Quiz 2

Section:

Name:

NOTE: There is one question on the back of the paper. You can quote any theorem in the book as long as you state it clearly.

1. Consider the following mapping from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \):
   \[ T(x_1, x_2, x_3) = (x_1 + x_3, 2x_1 - x_2, x_2 + x_3) \]

   (1) (5 points.) Find the standard matrix \( A \) of \( T \).

   \[
   A = \begin{bmatrix}
   T(e_1), T(e_2), T(e_3) \\
   \end{bmatrix}
   = \begin{bmatrix}
   1 & 0 & 1 \\
   2 & -1 & 0 \\
   0 & 1 & 1 \\
   \end{bmatrix}
   \]

   (2) (5 points.) Are the columns of the above matrix linearly independent? Justify your answer.

   \[
   A \sim \begin{bmatrix}
   1 & 0 & 1 \\
   0 & -1 & -2 \\
   0 & 1 & 1 \\
   \end{bmatrix} \sim \begin{bmatrix}
   1 & 0 & 1 \\
   0 & 1 & 1 \\
   0 & 0 & 1 \\
   \end{bmatrix}
   \]

   Yes, since \( A \) has a pivot in each column.
(3) (5 points.) Is it true that for every vector \( \mathbf{b} \) in \( \mathbb{R}^3 \), the equation \( A\mathbf{x} = \mathbf{b} \) has solution(s)? Explain.

Yes, since \( A \) has a pivot in each row.