1.2 Done in class.
1.5 \[ \frac{583333}{1000} = 58 + \frac{333}{1000} + 3 \times (1 + 1/10 + 1/100 + \ldots) = \frac{7}{12}. \] (Or note that \(0.583333 \times 3 = 1.7499999\ldots = \frac{7}{4}\).)
2.6 \[ 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{20} + \frac{1}{70} + \ldots \]
2.7 \[ \sum_{n=1}^{\infty} \frac{2n-1}{(2n+1)}. \]
4.1 Remainder \( |S - S_n| \leq \frac{1}{2^n} \) (in fact it is equal to \(1/2^n\)). So for example, choose \( \epsilon = 1/1000000 \). Then we can take \( N = 30 \) (say). (This is not the best possible value of \( N \).)
4.4 For \( n \geq 3 \), remainder is smaller than for the geometric series in question 1 by the hint, so we can use the same \( \epsilon, N \) as in 4.1.
4.7 Remainder \( S - S_n \) is less than \( 1/(n - 1) \) by integral test, so if we put (say) \( \epsilon = 1/1000000 \) we can take \( N = 1000002 \).
5.1 Divergent, as limit is not 0.
5.8 Limit of terms is 0, so preliminary test says nothing. (In fact series is divergent.)
6.1 \[ n! \geq 24 \times 5 \times 6 \times \cdots \times n > 16 \times 2 \times 2 \cdots \times 2 = 2^n \text{ if } n \geq 4. \]
6.2 \[ 1+(1/2)+(1/3+1/4)+(1/5+1/6+1/7+1/8)+\cdots \geq 1+(1/2)+(1/4+1/4)+(1/8+1/8+1/8+1/8)+\cdots = 1 + (1/2) + (1/2) + \cdots = +\infty. \]
6.3 \[ 1+(1/2^2+1/3^2)+(1/4^2+1/5^2+1/6^2+1/7^2)+\cdots \leq 1+(1/2^2+1/2^2)+(1/4^2+1/4^2+1/4^2+1/4^2)+\cdots = 1 + (2/2^2) + (4/4^2) + (8/8^2) + \cdots = 1 + 1/2 + 1/4 + 1/8 + \cdots = 2. \]
6.5 (a) Diverges; the terms are larger than those of the divergent series \( 1 + 1/2 + 1/3 + \ldots \). (b) Diverges; the terms are larger than those of the divergent series \( 1 + 1/2 + 1/3 + \ldots \).
6.7 Diverges; integral is \( \log \log(n) \) which tends to infinity with \( n \).
6.8 Diverges; integral is \( \log(n^2 + 4)/2 \) (substitute \( y = n^2 + 4 \)).
6.11 Converges; integral is \( -2(1 + \log(n))^{-1/2} \) which is bounded as \( n \) tends to infinity.
6.12 Converges; integral is \( -1/2(n^2 + 1) \).
6.15 Done in class; integral is \( \log(n) \) if \( p = 1 \), \( n^{1-p}/(1 - p) \) if \( p \neq 1 \).
6.16 The integral should have lower limit 1, not 0. The integral diverges because it is bad at 0, which has nothing to do with the convergence of the sum.
6.17 \[ \int_{1}^{\infty} e^{-n^2} \, dn \leq \int_{1}^{\infty} e^{-n} \, dn \] because \( n^2 \geq n \) for \( n \geq 1 \), and the latter integral converges (and has value \( e^{-1} \)). (By the way, the hint is slightly wrong: the integral \( \int_{0}^{\infty} e^{-n^2} \, dn \) can be evaluated explicitly, and it has value \( \sqrt{\pi}/2 \). However this is a very hard integral to do, and it is much easier just to bound it from above which is all that is needed.)