

Duration: 2 hours
Maximum Score: 60

Math 16B: Midterm

July 21, 2014

- (2 + 5 points) Suppose your score on this exam can be modelled by $S(x, y) = 20\sqrt{x} + y^2$ where x is the number of hours you spend weekly in class and y is the number of hours per week you study on your own.
 - What would be your score if you spent four hours a week in class and two hours studying on your own?
 - Determine the approximate **change** in your score if you spent half an hour less per week in class and studied on your own in that time.
- (8 points) Compute the double integral $\int \int_R \cos(y - x) dy dx$ over the region shown in Figure 1.

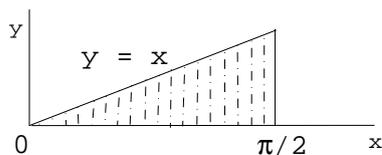


Figure 1

- (10 points) Let $f(x, y) = y^3 - x^2 + 6x - 12y + 5$. Find all possible relative extrema of f and determine their nature.
- (5 + 5 + 5 points) Evaluate:
 - $\int 3x\sqrt{12 - 5x^2} dx$.
 - $\int \frac{\cos(x)}{(1 - \sin(x))^2} dx$.
 - $\int_1^\infty \frac{\ln(x)}{x^5} dx$.
- (10 points) The company Victorinox manufactures Swiss Army knives. The number of knives it can produce in a day with L units of labour and K units of capital is given by the production function $P(L, K) = 500L^{3/5}K^{2/5}$. The expenses are \$100 per day for each unit of labour and \$200 per day for each unit of capital. Total expenses are limited to \$1000 per day. How many units of labour and capital should be utilized to maximize production?

6. (5 + 5 points) Let $f(x) = -\cos(x) \ln(\sec(x) + \tan(x))$.

(a) Show that

$$f'(x) = \sin(x) \ln(\sec(x) + \tan(x)) - 1.$$

(b) Verify that $y = f(x)$ is a solution of the differential equation

$$y'' + y = \tan(x).$$