This will be that last homework that I type out. If you have still not managed to get a copy of the textbook, you can order one from amazon.com; they have them in stock and will ship them 24 hours after getting your order.

3 Prove that a number is divisible by 2 only if its last digit (in decimal expansion) is divisible by 2. Prove that a number is divisible by 4 if and only if the number formed by its last 2 digits is divisible by 4. Prove that a number is divisible by 8 if and only if the number formed by its last 3 digits is divisible by 8.

7 Show that every positive integer n can be written uniquely as $n = ab$ where a is square-free (not divisible by any square greater than 1) and b is a square. Show that b is then the largest square dividing n.

10 Prove that any positive integer of the form $3k + 2$ has a prime factor of the same form; similarly for each of the forms $4k + 3$ and $6k + 5$.

16 Find a positive integer n such that $n/2$ is a square, $n/3$ is a cube, and $n/5$ is a fifth power.

24 Prove that if n is composite it must have a prime factor $p \leq \sqrt{n}$.

25 Find all primes less than 200 using the sieve of Eratosthenes (as in the lecture on Tuesday).

26 Prove that there are infinitely many primes of the form $4n + 3$, and of the form $6n + 5$.

1.4.1 Use the binomial theorem to show that

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

Give a combinatorial proof of this.

1.4.2 Show that if $n \geq 1$ then

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0.$$

1.4.3a Show that

$$\sum_{i=0}^{k} \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}.$$