Holography with broken Lorentz invariance

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Outline

- Why to break the Lorentz invariance (LI)?
- History - UHECR, Cosmology
- Lifshitz solutions
- Null Energy Condition (NEC) and Causality
- NEC and higher derivative Gravity

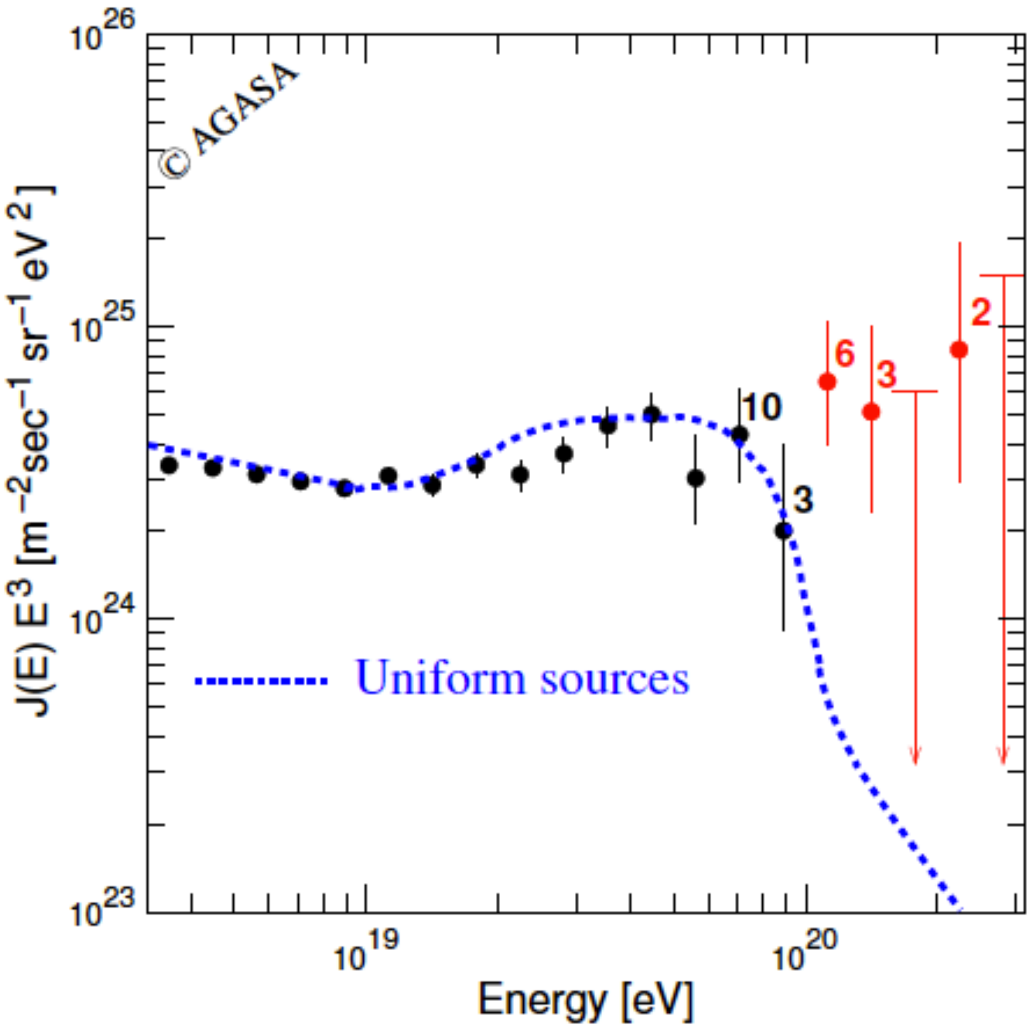
LI violation - Why?

- Does not have to be the fundamental principal of Nature
- LI is not well tested at high energies
- Is not excluded by some of cosmological observations
- In condensed matter systems it’s not there at all!

History - UHECR

1966 — K. Greisen, **PRL 16**, 748
G. Zatsepin, V. Kuzmin, **JETP Lett 4**, 78

AGASA: energy spectrum ($z < 45°$)



fit:

$dN/dE = F_0 \cdot E^{-\gamma}$

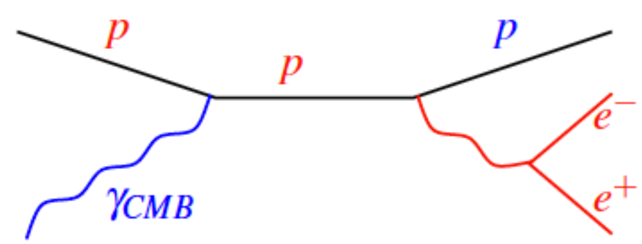
F_0, γ — low E

1.9 events expected

11 events observed

4.4σ deviation

$E_p \lesssim 5 \cdot 10^{19}$ eV



$L^{m.f.p.}_{p\gamma_{CMB} \to pe^+e^-} \sim 1$ Mpc

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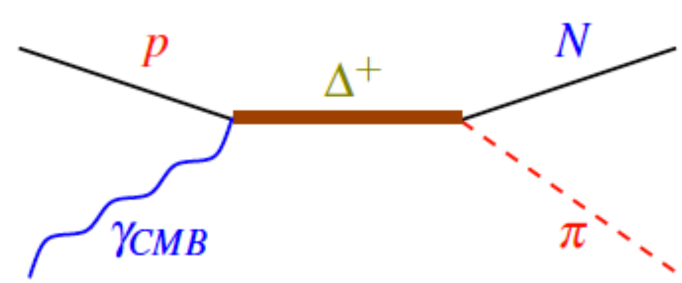

$L^{m.f.p.}_{p\gamma_{CMB} \to N\pi} \sim 10$ Mpc

LI violation (at high energies!)

Avoiding GZK cutoff

Renormalizable and gauge- invariant perturbations to the standard-model Lagrangian that are rotationally invariant in a preferred frame, but not Lorentz invariant, *lead to species-specific maximum attain- able velocities (MAV) for different particles.*∗

[Coleman, Glashow 99]

$$p + \gamma \to \Delta(1232)$$

$$4\omega \geq \delta(E)E + \frac{M_\Delta^2 - M_p^2}{E}$$

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Threshold can be raised once LI is broken!

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Threshold can be raised once LI is broken!

Later AGASA data were proved to be wrong (Pierre Auger experiment)

LI is preserved in all tests so far... but if it's broken then what?

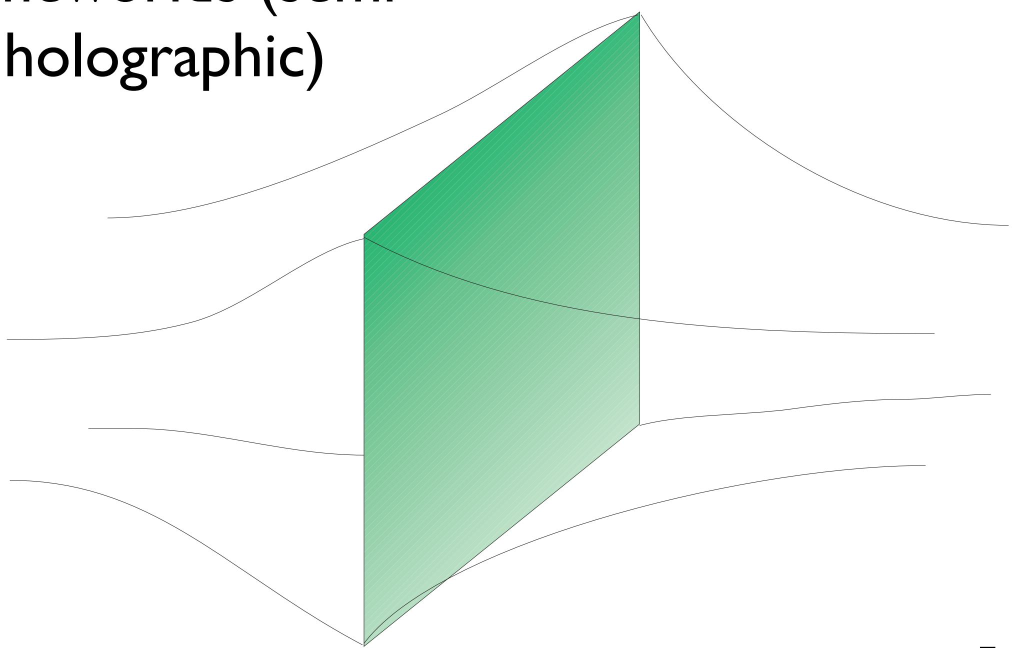
$$\omega^2 = p^2 + \mathcal{O}\left(\frac{\Lambda^2}{p^2}\right) \qquad\qquad \omega^2 = f(p^2)$$

- LI at low energies, but ~~LI~~ at high energies
  Modifications of gravity at low distances, massive gravitons, etc. Different cosmological scenarios
- ~~LI~~ at all energies Good for 'nonrelativistic' systems, e.g. studied in condensed matter physics

LI broken at high energies

[PK, Libanov]

Braneworlds (semi-holographic)



$$ds^2 = \mathrm{e}^{-2a(z)}dt^2 - \mathrm{e}^{-2b(z)}d\mathbf{x}^2 - dz^2$$

Our universe sits at fixed z

$$S = \int dt d\mathbf{x} \int\limits_{-\infty}^{+\infty} dz\, \sqrt{g}\, g^{AB} \partial_A \phi \partial_B \phi$$

Matter fields live in the bulk
(no brane localized matter assumed)

equation of motion

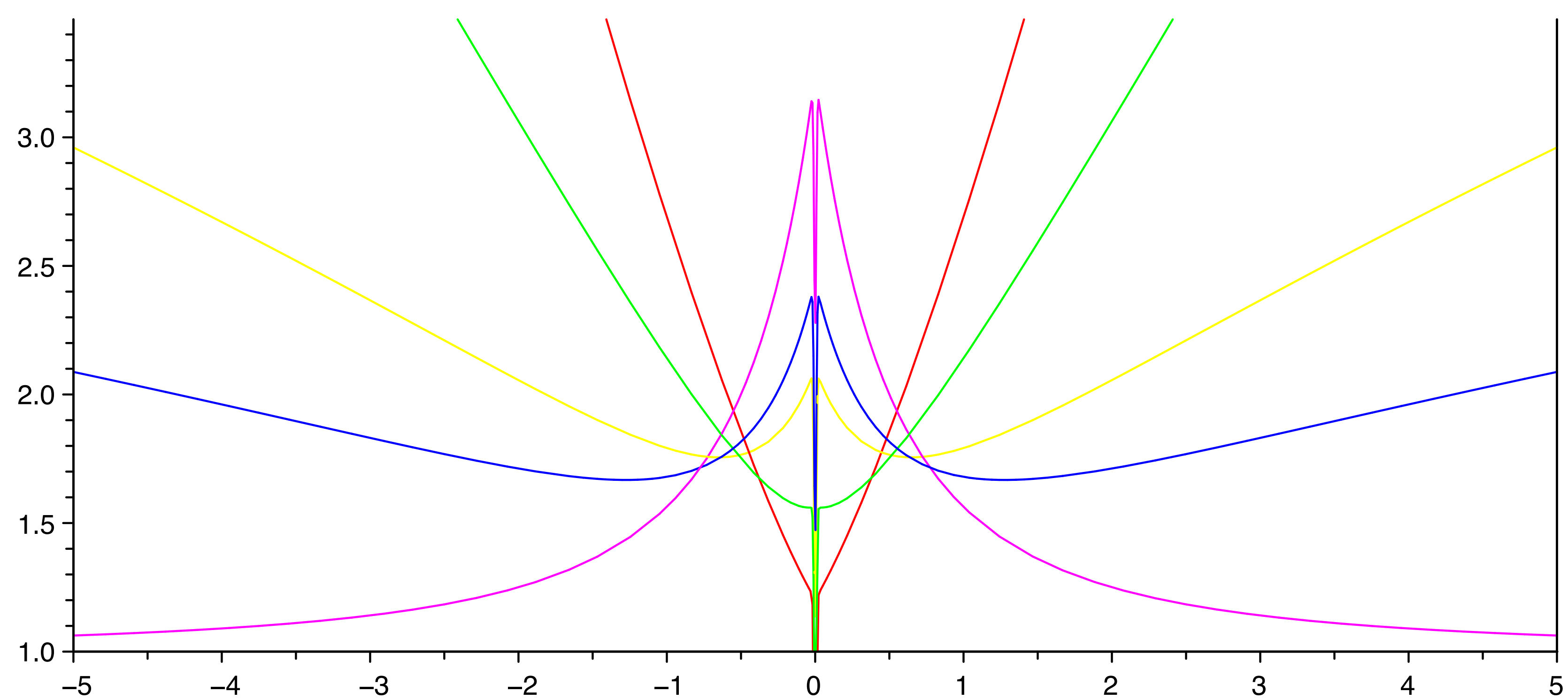
$$\left[\partial_z^2 + E^2 \mathrm{e}^{2a(z)} - p^2 \mathrm{e}^{2b(z)} + \frac{a'' + 3b''}{2} - \frac{(a' + 3b')^2}{4}\right] \chi = 0$$

reduces to Schroedinger equation with the potential

$$V = -\frac{1}{4}\left(\frac{\partial a}{\partial y}\right)^2 + \frac{9}{4}\left(\frac{\partial b}{\partial y}\right)^2 + p^2 \mathrm{e}^{2(b-a)} - \frac{1}{2}\left(\frac{\partial^2 a}{\partial y^2} + 3\frac{\partial^2 b}{\partial y^2}\right)$$

For simplicity we choose

$$a(z) = \xi k|z|, \quad b(z) = \zeta k|z|$$

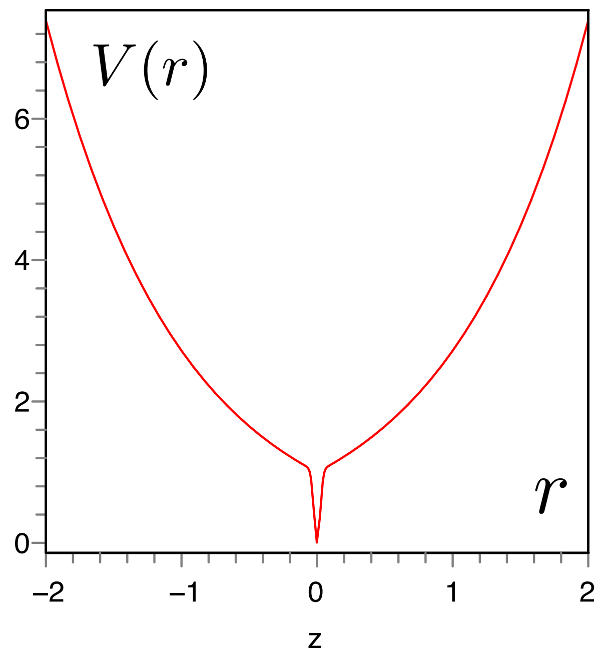


Spectrum of fluctuations

extremal example d=4 (Lifshitz z=0) $\xi = 0,\ \zeta = 1$

$$ds^2 = dt^2 - \mathrm{e}^{-2k|z|} d\mathbf{x}^2 - dz^2 \qquad \mathbb{R}^1 \times dS_{d+1}$$

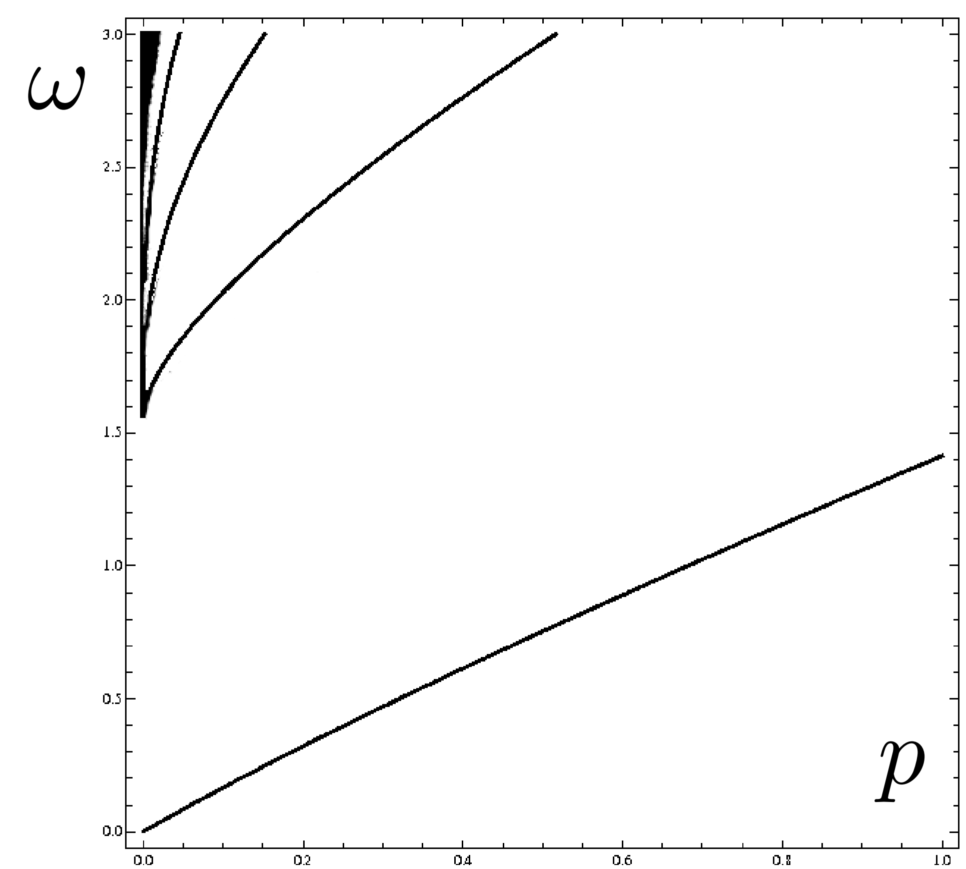
Gravitational Potential $$V(z) = p^2 \mathrm{e}^{2k|z|} + \frac{9}{4}k^2 - 3k\delta(z)$$



Solution $$\chi(z) = N\,\mathrm{K}_{\sqrt{\frac{9}{4}-\frac{E^2}{k^2}}}\left(\frac{p}{k}\mathrm{e}^{k|z|}\right)$$

Matching bc at the origin $$\frac{p}{k}\frac{\mathrm{K}_{\nu+1}\left(\frac{p}{k}\right)}{\mathrm{K}_{\nu}\left(\frac{p}{k}\right)} = \frac{3}{2} + \nu$$

Spectrum



Higher modes $$E_n^2 = \frac{9}{4}k^2 + \frac{\pi^2 k^2 n^2}{4\log^2 \frac{p}{k}}$$

Zero mode $$E^2 = 3p^2\left(1 - \frac{p}{k} + \mathcal{O}(p^2)\right)$$

Theories with dynamical scaling

$$\mathcal{L} = (\partial_t \phi)^2 - c^2 \ell^{2(z-1)} \phi (-\partial_{\mathbf{x}}^2)^z \phi \qquad t \to \lambda^z t,\ x \to \lambda x$$

Dispersion relation

$$\omega^2 = \frac{c^2}{\ell^2} (\ell k)^{2z}$$

Phase velocity

$$v_{\rm ph} = \frac{\omega}{k} = c(\ell k)^{z-1}$$

Physical dimensions

$$[\phi] = \frac{d-1}{2},\ [\omega] = 1,\ [k] = 1,\ [\ell] = 2(z-1)$$

Scaling dimensions

$$[[\phi]] = \frac{d-z}{2},\ [[\omega]] = z,\ [[k]] = 1,\ [[\ell]] = 0$$

Lifshitz metric

$$ds^2 = \frac{L^2}{r^2} \left(-\frac{\kappa^2 dt^2}{r^{2(z-1)}} + dr^2 + d\mathbf{x}^2 \right)$$

Speed of light

$$c(r) = \frac{\kappa}{r^{z-1}}$$

Same dependence of r

$$c(r) = c\,\ell^{(z-1)} r^{-(z-1)}$$

what is the difference between z>1 and z<1 from the gravitational perspective?

Lifshitz solution

Take perfect fluid in the bulk

$$T^{\mu}_{\nu} = (p+\rho)u^{\mu}u^{\nu} - p\delta^{\mu}_{\nu}$$

equation of state

$$p = w\rho$$

introduce anisotropy

$$T^0_0 = (1+\omega)\rho u_0 u^0 - p_{d+1}$$
$$T^1_1 = (1+w)\rho u_1 u^1 - p_1$$
$$T^{d+1}_{d+1} = (1+\omega)\rho u_{d+1} u^{d+1} - p_{d+1}$$
$$T^0_{d+1} = (1+\omega)\rho u^0 u_{d+1}$$

equations of state

$$p = \omega\rho$$
$$p = w\rho$$
$$p = \omega\rho$$
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[PK, Libanov]

d=3

$$S = -\int \frac{1}{e^2} F_{(2)} \wedge * F_{(2)} + F_{(3)} \wedge * F_{(3)} - c\int F_{(2)} \wedge B_{(2)}$$

$$F_{(2)} = dA_{(1)},\ F_{(3)} = dB_{(2)}$$

$$F_{(2)} = A\,\theta_r \wedge \theta_t\,, \quad F_{(3)} = B\,\theta_r \wedge \theta_x \wedge \theta_y$$

[Kachru, Liu, Mulligan]

give the Lifshitz solution

$$ds^2 = L^2\left(-\frac{dt^2}{r^{2\xi}} + \frac{d\mathbf{x}^2}{r^{2\zeta}} + \frac{dr^2}{r^2}\right)$$

$$\rho = -\Lambda - \frac{d(d-1)}{2L^2}\zeta^2$$
$$w = -1 + \frac{(\xi + (d-2)\zeta)(\xi - \zeta)}{L^2\rho}$$
$$\omega = -1 + \frac{(d-2)\zeta(\xi-\zeta)}{L^2\rho}$$

$$\Lambda = -\frac{z^2+z+4}{2L^2}$$
$$A^2 = \frac{2z(z-1)}{L^2}$$
$$B^2 = \frac{4(z-1)}{L^2}$$

Consider d+1 dimensional field theory with a D=d+2 gravity dual

In the holographic description the states created by a scalar operator correspond to classical normalizable solutions of a dual scalar field

Metric

$$ds^2 = du^2 + e^{2A(u)}(-e^{2B(u)}dt^2 + d\mathbf{x}^2)$$

Bulk scalar action

$$S = -\int d^{d+2}x\sqrt{-g}\left(\partial_M\Phi\,\partial^M\Phi + m^2\Phi^2\right)$$

Equation of motion

$$\phi'' + ((d+1)A' + B')\phi' + e^{-2A-2B}\omega^2\phi - e^{-2A}k^2\phi - m^2\phi = 0$$

after some redefinitions reduces to Schoedinger equation

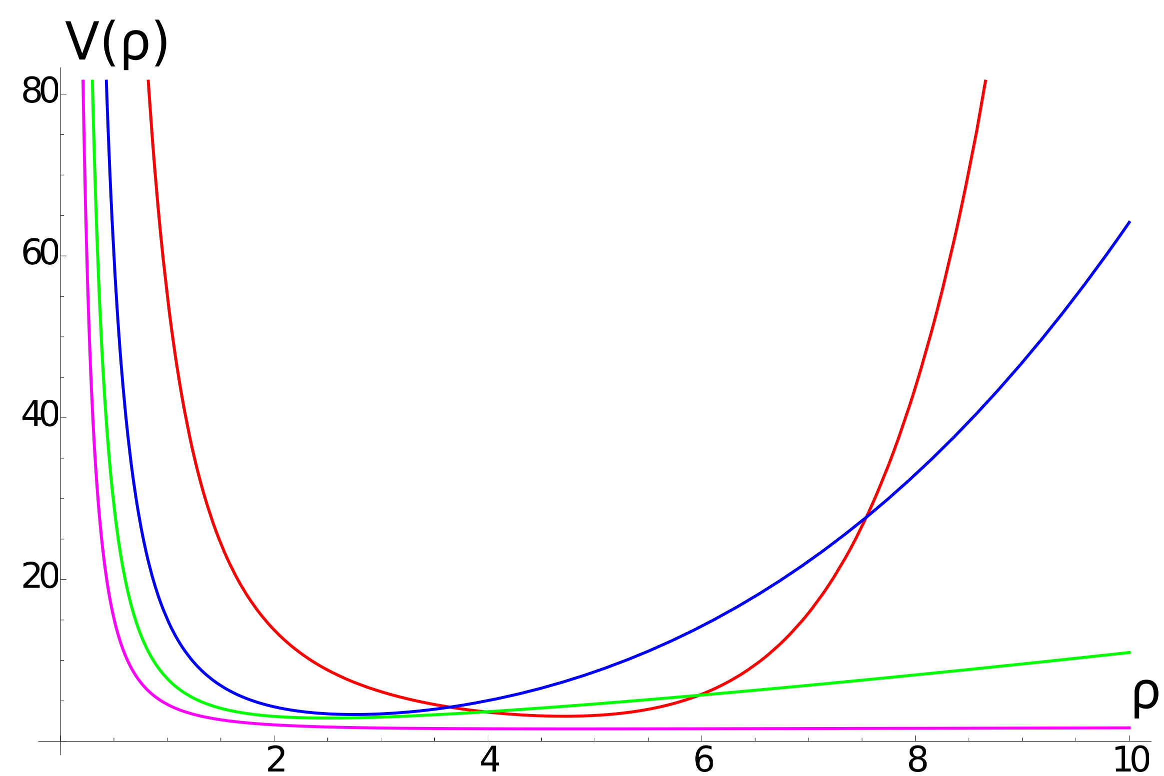
$$-\ddot{\psi} + V(\rho)\psi = \omega^2\psi$$

in the potential

$$V(\rho) = \frac{d^2 + 2dz + 4m^2L^2}{4\rho^2 z^2} + k^2\left(\frac{\rho z}{L}\right)^{\frac{2}{z}-2}$$

bottom of the potential

$$\rho_{min} = \frac{L}{z}\left(\frac{(1-z)(d^2 + 2dz + 4m^2L^2)}{4z(kL)^2}\right)^{\frac{z}{2}}$$



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z>1 continuous

z<1 discrete

in the potential

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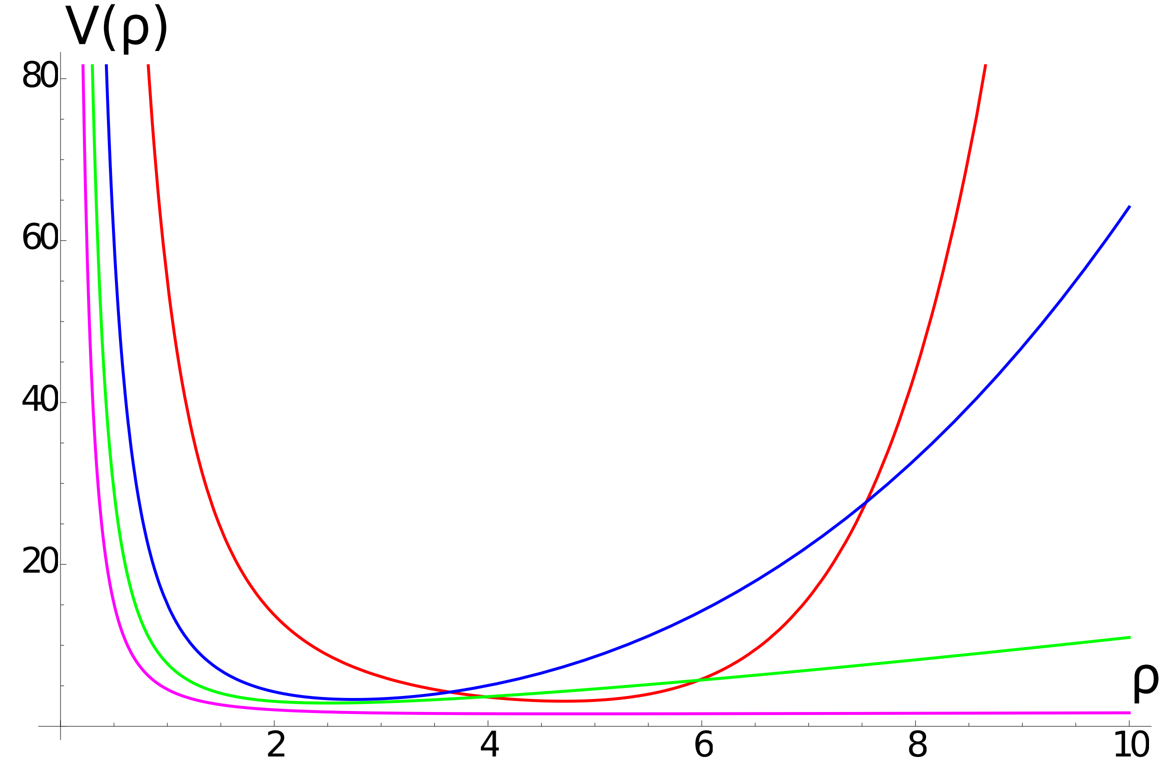

WKB analysis

Find the value of the turning point in the limit $\omega \to \infty$

The condition $V(\rho_0) = \omega^2$ leads to

Thus the wavefront velocity is given by the local speed of light at the turning point.

Therefore plane wave states created by a scalar operator in the field theory have wavefront velocities that are equal to the local speed of light in the holographic dual.

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$$v_{wf} \simeq v_{ph} = \frac{\omega}{k} \simeq e^{B(\rho_0)}$$

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Therefore plane wave states created by a scalar operator in the field theory have wavefront velocities that are equal to the local speed of light in the holographic dual.

Growing vs. decreasing s.o.l.

z<1

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 + d\mathbf{x}^2 - r^{2-2z}\kappa^2 dt^2\right)$$

Boundary is d-dimensional
Conical singularity for z=1/2

null geodesics tangent to boundary

$$\frac{dt}{dr} = -\frac{r^{z-1}}{\kappa}, \quad t(r_0) = 0$$

z>1

$$ds^2 = \frac{L^2}{z^2R^2}\left(-\kappa^2 dt^2 + dR^2 + R^{2-2/z}d\mathbf{x}^2\right)$$

Boundary goes along time direction
Conical singularity for z=2

null geodesics orthogonal to boundary

$$t(r) = \frac{r_0^z - r^z}{z\kappa}$$

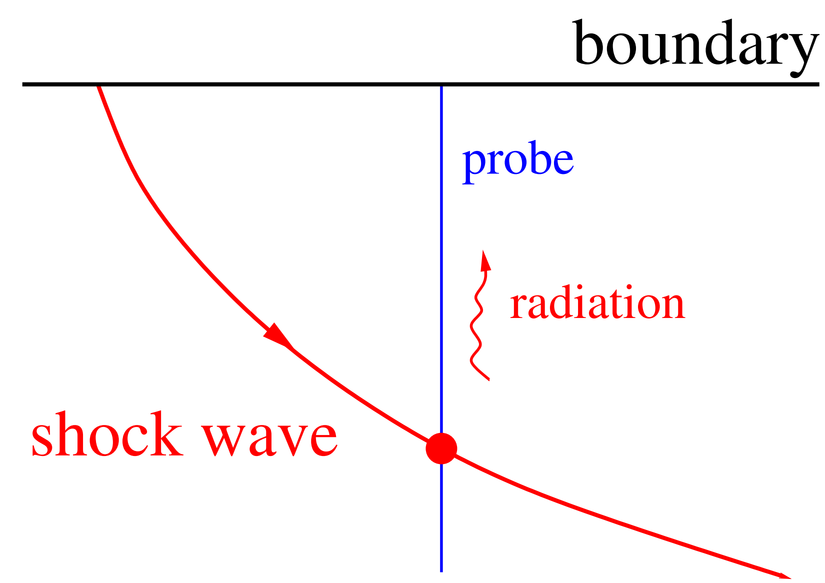
boundary singularity suggests UV completion

For our purpose it will be enough to introduce a cutoff, since the results we will obtain are independent on how the ultraviolet theory is defined

Causality from shock waves

[Hoffman,Maldacena]

source in the field theory localized in time
and in one of the spatial directions



Null geodesics

$$\frac{dt}{dr} = \frac{E r^{2(z-1)}}{\kappa^2 \sqrt{\frac{E^2 r^{2(z-1)}}{\kappa^2} - P^2}}\,, \qquad \frac{dx}{dr} = \frac{P}{\sqrt{\frac{E^2 r^{2(z-1)}}{\kappa^2} - P^2}}$$

for z>1

$$t \simeq \frac{r^z}{z\kappa} \to \infty\,, \qquad x \simeq \frac{\kappa P}{(2-z)E} r^{2-z} + x_0$$

z<1

$$\left(\frac{r_0}{\ell}\right)^{1-z} = \frac{E}{cP}$$

right turning point

The shock wave will be a source of radiation of gravitational fields that will then propagate along the radial direction to the boundary, producing a front of radiation that can be interpreted as the front of the perturbation in the dual theory.

Calculate the time and position of the shockwave travelled back to the boundary

thus the shock wave travels faster than light signals at the boundary

$$v_s > c$$

Null Energy Condition

$T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$

Example - perfect fluid $\quad p = w\rho \quad$ NEC $\quad w > -1$

cosmological constant $\quad w = 1$

Broken NEC is usually associated with *superluminal propagation, causality violation, etc*

From Einstein equations NEC $\quad R^t_t - R^x_x \leq 0\,, \quad R^t_t - R^u_u \leq 0$

$$ds^2 = du^2 + e^{2A(u)}(-e^{2B(u)}dt^2 + d\mathbf{x}^2)$$

Ricci tensor

$$R^t_t = -B'' - DA'B' - B'^2 - A'' - (D-1)A'^2$$
$$R^x_x = -A'B' - A'' - (D-1)A'^2$$
$$R^u_u = -B'' - (A'+B')^2 - (D-1)A'' - (D-2)A'^2$$

NEC I

$$B'' + B'(B' + (D-1)A') \geq 0$$

For Lifshitz $\quad$ Bulk NEC $\quad \mathbf{z \geq 1}$

Domain walls again - universality of NEC

Let the following conditions be satisfied

[PK, Libanov]

- Bulk NEC $T_{AB}\xi^A\xi^B \geq 0, \quad g_{AB}\xi^A\xi^B = 0$
- Brane NEC $T_{b,\,\mu\nu}\xi^\mu\xi^\nu \geq 0, \quad g_{b,\,\mu\nu}\xi^\mu\xi^\nu = 0$
- Spatial brane curvature vanishes
- Bulk LI is broken

Then a static smooth solution with symmetry

$SO(d) \times T^d \times \mathbb{Z}_2$ does not exist

NEC and speed of light

let's check our holographic construction: 1 Bulk NEC; 2 Boundary NEC

$$ds^2 = du^2 + e^{2A(u)}(-e^{2B(u)}dt^2 + d\mathbf{x}^2)$$

NEC 1 $$B'' + B'(B' + (D-1)A') \geq 0$$

define

$$B' = Ce^{-(D-1)A-B}$$

The derivative of the local speed of light is

$$(e^B)' = B'e^B = Ce^{-(D-1)A}$$

C>0 speed of light is monotonically increasing

C<0 speed of light is monotonically increasing

For Lifshitz $\mathbf{z} \geq 1$ implies both bulk and boundary NEC

Generically NEC is necessary in order to have a consistent holographic description

NEC and Higher Derivative Gravity

$$S = \int d^D x \sqrt{g} \left(R - 2\Lambda + L^2 \beta_1 R^2 + L^2 \beta_2 R_{\alpha\beta} R^{\alpha\beta} + L^2 \beta_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \right)$$

Represent higher derivative stuff as 'source' $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = L^2 \Theta_{\mu\nu}$

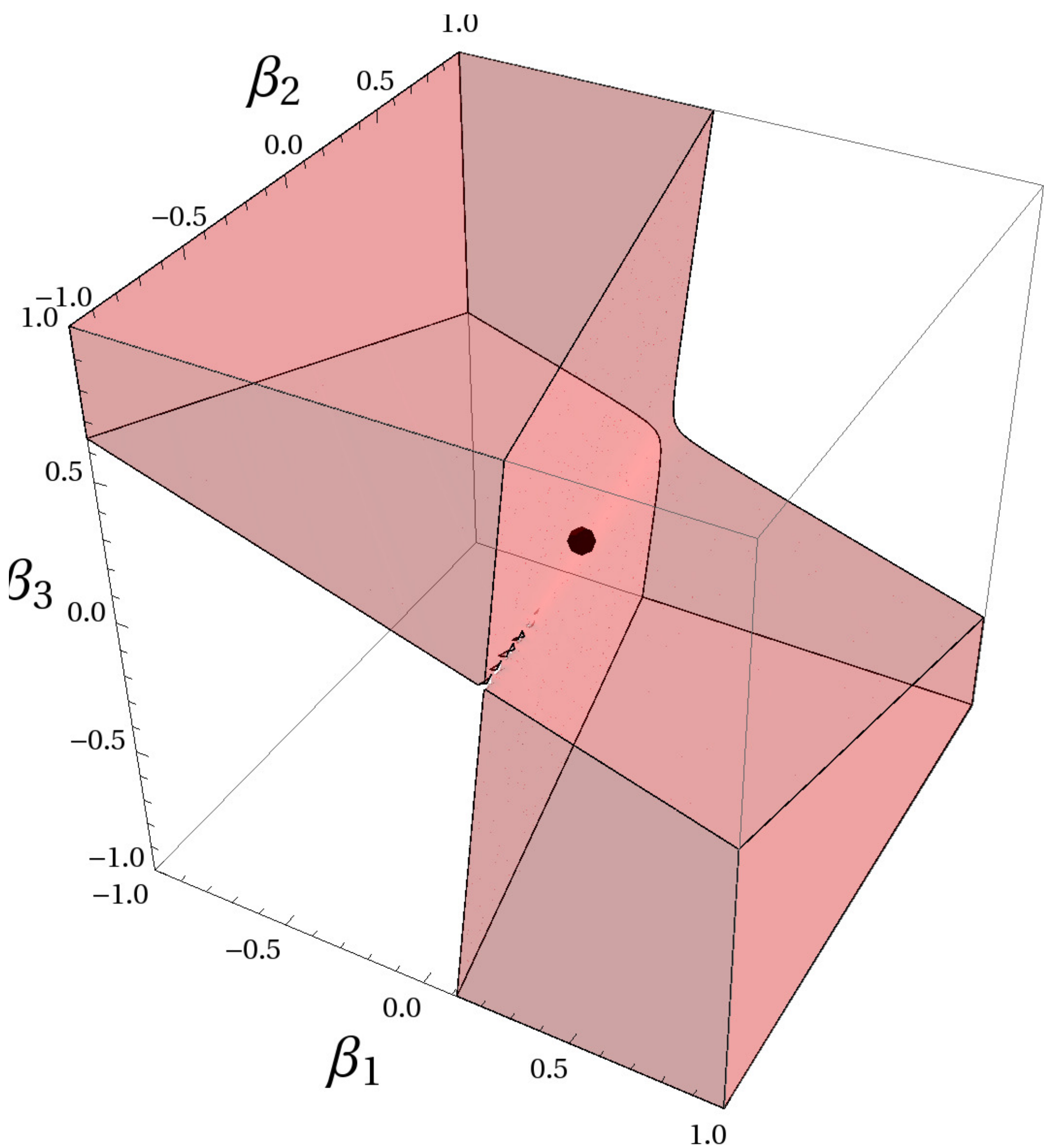
Constraints on existence of Lifshitz solutions

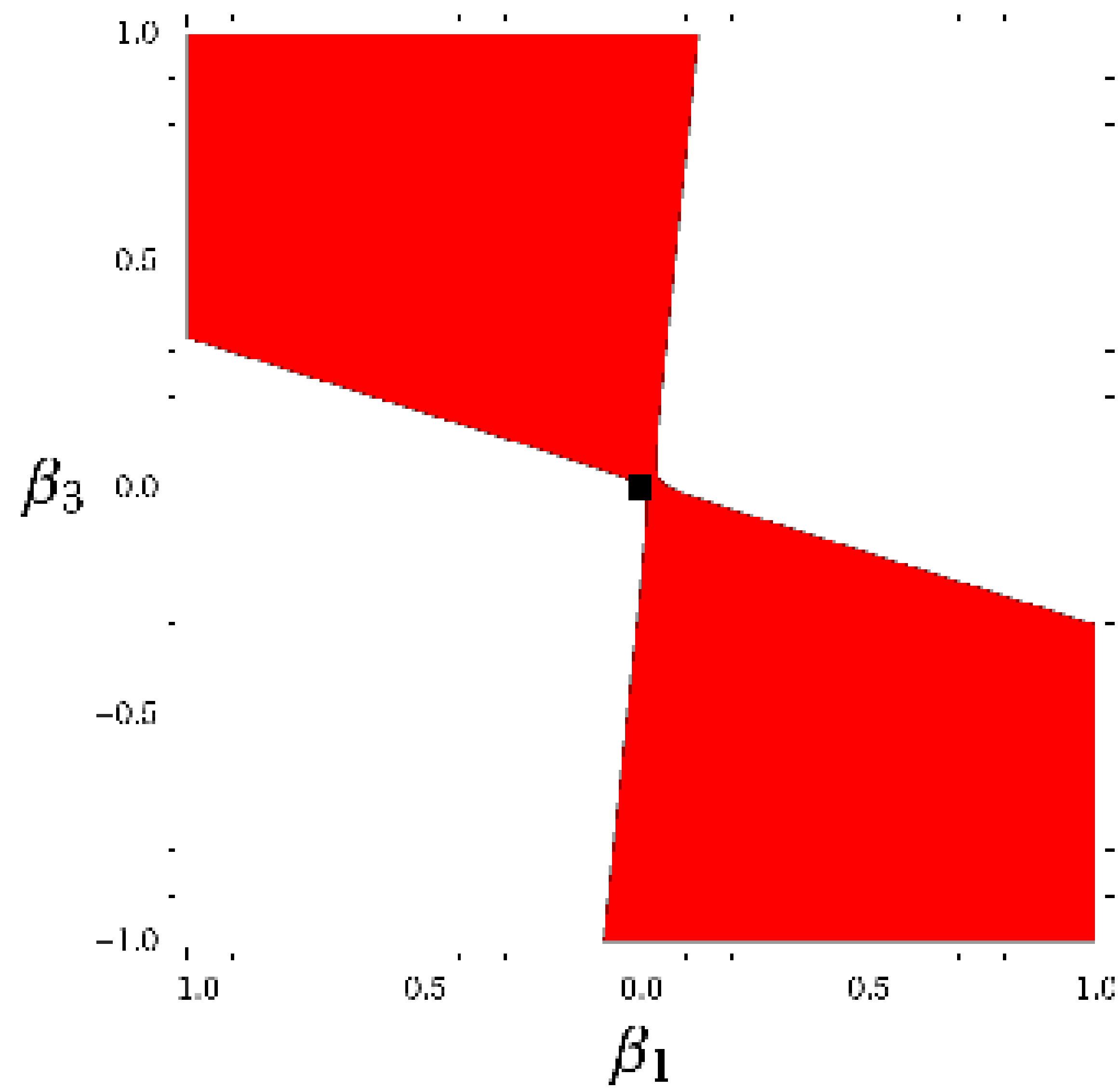
$$\Lambda = -\frac{1}{L^2} \left[1 + 2(\beta_1 - \beta_3) + 2z + \left(1 - 2z + \frac{1}{2} z^4 \right) (4\beta_1 + 2\beta_2 + 4\beta_3) + (3z^2 - 2z^3)(\beta_2 + 4\beta_3) \right]$$

$$2(2z^2 + (D-2)(2z + D - 1))\beta_1 + 2(z^2 + D - 2)\beta_2 + 4(z^2 - (D-2)z + 1)\beta_3 = 1$$

Impose NEC on the rhs of the Einstein equations treating is as a 'source' to Einstein Gravity

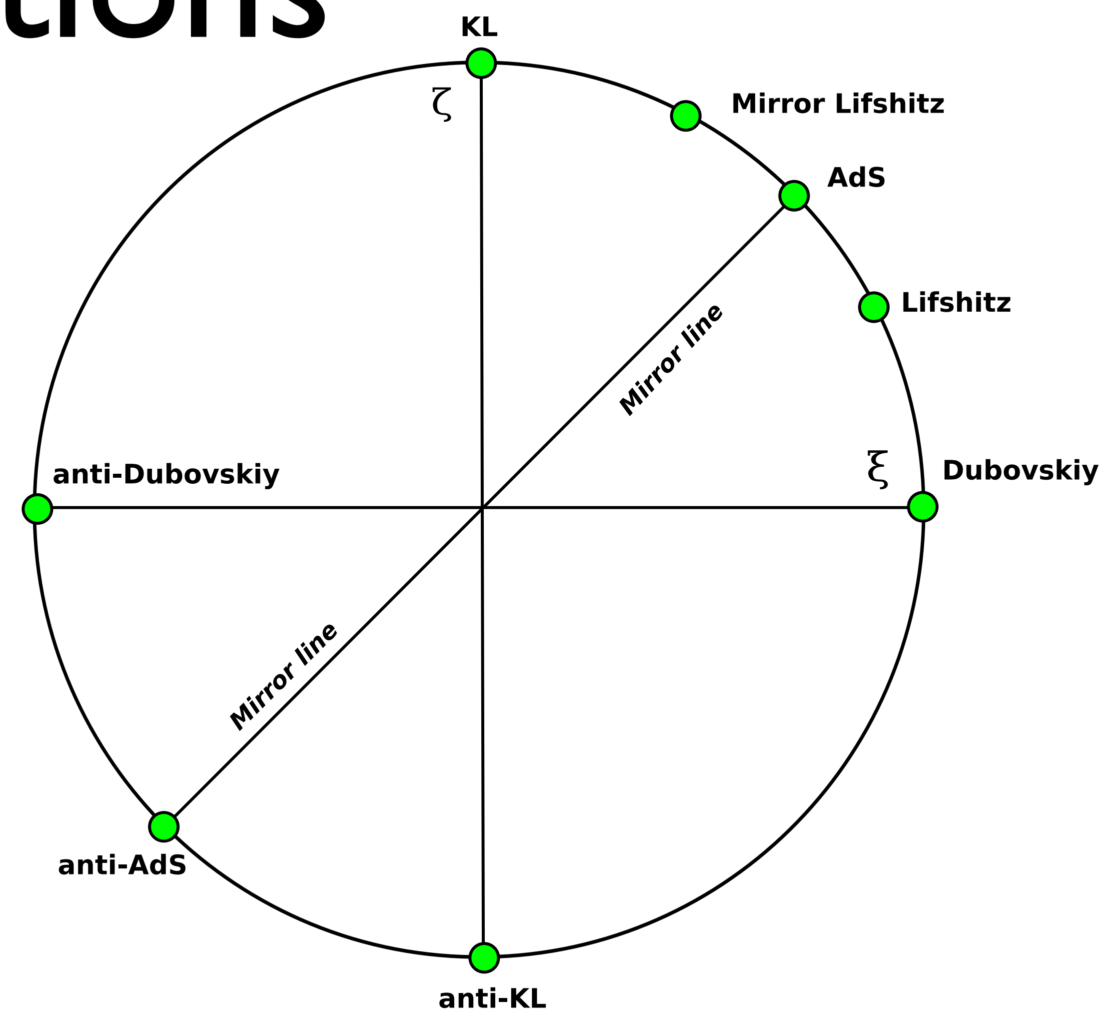
Solutions with z < 1 exist in the full region with fixed cosmological constant





violations of the NEC are possible in the full region !!

2-point functions



Scalar EOM

$$\varphi'' - \frac{z+d-1}{r}\varphi' + \frac{\omega^2}{\kappa^2} r^{2(z-1)}\varphi - k^2\varphi - \frac{m^2}{r^2}\varphi = 0$$

Correlator

$$G_2(\omega, \mathbf{k}) = - \lim_{\ell \to 0} \sqrt{-g} g^{rr} \varphi'_{\omega,\mathbf{k}}(r)\varphi_{\omega,\mathbf{k}}(r)\Big|_{r=\ell}$$

Scaling dimension $\quad m^2L^2 = \Delta(\Delta - d - z)$

z=2

$$G_2(\omega, k) \simeq \left(\frac{4\omega^2}{\kappa^2} + k^4\right)\left[\log(i\kappa\omega) + \psi\left(\frac{3}{2} - \frac{i\kappa k^2}{4\omega}\right) + i\Theta(\mathrm{Im}\,\omega)\pi\,\mathrm{sech}\left(\frac{\kappa k^2 \pi}{4\omega}\right)\right]$$

branch cut along the positive imaginary axis $\quad \omega_n = \frac{i\kappa k^2}{4n+6}\,, \quad n = 0, 1, 2, \ldots$

z=1/2

$$G_2(\omega, k) \simeq k^{5/2} \frac{\Gamma\left(\frac{7}{4} - \frac{\omega^2}{2k\kappa^2}\right)}{\Gamma\left(-\frac{3}{4} - \frac{\omega^2}{2k\kappa^2}\right)} \qquad \omega_n^2 = \left(2n + \frac{7}{2}\right)\kappa^2 k\,, \quad n = 0, 1, 2, \ldots$$

phase velocity $\quad (v_{ph})_n = \frac{\omega}{k_n} = \frac{c}{\omega\ell}\left(2n + \frac{7}{2}\right)$

leads to superluminal propagation

Further Constraints

Equation of state in scale invariant theory

$$z\langle T_{tt}\rangle - d\langle T_{xx}\rangle = 0$$

For the boundary theory which respects NEC

$$\langle T_{tt}\rangle + \langle T_{xx}\rangle = \langle T_{tt}\rangle\left(1 - \frac{z}{d}\right) \geq 0$$

Assuming

$$\langle T_{tt}\rangle \geq 0$$

we get more constraints

$$\mathbf{1 \leq z \leq d}$$

Bulk (holographic) NEC

Boundary NEC

Conclusions

- Geometries produced by matter that violates the NEC will produce superluminal propagation in the dual theory
- Further role of NEC in holography and RG dynamics of field theories (modifications of a-theorem?)