

Math 1A Final 2009-12-18 12:30-3:30pm

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the cover-sheet, and attach the cover-sheet to your bluebook.

1. Evaluate the limit $\lim_{x \rightarrow -4} \frac{\sqrt{x^2+9}-5}{x+4}$.
2. Differentiate $e^x / \sin(x)$.
3. Find the derivative of the function $y = \ln(\ln(\ln(x)))$.
4. Find dy/dx if $x \sin(y^2) = y \sin(x^2)$.
5. A bacteria culture initially contains 1000 cells, and grows at a rate proportional to its size. After 1 hour the population is 1100 cells. Find an expression for the approximate number of cells after t hours.
6. Find $\lim_{x \rightarrow \infty} x e^{1/x} - x$.
7. Sketch the graph of $y = \sin(x^2)$.
8. Explain why Newton's method does not converge to the root -1 of $2x^3 - x^2 + 3 = 0$ if the initial approximation is chosen to be $x_1 = 1$.
9. Find the most general anti-derivative of $(2 + x^2)/(1 + x^2)$.
10. A stone is dropped off a cliff and hits the ground after 2 seconds. What is the height of the cliff, assuming that acceleration due to gravity is $10ms^{-2}$.
11. Use the midpoint rule for 3 points to approximate the integral $\int_0^6 x^2 dx$.
12. Evaluate the integral $\int_{-3}^3 \sqrt{9 - x^2} dx$ by interpreting it as an area.
13. Find the derivative of the function $g(x) = \int_x^1 e^{-t^2} dt$.
14. The Fresnel integral is given by $S(x) = \int_0^x \sin(\pi t^2/2) dt$. Sketch its graph, and find the values of x at which it has local maximum values.
15. Evaluate the integral $\int_0^{3\pi/2} |\sin(x)| dx$.
16. Find the area of the finite region bounded by the lines $x = 0$, $y = 1$, $y = x^{1/4}$.
17. Prove that if n is a positive integer then $1 + 1/2 + 1/3 + \dots + 1/n > \ln(n)$.
18. Evaluate the indefinite integral $\int \frac{\cos(\ln(x))}{x} dx$.
19. Evaluate the definite integral $\int_0^2 (x - 1)^{10} dx$.
20. Evaluate the indefinite integral $\int \tan(x) dx$.
21. Find the volume of the region obtained by rotating the region bounded by the curves $y = x^3$, $y = 0$, $x = 1$, about the x -axis.