1. True or False. No justification is needed.

\((\neg)\) (1) The subset \(\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right\} \subseteq M_{2 \times 2}\) are linear independent.

\((\neg)\) (2) There is a \(6 \times 3 A\) such that for every \(b \in \mathbb{R}^6\), \(Ax = b\) has a solution.

\((\neg)\) (3) The subspace \(x + y + z = 0\) of \(\mathbb{R}^3\) has a basis \(\left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}\)

\((\neg)\) (4) The only basis of \(\mathbb{R}^3\) is \(\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}\)

\((\neg)\) (5) Let \(T\) be a linear transformation, if \(\{T(u), T(v)\}\) an linearly independent set, then \(\{u, v\}\) is also an linearly independent set.

2. (5 points.) Determine whether the following set \(X\) is a vector space, Justify your answer. \(X\) is the union of coordinate \(xyz\) plane in \(\mathbb{R}^3\), i.e, \(X = \{(x, y, z) \in \mathbb{R}^3 | xyz = 0\}\)

\(X\) is not a vector space since

\(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \in X\)

but \(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \notin X\).

For True or False

\((\neg)\) (2): \(A\) has at most 3 pivots, so can not be auto.

\((\neg)\) (4): Another basis can be \(\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}\)

\((\neg)\) (5): prove by contradiction: if \(\bar{u}, \bar{v}\) are linearly dependent, then there is \(c_1, c_2\) not all 0, such that 
\(c_1\bar{u} + c_2\bar{v} = \bar{0}\), apply \(T\), we have \(c_1T(\bar{u}) + c_2T(\bar{v}) = T(c_1\bar{u} + c_2\bar{v}) = \bar{0}\). this means \(\{T(\bar{u}), T(\bar{v})\}\) linearly dependent. Contradiction.