QUANTUM GROUPS

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Abstract. Lectures on quantum groups. Math 261B.

1. Lie bialgebras and Poisson Lie groups

1.1. **Elements of symplectic and Poisson geometry.** Symplectic manifolds (real, complex holomorphic, algebraic). Poisson manifolds (real, complex holomorphic, algebraic). Operations on Poisson manifolds. Symplectic leaves. Examples: symplectic $\mathbb{R}^{2n}$, symplectic T^*M, Poisson $\mathfrak{g}^*$, symplectic: coadjoint orbits in $\mathfrak{g}^*$.

1.2. **Lie bialgebras.** The definition of Poisson Lie groups and Lie bialgebras. tangent Lie bialgebra to a Poisson Lie group. Duality for Lie bialgebras and Poisson Lie groups. Examples: Standard Poisson Lie structures on SL_2 and SL_n. Chevalley complex for Lie algebras and for Lie bialgebras. Factorizable Lie bialgebras and Poisson Lie group. The double of a Lie bialgebra and of a Poisson Lie group.

1.3. **Poisson Lie groups.** Crush course on symplectic and Poisson geometry. The definition of Poisson Lie groups. Tangent Lie bialgebra to a Poisson Lie group . Dual pairs of Poisson Lie groups. Standard Poisson Lie structure on SL_2. Standard Poisson Lie structures on SL_n and on simple Lie groups. Examples: Lie bialgebra structures on Borel subalgebras of simple Lie algebras and their doubles (SL_2, SL_n and general simple Lie algebras).

1.4. **Symplectic leaves of Poisson Lie groups.** The description in terms of orbits. The explicit description for $SU(2)$, $SU(n)$ and their complex algebraic counterparts.

1.5. **Poisson Hopf algebras and their duals.** Functions on Poisson Lie groups and Poisson Hopf algebras. Duality with the universal enveloping algebra of the tangent Lie bialgebra.

2. Quantum groups as deformation quantization of functions on Poisson-Lie groups

2.1. **Deformations of associative algebras.**

2.2. **Deformations of commutative algebras and Poisson algebras.**

2.3. **Deformations of Hopf algebras.**

2.4. **Deformations of Hopf Poisson algebras.**

3. Quantized universal enveloping algebras

3.1. **Deformations of universal enveloping algebras on Lie bialgebras.**

3.2. **The duality.I.** In this section we will see that quantized universal enveloping algebras on Lie bialgebras and quantized algebras of functions on Poisson Lie groups are in natural duality.

3.3. **The duality.II.** In this section we will see that in certain sense a quantized universal enveloping algebra of a Lie bialgebra can also be regarded as a quantized algebra of functions on the dual Poisson Lie group.

4. Categories of modules over quantized universal enveloping algebras

Hopf algebras and basic constructions with Hopf algebras. Monoidal categories. Category of vector spaces. Category of modules over a Hopf algebra. Duality in a monoidal category (dual vector spaces for the category of vector spaces).

5. Braid group and quantum groups

Braiding in monoidal categories. The Drinfeld double construction. The double construction of quantized universal enveloping algebras for simple Lie algebras. Corresponding representations of the braid group.

6. Integral forms and the specialization at roots of unity

Integral forms of the quantized universal enveloping algebra of sl_2. Their specializations at roots of unity.

7. The category of finite modules over $U_q(g)$ as a braided monoidal category

Elements of the representation theory of quantized universal enveloping algebras. Finite dimensional irreducible representations. The category of finite dimensional modules as a braided monoidal category.

8. Elements of harmonic analysis on quantum groups.

If time permits.

9. Literature

There are several books on the subject. They all can be used as a supplementary reading material.

References

[1] V. Chari, A. Pressley. , A guide to quantum groups, Cambridge University Press, 1994.
[2] P. Etingof and O. Schiffman. Lectures on Quantum Groups. International Press, 1998.
[3] C. Kassel. Quantum Groups (Springer: 1994). ISBN 0-387-94370-6
[4] L. Korogodsky, Y. Soibelman, Alegbras of functions on quantum groups. I, Amer. Math. Soc., 1997.