This will be the last homework that I type out. If you have still not managed to get a copy of the textbook, you can order one from amazon.com; they have them in stock and will ship them 24 hours after getting your order.

3 Prove that a number is divisible by 2 only if its last digit (in decimal expansion) is divisible by 2. Prove that a number is divisible by 4 if and only if the number formed by its last 2 digits is divisible by 4. Prove that a number is divisible by 8 if and only if the number formed by its last 3 digits is divisible by 8.

7 Show that every positive integer \( n \) can be written uniquely as \( n = ab \) where \( a \) is square-free (not divisible by any square greater than 1) and \( b \) is a square. Show that \( b \) is then the largest square dividing \( n \).

10 Prove that any positive integer of the form \( 3k + 2 \) has a prime factor of the same form; similarly for each of the forms \( 4k + 3 \) and \( 6k + 5 \).

16 Find a positive integer \( n \) such that \( n/2 \) is a square, \( n/3 \) is a cube, and \( n/5 \) is a fifth power.

24 Prove that if \( n \) is composite it must have a prime factor \( p \leq \sqrt{n} \).

25 Find all primes less than 200 using the sieve of Eratosthenes (as in the lecture on Tuesday).

26 Prove that there are infinitely many primes of the form \( 4n + 3 \), and of the form \( 6n + 5 \).

1.4.1 Use the binomial theorem to show that

\[
\sum_{k=0}^{n} \binom{n}{k} = 2^n.
\]

Give a combinatorial proof of this.

1.4.2 Show that if \( n \geq 1 \) then

\[
\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0.
\]

1.4.3a Show that

\[
\sum_{i=0}^{k} \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}.
\]