**Problem 1.** You may use a calculator for these steps. This is to get an intuitive understanding of exponents and logarithms.

1) Graph $e^x$ and $\ln(x)$. What is the domain/range of these functions?

2) Explain in words and convince yourself of the following facts:
   For any $x, y \in \mathbb{R}$, we have $e^{x+y} = e^x e^y$ and $e^{xy} = (e^x)^y = (e^y)^x$

3) Calculate $c = \ln(2)$. Now, take the number $c$ and calculate $e^c$. In words, can you explain what the function $\ln(x)$ returns when you feed in the input 2?

4) Now, explain in words and convince yourself of the following facts:
   (a) $\ln(4) = \ln(2) + \ln(2)$
   (b) $\ln(6) = \ln(2) + \ln(3)$
   (c) $\ln(2/3) = \ln(2) - \ln(3)$
   (d) $\ln(2^x) = x \ln(2)$ for any $x$

5) Finally, convince yourself that you can change bases via:
   \[
   \log_b a = \frac{\log_c a}{\log_c b} \quad \text{for any new base } c \text{ and old base } b
   \]

(6) Graph $2^x$ and $\log_2(x)$. Compare the graphs of (6) and (1). How are they related? (in terms of horizontal/vertical stretches)

**Problem 2.** Solve for $x$:

(a) $\ln(\ln(x)) = 1$

**Solution:** $\ln(\ln(x)) = 1 \Rightarrow \ln(x) = e \Rightarrow x = e^e$

(b) $\ln(x) + \ln(x + 1) = 2$

**Solution:** Using the product rule:
$\Rightarrow \ln(x(x + 1)) = 2$
\[ x(x + 1) = e^2 \]
\[ x^2 + x - e^2 = 0 \]
\[ x = \frac{-1 \pm \sqrt{1 + 4e^2}}{2} \]

We see that the negative solution is discarded, so \( x = \frac{-1 + \sqrt{1 + 4e^2}}{2} \).

(c) \( e^{2x} - 3e^x + 2 = 0 \)

**Solution:** Let \( u = e^x \), then
\[ u^2 - 3u + 2 = 0 \Rightarrow (u - 1)(u - 2) = 0 \Rightarrow u = 1, 2 \]
\[ \Rightarrow x = \ln(1), \ln(2) = 0, \ln(2) \]

(d) \( \ln(x^2) + \log_2(x^2) = 3 \)

**Solution:** By change of bases,
\[ \ln(x^2) + \frac{\ln(x^2)}{\ln(2)} = 3 \]
\[ \Rightarrow \ln(x^2)(1 + \frac{1}{\ln(2)}) = 3 \]
\[ \Rightarrow \ln(x^2)(\frac{\ln(2) + 1}{\ln(2)}) = 3 \]
\[ \Rightarrow \ln(x^2)(\frac{\ln(2e)}{\ln(2)}) = 3 \]
\[ \Rightarrow \ln(x^2) = \frac{3\ln(2)}{\ln(2e)} \]
\[ \Rightarrow x^2 = e^{\frac{3\ln(2)}{\ln(2e)}} \]
\[ \Rightarrow x = \pm e^{\frac{3\ln(2)}{\ln(2e)}} \]

Note to be careful when using \( \ln(x^2) = 2 \ln x \).

(e) \( (\log_2 x)(\log_x x^2) = \log_{x^3} x^2 \)

**Solution:** By change of bases,
\[
\frac{\ln x \ln x^2}{\ln 2 \ln x} = \frac{\ln x^2}{\ln x^3}
\]
\[
\Rightarrow \frac{1}{\ln 2} = \frac{1}{\ln x^3}
\]
\[
\Rightarrow \ln 2 = \ln x^3
\]
\[
\Rightarrow 2 = x^3 \Rightarrow x = 2^{1/3}
\]

**Problem 3.** Compute the following limits (via squeeze):

(a) \(\lim_{x \to \infty} \frac{\cos(\tan(1/x))}{\ln(\ln(x))}\)

**Solution:** Since \(\cos x \in [-1, 1]\),
\[
-\frac{1}{\ln(\ln(x))} \leq \frac{\cos(\tan(1/x))}{\ln(\ln(x))} \leq \frac{1}{\ln(\ln(x))}
\]
\[
\lim_{x \to \infty} -\frac{1}{\ln(\ln(x))} \leq \lim_{x \to \infty} \frac{\cos(\tan(1/x))}{\ln(\ln(x))} \leq \lim_{x \to \infty} \frac{1}{\ln(\ln(x))}
\]
Since \(\lim_{x \to \infty} -\frac{1}{\ln(\ln(x))} = 0 = \lim_{x \to \infty} \frac{1}{\ln(\ln(x))}\), we use the Squeeze Theorem to conclude that
\[
\lim_{x \to \infty} \frac{\cos(\tan(1/x))}{\ln(\ln(x))} = 0
\]

(b) \(\lim_{x \to 0} x^2 \sin(1/x)\)

**Solution:** Since \(\sin x \in [-1, 1]\),
\[-x^2 \leq x^2 \sin(1/x) \leq x^2 \]
\[
\Rightarrow \lim_{x \to 0} -x^2 \leq \lim_{x \to 0} x^2 \sin(1/x) \leq \lim_{x \to 0} x^2
\]
Since \(\lim_{x \to 0} -x^2 = 0 = \lim_{x \to 0} x^2\), we use the Squeeze Theorem to conclude that
\[
\lim_{x \to 0} x^2 \sin(1/x) = 0
\]

(c) \(\lim_{x \to \infty} \frac{(2x + \sin(x))^2}{x^2}\)

**Solution:** Since \(\sin x \in [-1, 1]\), for \(x > 0\),
\[
\frac{(2x - 1)^2}{x^2} \leq \frac{(2x + \sin(x))^2}{x^2} \leq \frac{(2x + 1)^2}{x^2}
\]
\[
\lim_{x \to \infty} \frac{(2x - 1)^2}{x^2} \leq \lim_{x \to \infty} \frac{(2x + \sin(x))^2}{x^2} \leq \lim_{x \to \infty} \frac{(2x + 1)^2}{x^2}
\]

Since \( \lim_{x \to \infty} \frac{(2x - 1)^2}{x^2} = 4 = \lim_{x \to \infty} \frac{(2x + 1)^2}{x^2} \), we use the Squeeze Theorem to conclude that

\[
\lim_{x \to \infty} \frac{(2x + \sin(x))^2}{x^2} = 4
\]

(d) \[
\lim_{x \to \infty} \frac{\ln(x + 1) - \ln(x)}{2 + \sin(x)}
\]

**Solution:** Note that \( \ln(x + 1) - \ln(x) = \ln \left( \frac{x + 1}{x} \right) = \ln(1 + 1/x) \), so

\[
\lim_{x \to \infty} [\ln(x + 1) - \ln(x)] = \lim_{x \to \infty} \ln(1 + 1/x) = \ln(1) = 0
\]

So, since \( 2 + \sin(x) \in [1, 3] \), we see that

\[
\frac{\ln(x + 1) - \ln(x)}{3} \leq \frac{\ln(x + 1) - \ln(x)}{2 + \sin(x)} \leq \frac{\ln(x + 1) - \ln(x)}{1}
\]

Since \( \lim_{x \to \infty} \frac{\ln(x + 1) - \ln(x)}{3} = 0 = \lim_{x \to \infty} \frac{\ln(x + 1) - \ln(x)}{1} \), by the Squeeze Theorem,

\[
\lim_{x \to \infty} \frac{\ln(x + 1) - \ln(x)}{2 + \sin(x)} = 0
\]