Math 54 Worksheet 4

1. Determine whether the following subsets of $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.
   a. $\{(x, y)|xy = 0\}$.
   b. $\{(x, y)|x + y = 1\}$.
   c. $\{(x, y)|x + y = 0\}$.
   d. $\{(x, y)|y = \sin x\}$.
   e. The column space of a $2 \times 2$ matrix.
   f. The null space of a $2 \times 2$ matrix.
   g. What does a general subspace of $\mathbb{R}^2$ look like? How about a subspace of $\mathbb{R}^3$.

*2. Let $V, W$ be two subspace of $\mathbb{R}^n$, Are their intersection, union again subspaces?

3. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $v_4 = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$.
   a. Is $\{v_1, v_2, v_3\}$ a basis for $\mathbb{R}^3$? Is $\{v_1, v_2, v_3\}$ a basis for Col $A$? Where $A$ is the matrix $[v_1 \ v_2 \ v_3]$.
   b. Is $\{v_1, v_2, v_4\}$ a basis for $\mathbb{R}^3$? Is $\{v_1, v_2, v_4\}$ a basis for Col $A$? Where $A$ is the matrix $[v_1 \ v_2 \ v_4]$.
   c. Is $\{v_1, v_2\}$ a basis for $\mathbb{R}^3$? Is $\{v_1, v_2\}$ a basis for Col $A$? Where $A$ is the matrix $[v_1 \ v_2]$.
   d. Is $\{v_1, v_2, v_3, v_4\}$ a basis for $\mathbb{R}^3$? Is $\{v_1, v_2, v_3, v_4\}$ a basis for Col $A$? Where $A$ is the matrix $[v_1 \ v_2 \ v_3 \ v_4]$.

4. Find basis for Col $A$ and Null $A$. And check the Rank Theorem for $A$.
   a. $A = \begin{bmatrix} 3 & -4 & 2 \\ -9 & 12 & -6 \\ -6 & 8 & -4 \end{bmatrix}$
   b. $A$ is the matrix in problem 3.

Reference:
Lay, Nagle, Saff, Snider - Linear algebra and differential equations