1. False.
See Note 2 after Theorem 12.2.6.

2. False.
The series \( \sum_{n=1}^{\infty} n^{-\sin 1} = \sum_{n=1}^{\infty} \frac{1}{n^{\sin 1}} \) is a \( p \)-series with \( p=\sin 1 \approx 0.84 \leq 1 \), so the series diverges.

3. True.
If \( \lim_{n \to \infty} a_n = L \), then given any \( \varepsilon > 0 \), we can find a positive integer \( N \) such that \( |a_n - L| < \varepsilon \) whenever \( n > N \). If \( n > N \), then \( 2n+1 > N \) and \( |a_{2n+1} - L| < \varepsilon \). Thus, \( \lim_{n \to \infty} a_{2n+1} = L \).

4. True by Theorem 12.8.3.

Or: Use the Comparison Test to show that \( \sum c_n (-2)^n \) converges absolutely.

5. False.
For example, take \( c_n = (-1)^n / \left( n6^n \right) \).

6. True by Theorem 12.8.3.

7. False, since
\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{1}{(n+1)^3} \cdot \frac{n^3}{1} \right| = \lim_{n \to \infty} \left| \frac{n^3}{(n+1)^3} \cdot \frac{1/n^3}{1/n^3} \right| = \lim_{n \to \infty} \frac{1}{(1+1/n)^3} = 1.
\]

8. True, since \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right| = \lim_{n \to \infty} \frac{1}{n+1} = 0 < 1 \).

See the note after Example 2 in Section 12.4.

10. True, since \( \frac{1}{e} = e^{-1} \) and \( e = \sum_{n=0}^{\infty} \frac{x^n}{n!} \), so \( e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \).

11. True.
See (8) in Section 12.1.

12. True, because if \( \sum |a_n| \) is convergent, then so is \( \sum a_n \) by Theorem 12.6.3.
13. True.

By Theorem .10.5 the coefficient of $x^3$ is $f'''(0) = \frac{1}{3} \Rightarrow f'''(0)=2$.

Or: Use Theorem 12.9.2 to differentiate $f$ three times.

14. False.

Let $a_n=n$ and $b_{-n}$. Then $\{a_n\}$ and $\{b_n\}$ are divergent, but $a_n+b_{-n}=0$, so $\{a_n+b_{-n}\}$ is convergent.

15. False.

For example, let $a_n=b_n=(-1)^n$. Then $\{a_n\}$ and $\{b_n\}$ are divergent, but $a_n b_{-n}=1$, so $\{a_n b_{-n}\}$ is convergent.

16. True by the Monotonic Sequence Theorem, since $\{a_n\}$ is decreasing and $0 \leq a_n \leq a_1$ for all $n \Rightarrow \{a_n\}$ is bounded.

17. True by Theorem 12.6.3. $\sum (-1)^n a_n$ is absolutely convergent and hence convergent.

18. True.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} < 1 \Rightarrow \sum a_n \text{ converges (Ratio Test)} \Rightarrow \lim_{n \to \infty} a_n =0 \text{ [Theorem 12.2.6]}.$$