“Do not be anxious about anything. Can all your worries add a single hour to your life?”

**Problem 1.** Circle True or False. (1pt each)

a. (True or False) If \( f(0) = 0 \) and \( f'(x) \geq 1 \) for all \( x \geq 0 \), then \( f(x) \geq x \) for all \( x \geq 0 \).

**Solution:** True, by the Mean Value Theorem, as \( f(x) = f(0) + f'(c)(x - 0) \geq x \).

b. (True or False) If \( f(0) = 0 \) and \( f(x) \geq x \) for all \( x \geq 0 \), then \( f'(x) \geq 1 \) for all \( x \geq 0 \).

**Solution:** False, consider \( f(x) = x + 2 + \cos(x) \geq x \). However, \( f'(x) = 1 + \sin(x) \) can be less than 1.

**Problem 2.** Evaluate the following limits: (2 pts each)

(a) \( \lim_{x \to 1} \frac{x^{3/5} - x}{x^2 - 1} \)

**Solution:** Using LHopital’s, we have
\[
\lim_{x \to 1} \frac{x^{3/5} - x}{x^2 - 1} = \lim_{x \to 1} \frac{3/5x^{-2/5} - 1}{2x} = \frac{3/5 - 1}{2} = -1/5
\]

(b) \( \lim_{x \to 1} \frac{x^{1/x} - x^2}{x - 1} \)

**Solution:** Using LHopital’s, we have
\[
\lim_{x \to 1} \frac{x^{1/x} - x^2}{x - 1} = \lim_{x \to 1} x^{1/x} \frac{1 - \ln(x)}{x^2} - 2x = 1 - 2 = -1
\]
Problem 3. Use the MVT to prove that $x^3 \geq 3x - 2$ for all $x \geq 1$.

Solution: Note that when $x = 1$, this inequality holds. Now, $f(x) = x^3$ is a polynomial and is cts/diffable everywhere, specifically on the interval $[1, x]$ for any $x > 1$. So, by MVT, there exists $c \in (1, x)$ such that

$$f(x) = f(1) + f'(c)(x - 1) = 1 + 3c^2(x - 1) \geq 1 + 3(x - 1) = 3x - 2,$$

as desired.