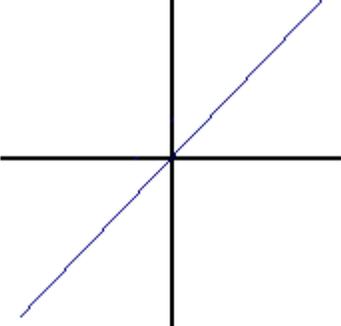
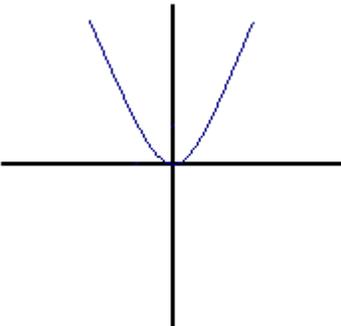
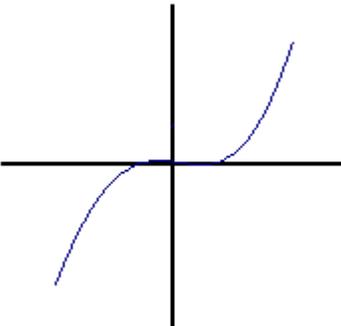
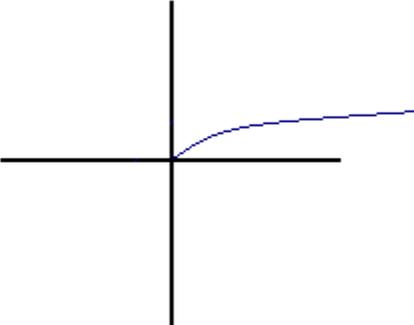
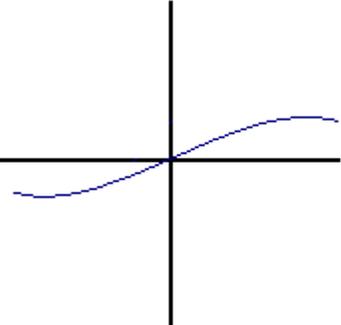
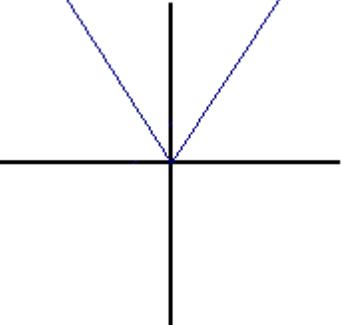


**Notes for EER #4 – Graph transformations (vertical & horizontal shifts, vertical stretching & compression, and reflections) of basic functions.**

**Basic Functions**

In several sections you will be applying shifts and transformations to these basic functions. So, it's very important that you put these graphs and their equations to memory.

<p><b>Identity Function</b> <math>f(x) = x</math></p> 	<p><b>Squaring Function</b> <math>f(x) = x^2</math></p> 	<p><b>Cubing Function</b> <math>f(x) = x^3</math></p> 
<p><b>Square Root Function</b> <math>f(x) = \sqrt{x}</math></p> 	<p><b>Cube Root Function</b> <math>f(x) = \sqrt[3]{x}</math></p> 	<p><b>Absolute Value Function</b> <math>f(x) =  x </math></p> 

**Vertical & Horizontal Shifts**

Above, you were presented with some Basic Functions. Next, you will apply vertical and horizontal shifts to those functions to create new ones. These shifts are called transformations.

\*Note-The identity function was one of the basic functions listed, but we won't be applying any shifts to that one. The graph is just a straight line, and we've learned more proficient ways to graph lines in chapter 1.

**Vertical Shifts**

(C is a real #)

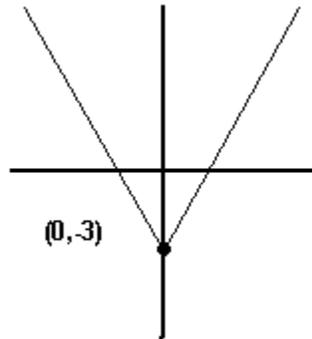
$f(x) + C$	Vertical shift <u>UP</u> C units
$f(x) - C$	Vertical shift <u>DOWN</u> C units

Let's apply some vertical shifts to a couple of basic functions.

Ex: Graph the following.

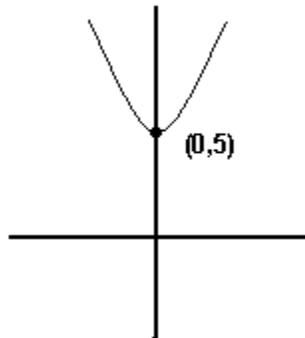
1.  $f(x) = |x| - 3$

This function comes from the basic function  $f(x) = |x|$  with the constant 3 subtracted on the outside. This gives the basic function a vertical shift **DOWN** 3 units. So, take the basic graph of  $f(x) = |x|$  and shift it **DOWN** 3 units. You will get the following graph:



1.  $f(x) = x^2 + 5$

This function comes from the basic function  $f(x) = x^2$  with the constant 5 added to the outside. This gives the basic function a vertical shift **UP** 5 units. So, take the basic graph of  $f(x) = x^2$  and shift **UP** 5 units. You will get the following graph:



Now let's take a look at the Horizontal Shifts.

### Horizontal Shifts

(C is a real #)

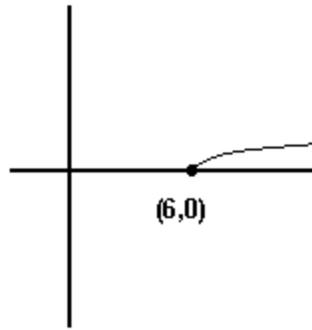
$f(x + C)$	Vertical shift <u>LEFT</u> C units
$f(x - C)$	Vertical shift <u>RIGHT</u> C units

Let's apply some horizontal shifts to some basic functions.

**Ex:** Graph the following.

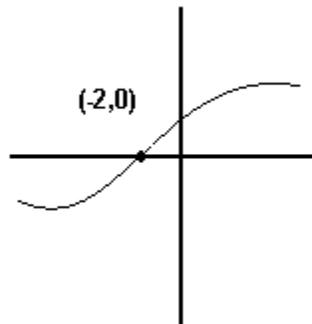
3.  $f(x) = \sqrt{x - 6}$

This function comes from the basic function  $f(x) = \sqrt{x}$  with the constant 6 subtracted on the inside. This gives the basic function a horizontal shift **RIGHT** 6 units. So, take the basic graph of  $f(x) = \sqrt{x}$  and shift **RIGHT** 6 units. You will get the following graph:



4.  $f(x) = \sqrt[3]{x+2}$

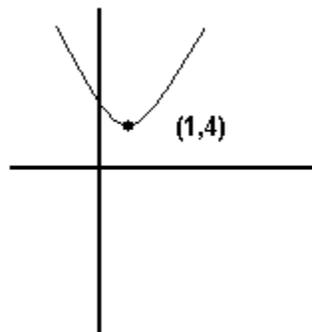
This function comes from the basic function  $f(x) = \sqrt[3]{x}$  with the constant 2 added on the inside. This gives the basic function a horizontal shift LEFT 2 units. So, take the basic graph of  $f(x) = \sqrt[3]{x}$  and shift LEFT 2 units. You will get the following graph:



You can also have combinations of vertical and horizontal shifts as well. Let's consider the following example.

5. Graph:  $f(x) = (x - 1)^2 + 4$

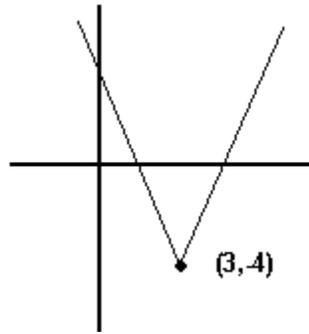
This function comes from the base function  $f(x) = x^2$ . Since 1 is subtracted on the inside, this is a horizontal shift RIGHT 1 unit, and since 4 is added on the outside, this is a vertical shift UP 4 units. Apply the transformations, and we get the following graph:



On some examples, they may not ask you to graph at all, but only list the transformations. So on example 5, the transformations would be "shifted right 1 unit and up 4 units."

You could also be asked to go in the opposite direction. For example, you could be given the graph and asked to write the corresponding equation. Let's look at an example such as this one.

6. Given the graph, write the corresponding function.

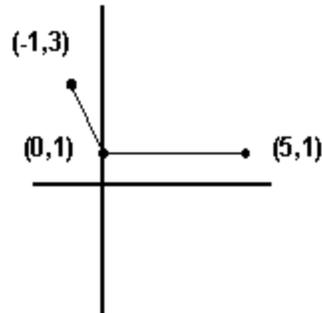


Well, looking at the graph, we know that it's a transformation on the function  $f(x) = |x|$ . Since the base of the function has been shifted right 3 units and down 4 units, that tells us we have "- 3" inside the function and "- 4" outside the function. Thus, the function is

$$f(x) = |x - 3| - 4$$

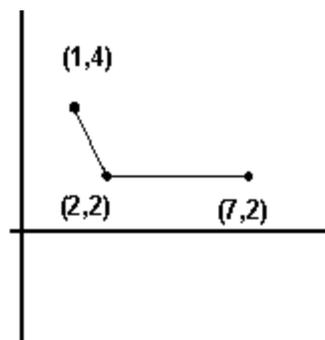
You could also be asked to apply transformations to functions that ARE NOT basic functions. The process is still the same. Let's look at one of these examples.

7. Given the function of  $f(x)$  below:



Graph the function  $f(x-2) + 1$ .

To graph  $f(x-2)+1$ , we need to shift the function of  $f(x)$  right 2 units and up 1 unit. The easiest way to do this is to do it point by point. Once we have applied the transformations to each point, we get the following graph:



## Stretching, Shrinking & Reflecting

Here, you will learn three more transformations to apply to the basic functions, stretching, shrinking and reflecting.

### Reflecting

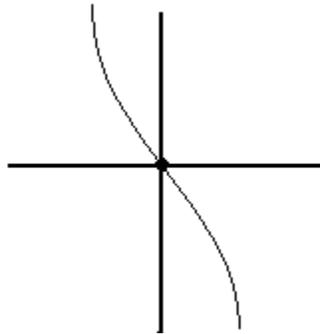
Reflecting causes the graph to flip to a mirror image of its original.

$-f(x)$	Reflection about the x-axis.
$f(-x)$	Reflection about the y-axis.

Let's look at a couple of examples.

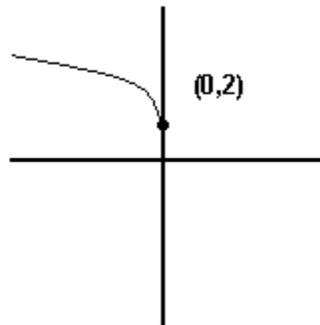
1. Graph:  $f(x) = -x^3$

This is a transformation on the basic function  $f(x) = x^3$ . Since the negative is on the outside of the function, there is a reflection about the x-axis. This gives us the following graph:



2. Graph:  $f(x) = \sqrt{-x} + 2$

This function is a transformation on the basic function  $f(x) = \sqrt{x}$ . Since we have a negative on the inside of the radical that gives us a reflection about the y-axis. We also have "+2" on the outside of the function, so this gives us a shift UP 2 units. When we apply the transformations to the basic graph, we get the following:



Now let's look at the last two transformations, stretching and shrinking. Stretching and shrinking occurs when you multiply the function by a constant.

### Stretching & Shrinking

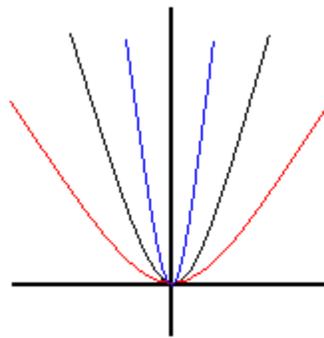
$C \cdot f(x)$ , where  $C$  is a real number  $> 0$

If  $C > 0$ , then  $f(x)$  has a vertical STRETCH by a factor of  $C$  units.  
If  $0 < C < 1$ , then  $f(x)$  has a vertical SHRINK by a factor of  $C$  units.

Let's look at these transformations by comparing a few graphs in your calculator. Let's sketch the graphs of the following in the calculator:

$$Y_1 = x^2$$
$$Y_2 = (1/3)x^2$$
$$Y_3 = 10x^2$$

You get the following graphs (in their respective colors):



As you can see, when you multiply by a constant less than 1, you get a vertical shrink (the graph become wider). When you multiply by a constant greater than 1, you get a vertical stretch (the graph becomes skinnier).

\*You may also have to apply these transformations given the function of  $f(x)$ , as we did before.