You do not need to simplify expressions that only involve arithmetic evaluations of numbers. Note: Formulas will be given to you on the quiz.

“Love is patient and kind; love does not envy or boast; it is not arrogant or rude. It does not insist on its own way; it is not irritable or resentful; it does not rejoice at wrongdoing, but rejoices with the truth.”

Problem 1. Circle True or False.

a. (True or False) If \( f(x) \) is Lipschitz on \((-\infty, \infty)\), then so is \( f(x)^2 \) on \((-\infty, \infty)\).

b. (True or False) Integrating a cubic polynomial with Gaussian quadrature on two nodes gives no approximation error.

Problem 2. a. Use Gaussian quadrature on two nodes to evaluate \( \int_0^1 \frac{1}{x} \, dx \)
Problem 3. Consider the IVP $y'(t) = yt^2$, $t \in [0, 1]$ with the initial values $y(0) = 1$.

a. Show that this IVP is well-posed.

b. Run Euler’s method with $h = 0.5$ to find $y(1)$.

c. Assuming that $|y(t)| \leq 2$ for $t \in [0, 1]$, find the stepsize $h$ such that running Euler’s method to approximate $y(1)$ would guarantee an error of at most $10^{-3}$.  