Math 53: Quiz 1

September 2, 2014

1. (1 point) Consider a body falling from rest on Mars. The speed (in m/s) after \( t \) seconds is given by \( s(t) = 3.7t \). Find the distance covered by the body in 10 seconds.

The distance covered is given by

\[
\int_0^{10} 3.7t \, dt = \left[ \frac{3.7t^2}{2} \right]_0^{10} = \left[ \frac{3.7(10)^2}{2} - 0 \right] = 185 \text{ m.}
\]

2. (1 point) Find the equation of the tangent to the curve \( y = \sin^2(x) \) at \( x = \pi/4 \).

The slope at \( x = \pi/4 \) is

\[
m = 2 \sin(x) \cos(x) \bigg|_{x=\pi/4} = 2 \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{\sqrt{2}} \right) = 1.
\]

At \( x = \pi/4 \), we get \( y = (1/\sqrt{2})^2 = 1/2 \). Hence, the equation of the tangent is

\[
y - \frac{1}{2} = 1 \left( x - \frac{\pi}{4} \right) \Rightarrow y = x + \frac{1}{2} - \frac{\pi}{4}.
\]

3. (1 point) Determine whether the series converges or diverges:

\[
\sum_{n=1}^{\infty} \frac{1}{2n-1}
\]

Let \( b_n = \frac{1}{2n-1} \) and \( a_n = \frac{1}{2n} \). Then, \( a_n < b_n \). Furthermore, since \( \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{2n} \) diverges, by the comparison test so does the “larger” series \( \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{2n-1} \).