1. (a) \( ay'' + by' + cy = 0 \) where \( a, b, \) and \( c \) are constants.
(b) \( ar^2 + br + c = 0 \)

(c) If the auxiliary equation has two distinct real roots \( r_1 \) and \( r_2 \), the solution is \( y = c_1 e^{r_1 x} + c_2 e^{r_2 x} \). If the roots are real and equal, the solution is \( y = c_1 e^{rx} + c_2 xe^{rx} \) where \( r \) is the common root. If the roots are complex, we can write \( r_1 = \alpha + i\beta \) and \( r_2 = \alpha - i\beta \), and the solution is \( y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x) \).

2. (a) An initial-value problem consists of finding a solution \( y \) of a second-order differential equation that also satisfies given conditions \( y(x_0) = y_0 \) and \( y'(x_0) = y_1 \), where \( y_0 \) and \( y_1 \) are constants.
(b) A boundary-value problem consists of finding a solution \( y \) of a second-order differential equation that also satisfies given boundary conditions \( y(x_0) = y_0 \) and \( y(x_1) = y_1 \).

3. (a) \( ay'' + by' + cy = G(x) \) where \( a, b, \) and \( c \) are constants and \( G \) is a continuous function.
(b) The complementary equation is the related homogeneous equation \( ay'' + by' + cy = 0 \). If we find the general solution \( y_c \) of the complementary equation and \( y_p \) is any particular solution of the original differential equation, then the general solution of the original differential equation is \( y(x) = y_p(x) + y_c(x) \).
(c) See Examples 1—5 and the associated discussion in Section 18.2 [ET 17.2].
(d) See the discussion on pages 1188—1190 [ET 1152—1154].

4. Second-order linear differential equations can be used to describe the motion of a vibrating spring or to analyze an electric circuit; see the discussion in Section 18.3 [ET 17.3].

5. See Example 1 and the preceding discussion in Section 18.4 [ET 17.4].