1. (1 point) Find the length of the curve: \( r(t) = (e^t \sin(t)) \mathbf{i} + (e^t \cos(t)) \mathbf{j} + e^t \mathbf{k} \) for \(-\ln(4) \leq t \leq 0\).

The arc length is given by

\[
\int_{-\ln(4)}^{0} \sqrt{(e^t \cos(t) + e^t \sin(t))^2 + (-e^t \sin(t) + e^t \cos(t))^2 + (e^t)^2} \, dt
\]

\[
= \int_{-\ln(4)}^{0} e^t \sqrt{\cos^2(t) + \sin^2(t) + 2 \sin(t) \cos(t) \cos^2(t) + \sin^2(t) - 2 \sin(t) \cos(t) + 1} \, dt
\]

\[
= \int_{-\ln(4)}^{0} e^t \sqrt{3} \, dt
\]

\[
= \sqrt{3} \left[ e^t \right]_{-\ln(4)}^{0}
\]

\[
= \sqrt{3} \left[ 1 - \frac{1}{4} \right]
\]

\[
= \frac{3}{4} \sqrt{3}.
\]

2. (1 point) A particle’s position at time \( t \) is given by \( r(t) = \frac{4}{9} (1 + t)^{3/2} \mathbf{i} + \frac{4}{9} (1 - t)^{3/2} \mathbf{j} \).

Find the angle between the velocity and acceleration vectors at \( t = 0 \).

The velocity vector is \( r'(t) = \frac{2}{3} (1 + t)^{1/2} \mathbf{i} - \frac{2}{3} (1 - t)^{1/2} \mathbf{j} \) and the acceleration vector is \( r''(t) = \frac{1}{3} (1 + t)^{1/2} \mathbf{i} + \frac{1}{3} (1 - t)^{1/2} \mathbf{j} \). Hence, \( r'(0) = \frac{2}{3} \mathbf{i} - \frac{2}{3} \mathbf{j} \) and \( r''(0) = \frac{1}{3} \mathbf{i} + \frac{1}{3} \mathbf{j} \). Since \( r'(0).r''(0) = \frac{2}{3} - \frac{2}{3} = 0 \), the angle between the vectors is \( \frac{\pi}{2} \).

3. (1 point) Find the limit, if it exists, or show that it does not exist:

\[
\lim_{(x,y) \to (0,0)} \frac{x^4 - y^4}{x^2 + y^2}.
\]

We have

\[
\lim_{(x,y) \to (0,0)} \frac{x^4 - y^4}{x^2 + y^2} = \lim_{(x,y) \to (0,0)} \frac{(x^2 - y^2)(x^2 + y^2)}{x^2 + y^2} = \lim_{(x,y) \to (0,0)} (x^2 - y^2) = 0
\]

since \( x^2 - y^2 \) is continuous at \((0,0)\).