

Math 1A Final 2006-12-13 12:30-3:30pm

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the coversheet, and attach the coversheet to your bluebook.

1. Evaluate the limit $\lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$.
2. Differentiate $x/(1+x^2)$.
3. Find the derivative of the function $y = \sin(\cos(\sqrt{x}))$.
4. Find dy/dx if $x^3 + x^2y + y^2 = 6$.
5. Find the derivative $D^{57}e^{3x}$. (D means d/dx)
6. Find $\lim_{x \rightarrow 0^+} x^{x^2}$.
7. Find a positive number x such that $x + 1/x$ is as small as possible.
8. Use one iteration of Newton's method applied to the initial approximation $x_1 = 5$ to estimate $\sqrt{26}$.
9. Find the most general antiderivative of $\sin(\theta)/\cos^2(\theta)$.
10. Find f given that $f''(x) = 1/x^2$, $f(1) = 1$, $f(2) = 0$.
11. If $\int_1^5 f(x)dx = 1$ and $\int_1^5 g(x)dx = 2$ find $\int_1^5 2f(x) - 3g(x)dx$.
12. Evaluate the integral $\int_{-1}^2 |x|dx$ by interpreting it as an area.
13. Find the derivative of the function $g(x) = \int_1^x t^2 \ln(t)dt$.
14. Find the derivative of $y = \int_{\cos(x)}^x \cos(t^2)dt$.
15. Evaluate the integral $\int_1^{64} \frac{1+x^{1/3}}{\sqrt{x}} dx$.
16. Evaluate the integral $\int_0^{\pi/4} \frac{1+\cos^2(\theta)}{\cos^2(\theta)} d\theta$.
17. Evaluate the indefinite integral $\int y^3 \sqrt{2y^4 - 1} dy$.
18. Evaluate the indefinite integral $\int \tan(x) \ln(\cos(x)) dx$.
19. Evaluate the definite integral $\int_1^e \frac{\ln(x)^3}{x} dx$.
20. Evaluate the indefinite integral $\int \frac{\cos(x)}{\sqrt{1+\sin(x)}} dx$.
21. Find the area enclosed by the curves $y = 1/x$, $y = 1/x^2$, $x = 3$.
22. Find the volume of the region obtained by rotating the region bounded by the curves $y = 1/x$, $y = 0$, $x = 1$, $x = 3$, about the x -axis.
23. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = x^4$, $y = 0$, $x = 1$ about the y -axis.
24. Find the average value of $\cos(x) \sin(x)^4$ on $[0, \pi]$.