

## Math 1A Final 2004-12-15 12:30-3:30

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 1 mark, which will be given only for a clear correct answer and correct working. There is no partial credit for wrong answers.

1. Find a formula for the inverse of the function  $y = 3x^3 + 2$ .
2. Evaluate the limit  $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$ .
3. Prove that  $e^x = 2 + x$  has at least one real root.
4. Differentiate  $f(x) = (ax + b)/(cx + d)$ .
5. Find the derivative of the function  $y = \tan^3(2x)$ .
6. Find  $dy/dx$  if  $4 \cos(x) \sin(y) = 1$ .
7. Find the critical numbers of the function  $f(x) = xe^{2x}$ .
8. Show that the equation  $x^4 + 4x + c = 0$  has at most two real roots.
9. Find  $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$ .
10. Find the points on the ellipse  $4x^2 + y^2 = 4$  that are farthest away from the point  $(1, 0)$ .
11. Explain why Newton's method does not work for finding the root of the equation  $x^3 - 3x + 6 = 0$  if the initial approximation is chosen to be  $x_1 = 1$ .
12. Use one iteration of Newton's method applied to the initial approximation  $x_1 = 2$  to find  $36^{1/5}$  correct to two decimal places.
13. Find a function  $f$  such that  $f'(x) = x^3$  and the line  $x + y = 0$  is tangent to the graph of  $f$ .
14. Find  $f$  given that  $f''(x) = x^{-2}$ ,  $x > 0$ ,  $f(1) = 0$ ,  $f(2) = 0$ .
15. Estimate the area under the graph of  $f(x) = 1/x$  from  $x = 1$  to  $x = 4$  using three rectangles and right endpoints. Sketch the graph and rectangles.
16. If  $\int_1^5 f(x)dx = 12$  and  $\int_2^5 f(x)dx = 14$  find  $\int_1^2 f(x)dx$ .
17. Evaluate the integral  $\int_{-2}^2 \sqrt{4 - x^2} dx$  by interpreting it as an area.
18. Prove that  $2 \leq \int_{-1}^1 \sqrt{1 + x^2} dx \leq 2\sqrt{2}$ .
19. Find the derivative of the function  $g(x) = \int_1^x \ln(t) dt$ .
20. Find the derivative of  $y = \int_{\sin(x)}^x \cos(t) dt$ .
21. Evaluate the integral  $\int_{-1}^0 (2x - e^x) dx$ .
22. Evaluate the integral  $\int_0^{\pi/4} \frac{1 + \cos^2(\theta)}{\cos^2(\theta)} d\theta$ .
23. Evaluate the indefinite integral  $\int y^3 \sqrt{2y^4 - 1} dy$ .
24. Evaluate the indefinite integral  $\int \tan(x) dx$ .
25. Evaluate the definite integral  $\int_0^4 (x - 2)^7 dx$ .
26. By comparing areas, show that  $1 + 1/2 + 1/3 + \dots + 1/(n - 1) > \ln(n)$  if  $n \geq 2$ .
27. Find the area enclosed by the curves  $4x + y^2 = 12$ ,  $x = y$ .
28. Find the volume of the region obtained by rotating the region bounded by the curves  $y = e^x$ ,  $y = 0$ ,  $x = 0$ ,  $x = 1$ , about the  $x$ -axis.
29. Use the method of cylindrical shells to find the volume of a sphere of radius  $r$ .
30. Find the average value of  $\sqrt{x}$  on  $[0, 4]$ .