln x - x + C_1 \). Thus,

\[
I = x(\ln x)^2 - 2 \left(x \ln x + x + C_1 \right) = x(\ln x)^2 - 2x \ln x + 2x + C,
\]

where \( C = -2C_1 \).

18. Let \( u = y \), \( dv = \cosh ay \, dy \Rightarrow du = dy \), \( v = \frac{\sinh ay}{a} \). Then

\[
\int y \cosh ay \, dy = \frac{y \sinh ay}{a} - \frac{1}{a} \int \sinh ay \, dy = \frac{y \sinh ay}{a} - \frac{\cosh ay}{a^2} + C.
\]

19. Let \( u = t \), \( dv = \sin 3t \, dt \Rightarrow du = dt \), \( v = -\frac{1}{3} \cos 3t \). Then
$\int_0^\pi t \sin 3t \, dt = \left[ -\frac{1}{3} t \cos 3t \right]_0^\pi + \frac{1}{3} \int_0^\pi \cos 3t \, dt = \left( \frac{1}{3} \pi - 0 \right) + \frac{1}{9} \left[ \sin 3t \right]_0^\pi = \frac{\pi}{3}.$

21. Let $u = \ln x$, $dv = x^{-2} \, dx \Rightarrow du = \frac{1}{x} \, dx$, $v = -x^{-1}$. By (6),

$$\int_1^2 \frac{\ln x}{x^2} \, dx = \left[ -\frac{\ln x}{x} \right]_1^2 + \int_1^2 x^{-2} \, dx = -\frac{1}{2} \ln x + 1 + \left[ -\frac{1}{x} \right]_1^2 = -\frac{1}{2} \ln 2 + 0 - \frac{1}{2} + 1 = \frac{1}{2} - \frac{1}{2} \ln 2.$$  

27. Let $u = \ln (\sin x)$, $dv = \cos x \, dx \Rightarrow du = \frac{\cos x}{\sin x} \, dx$, $v = \sin x$. Then

$I = \int \cos x \ln (\sin x) \, dx = \sin x \ln (\sin x) - \int \cos x \, dx = \sin x \ln (\sin x) - \sin x + C$.

Another method: Substitute $t = \sin x$, so $dt = \cos x \, dx$. Then $I = \int \ln t \, dt = t \ln t - t + C$ (see Example 2) and so $I = \sin x (\ln \sin x - 1) + C$.

33. Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w \, dw$. Thus, $\int \sin \sqrt{x} \, dx = \int 2w \sin w \, dw$. Now use parts with $u = 2w$, $dv = \sin w \, dw$, $du = 2 \, dw$, $v = -\cos w$ to get

$$\int 2w \sin w \, dw = -2w \cos w + \int 2 \cos w \, dw = -2w \cos w + 2 \sin w + C = -2 \sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} + C = 2 \left( \sin \sqrt{x} - \sqrt{x} \cos \sqrt{x} \right) + C$$

47. Let $u = \left( \frac{x + a}{2} \right)^n$, $dv = dx \Rightarrow du = n \left( \frac{x + a}{2} \right)^{n-1} x \, dx$, $v = x$. Then

$$\int \left( \frac{x + a}{2} \right)^n \, dx = x \left( \frac{x + a}{2} \right)^n - 2n \int \left( \frac{x + a}{2} \right)^{n-1} \, dx$$

$$= x \left( \frac{x + a}{2} \right)^n - 2n \left[ \int \left( \frac{x + a}{2} \right)^n \, dx - a^2 \int \left( \frac{x + a}{2} \right)^{n-1} \, dx \right]$$

$$= x \left( \frac{x + a}{2} \right)^n - 2n \left[ \int \frac{1}{2} \left( \frac{x + a}{2} \right)^n \, dx - a^2 \int \left( \frac{x + a}{2} \right)^{n-1} \, dx \right]$$

$$= x \left( \frac{x + a}{2} \right)^n - 2n \left[ \int \frac{1}{2} \left( \frac{x + a}{2} \right)^n \, dx - a^2 \int \left( \frac{x + a}{2} \right)^{n-1} \, dx \right]$$

$$\Rightarrow (2n+1) \int \left( \frac{x + a}{2} \right)^n \, dx = x \left( \frac{x + a}{2} \right)^n + 2na^2 \int \left( \frac{x + a}{2} \right)^{n-1} \, dx$$.

$$\int \left( \frac{x + a}{2} \right)^n \, dx = \frac{x \left( \frac{x + a}{2} \right)^n}{2n+1} + \frac{2na^2}{2n+1} \int \left( \frac{x + a}{2} \right)^{n-1} \, dx$$ [ provided $2n+1 \neq 0$ ].