5.1 $dx/dt = \sinh(t)$, $dy/dt = -\sin(t)$, $dz/dt = \partial(z/\partial x)dx/dt + \partial(z/\partial y)dy/dt = e^{-y}\sinh(t) + (-x e^{-y})/(-\sin(t))$

5.4 $dx/dt = (2u/(u^2 - v^2)) \times 2t + (-2v/(u^2 - v^2)) \times (-\sin(t))$.

6.1 Differential with respect to $p$: $v^a \partial v^{a-1}(dv/dp) = 0$. So $dv/dp = -v/ap$. Differentiating $v + ap(dv/dp) = 0$ again with respect to $p$ gives $dv/dp + adv/dp + ap(d^2 v/dp^2) = 0$. So $d^2 v/dp^2 = -(1 + a)(-v/ap)/ap$.

6.2 $y e^{x+y} + y^2 e^{x+y} + xy e^{x+y} = \cos(x)$, so $y' = 1$ at $x = y = 0$. This gives $e^{x+y}(y' + y^2 + yxy') = \cos(x)$, and differentiating this gives $(y + xy')(e^{x+y}(y' + y^2 + yxy') + e^{x+y}(y'' + 2yy' + y'y + xy'^2 + yxy'')) = -\sin(x)$

For $x = y = 0, y' = 1$ this gives $y'' = 0$.

6.3 Taking differential we have $xy y^{-1} dx + \log(x)y dx = x y^{-1} dy + \log(y)y dx$, which for $x = 2, y = 4$ gives $32 dx + 16 \log(2) dy = 8dy + 16 \log(4) dx$ so $dy/dx = (16 \log(2) - 8)/32 \log(2) - 1$.

6.5 $y^3 dx + 3x y^2 dy - x^3 dy - 3x^2 y dx = 0$, so at $(1, 2)$ we have $dy/dx = -2/11$. So the equation is $2x + 11y = 24$.

6.6 We have $y^3 + 3x y^2 - x^3 y^2 - 3x^2 y = 0$. Differentiating again gives $3y^2 + 3y y'/y' - 6y y'/y' + 2x y y'' - 3x^2 y' - x^3 y'' - 6x y^2 y' = 0$. For $x = 1, y = 2, y' = 4$ this gives $y'' = 1800/113$.

7.1 $dx + y dy + zd y = \cos(y+z)(dy + dz)$. Eliminating $dz$ gives $dy = \cos(y+z) dy + (dx - zd y)/y)$. So $dx/dy = 1/2(\cos(y+z) - y + z) = \tan(y+z) - y + z$. Differentiating again gives $d^2 x/dy^2 = \sec^2(y+z)(1 + dz/dy) - 1 + (dz/dy)$. Since $dz/dy = 1/2\cos(y+z) - 1$ this gives $d^2 x/dy^2 = \sec^2(y+z)/(2 + \sec(y+z))/2 - 2$.

7.2 $dP/dt = \cos(t)dr/dt - r\sin(t)$ from the first equation, and the second gives $dr \sin(t) + r \cos(t)dt - 2e^{-r}dt = 0$ so $dr/dt = (2e^{-r} - r \cos(t))/\sin(t) - 2e^{t}$ and hence $dP/dt = \cos(t)(2e^{-r} - r \cos(t))/\sin(t) - 2e^{t} - r \sin(t)$.

7.4 $dw = (-2dr - 2ds) w$, $dr = u dv + v du$, $ds = du + 2dv$, so $dw = (-2r(udv + vdu) - 2s(du + 2dv)) w$. So $\partial w/\partial u = (-2r - 2s) w$ and $\partial w/\partial v = (-2r + 4s) w$.

7.5 $du = 2x y^2 z dx + 3x^2 y^2 dz + x^2 y^3 dz, dx = \cos(s+t)(ds + dt), dy = -\sin(s+t)(ds + dt), dz = (sdt + tds)z$.

So $du = 2x y^2 z \cos(s+t)(ds + dt) + 3x^2 y^2 z (-\sin(s+t)(ds + dt)) + x^2 y^3 (sdt + tds)z$, which implies $\partial u/\partial s = 2x y^2 z \cos(s+t) + 3x^2 y^2 z (-\sin(s+t)) + x^2 y^3 z (\partial u/\partial t = 2xy y z \cos(s+t) + 3x^2 y^2 z (-\sin(s+t)) + x^2 y^3 z$.

7.8 $s^2 dx + 2xs ds + t^2 dy + 2tdt = 0$ and $2x dx + x^2 ds + 2ydy + 2dy = x dy - y dx$. For $(x, y, s, t) = (1, -3, 2, -1)$ this gives $4dx + 4ds + dy + 6dt = 0$ and $4dx + ds + 6dy + 9dt = dy - 3dx$, so $7dx + ds + 5dy + 9dt = 0$. Eliminating $dy$ gives $-13dx - 19ds - 21dt = 0$ so $dx/ds = -19/13, dx/dt = -21/13$. Eliminating $dx$ gives $24ds - 13dy + 6dt = 0$ so $dy/ds = 24/13, dy/dt = 6/13$.

7.10 $2x dx + 2y dy = 2ds + 2ds$ and $2x dx + 2y dy = 2ds - 2tdt$. For $(x, y, s, t) = (4.2, 5, 3, 3)$ this gives $8dx + 4dy = 10dt + 6ds$ and $8dy + 4dx = 10ds - 6dt$. Eliminating $dx$ gives $-12dx = -2ds - 26dt$ so $dx/ds = 2/12 = 1/6, dx/dt = 26/12 = 13/6$. Eliminating $dx$ gives $-12dy = 22dt - 14ds$ so $dy/ds = 14/12 = 7/6, dy/dt = 22/12 = 11/6$.

7.15 $2x dx + x^2 du - 2y dy = 0$ and $dx + dy = u dv + vdu$. If we put $dv = 0$ and eliminate $dy$ we find $2x du + x^2 du = -2y v du = 0$ so $dx/\partial u = (-x^2 + 2y^2 v)/(2x u + 2y v)$. If we put $dy = 0$ and eliminate $dv$ we find $2x du = x^2 du - y^2 (dx - du)/u = 0$, so that $(dx/\partial u)_y = -(x^2 + 2y^2 v/u)/(2x u - y^2/u) = (x^2 u + y^2 v)/(2x u - x^2 u)$

7.19 $dz = dr + 2ds, dx + dy = 3x^2 ds + 3r^2 dr, x dy + y dx = 2ds - 2dr$, so we have $dz = dr + 4ds, dz + dy = 12ds + 3dr, dy + dx = 4ds + 2dr$ at $(r, s, x, y, z) = (-1, 2, 3, 1, 3)$. For $(dx/\partial dz)_y$ we put $ds = 0$ and eliminate $dr, dy$ from the equations using $dz = dr, dz = dy$ to find that $9dz - 3dx + dx = 2dz$, so $(dx/\partial dz) = 7/2$. For $(dx/\partial dz)_y$ we put $dr = 0$ and eliminate $ds, dy$ from the equations using $ds = dz/4, dy = 12ds - dx$ to find that $9dz - 3dx + dx = dz$, so $(dx/\partial dz)_y = 4$. For $(dx/\partial dz)_y$ we put $dy = 0$ and eliminate $dr$, $dz$ from the equations to find $dx = dz, 9dz - 3dx + dx = dz$, so $(dx/\partial dz)_y = 3$. 

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