Problem 1. Find the range and zeros of:

(a) \( x^2 + 2x - 3 \)

**Solution:** Completing the square gives: \( x^2 + 2x - 3 = (x + 1)^2 - 4 \). The range is \([-4, \infty)\) and the roots are \((x + 1)^2 = 4 \Rightarrow x = 1, -3\).

(b) \( x^4 - 4x^2 + 1 \)

**Solution:** Let \( u = x^2 \) to get \( u^2 - 4u + 1 \). Completing the square gives: \( u^2 - 4u + 1 = (u - 2)^2 - 3 = (x^2 - 2)^2 - 3 \). The range is \([-3, \infty)\) and the roots are \((x^2 - 2)^2 = 3 \Rightarrow x^2 = 2 + \sqrt{3}, 2 - \sqrt{3} \Rightarrow x = \sqrt{2 + \sqrt{3}}, \sqrt{2 - \sqrt{3}}\).

(c) \( 2 \cos^2(x) + \sin(x) - 2 \)

**Solution:** Using \( \cos^2(x) = 1 - \sin^2(x) \), we get: \( 2 - 2\sin^2(x) + \sin(x) - 2 = -2\sin^2(x) + \sin(x) = -2u^2 + u = u(-2u + 1) \). So, the zeros are \( u = 0, u = 1/2 \). Therefore, \( \sin(x) = 0, 1/2 \Rightarrow x = 0, \pi/6 (+2\pi z \text{ for any integer } z) \)

Completing the square gives: \( -2(u^2 - \frac{1}{2}u) = -2((u - \frac{1}{4})^2 - \frac{1}{16}) = -2(u - \frac{1}{4})^2 + \frac{1}{8} \). Note that \( u = \sin(x) \) can only take values in \([-1, 1]\). So the minimal value is \( -2(-1-1/4)^2+1/8 = -25/8 + 1/8 = -3 \). Therefore, the range is \([-3, 1/8]\).

(d) \( x^2 + 2|x| + 2 \)

**Solution:** Note that \( u = |x| \) gives \( u^2 + 2u + 2 = (u + 1)^2 + 1 \). This has no zeros. Furthermore, note that \( u = |x| \geq 0 \), so \((u + 1)^2 \geq 1\). Therefore, the range is \([2, \infty)\).
Problem 2. Compute the following trigonometric values:

(a) $\sin(\pi/3 + \pi/4)$

Solution: By the addition formula, $\sin(\pi/3 + \pi/4) = \sin(\pi/3 \cos(\pi/4) + \sin(\pi/4) \cos(\pi/3)$

\[= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2} = \frac{\sqrt{2} + \sqrt{6}}{4} \]

(b) $\cos(\pi/4 - \pi/6)$

Solution: By the addition formula, $\cos(\pi/4 + (-\pi/6)) = \cos(\pi/4) \cos(-\pi/6) - \sin(\pi/4) \sin(-\pi/6)$

\[= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \left(-\frac{1}{2}\right) = \frac{\sqrt{2} + \sqrt{6}}{4} \]

(c) $\sin(\sin^{-1}(x) + \cos^{-1}(x))$

Solution: By the addition formula, $\sin(\sin^{-1}(x) + \cos^{-1}(x)) = \sin(\sin^{-1}(x)) \cos(\cos^{-1}(x)) + \sin(\cos^{-1}(x)) \cos(\sin^{-1}(x))$

Note that $\sin(\cos^{-1}(x)) = \sin \theta$, where $\theta = \cos^{-1}(x)$ is the angle of a triangle whose cosine is $x$. Therefore, the ratio of the adjacent to hypotenuse is $x$, so if the hypotenuse has length 1, then the adjacent edge has length $x$. By Pythagorean, the opposite edge has length $\sqrt{1-x^2}$. Therefore, $\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$.

Similarly, $\cos(\sin^{-1}(x)) = \sin \theta$, where $\theta = \sin^{-1}(x)$ is the angle of a triangle whose sine is $x$. The ratio of the opposite to hypotenuse is $x$, so going through the steps, we get $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$.

\[\sin(\sin^{-1}(x) + \cos^{-1}(x)) = \sin(\sin^{-1}(x)) \cos(\cos^{-1}(x)) + \sin(\cos^{-1}(x)) \cos(\sin^{-1}(x))\]

\[= (x)(x) + (\sqrt{1-x^2})(\sqrt{1-x^2}) = x^2 + 1 - x^2 = 1\]

(d) $\cos(\sin^{-1}(x) + \cos^{-1}(x))$

Solution: Let $\theta = \sin^{-1}(x) + \cos^{-1}(x)$ and we know from (c) that $\sin \theta = 1$. Therefore, $\cos^2 \theta = 1 - \sin^2 \theta = 0 \Rightarrow \cos \theta = 0$. 