Problem 1. You may use a calculator for these steps. This is to get an intuitive understanding of exponents and logarithms.

1) Graph \( e^x \) and \( \ln(x) \). What is the domain/range of these functions?

2) Explain in words and convince yourself of the following facts:
   For any \( x, y \in \mathbb{R} \), we have \( e^{x+y} = e^x e^y \) and \( e^{xy} = (e^x)^y = (e^y)^x \)

3) Calculate \( c = \ln(2) \). Now, take the number \( c \) and calculate \( e^c \). In words, can you explain what the function \( \ln(x) \) returns when you feed in the input \( 2 \)?

4) Now, explain in words and convince yourself of the following facts:
   (a) \( \ln(4) = \ln(2) + \ln(2) \)
   (b) \( \ln(6) = \ln(2) + \ln(3) \)
   (c) \( \ln(2/3) = \ln(2) - \ln(3) \)
   (d) \( \ln(2^x) = x \ln(2) \) for any \( x \)

5) Finally, convince yourself that you can change bases via:

   \[
   \log_b a = \frac{\log_c a}{\log_c b}
   \]
   for any new base \( c \) and old base \( b \)

6) Graph \( 2^x \) and \( \log_2(x) \). Compare the graphs of (6) and (1). How are they related? (in terms of horizontal/vertical stretches)

Problem 2. Solve for \( x \):

(a) \( \ln(\ln(x)) = 1 \)

(b) \( \ln(x) + \ln(x + 1) = 2 \)

(c) \( e^{2x} - 3e^x + 2 = 0 \)

(d) \( \ln(x^2) + \log_2(x^2) = 3 \)
(e) \((\log_2 x)(\log_x x^2) = \log_{x^3} x^2\)

**Problem 3.** Compute the following limits (via squeeze):

(a) \(\lim_{x \to \infty} \frac{\cos(\tan(1/x))}{\ln(\ln(x))}\)

(b) \(\lim_{x \to 0} x^2 \sin(1/x)\)

(c) \(\lim_{x \to \infty} \frac{(2x + \sin(x))^2}{x^2}\)

(d) \(\lim_{x \to \infty} \frac{\ln(x + 1) - \ln(x)}{2 + \sin x}\)