Holography and Lorentz Invariance Violation

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0712.1136 0904.0509 0909.2551 work in progress

GLSPOCK, March 21st 2010

Plan

- Lifshitz field theory and Lifshitz geometry (see also Sumit's talk)

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- Lifshitz holography. First tests and observations

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- Wilson loops in Lifshitz backgrounds (see also Sasha's talk)

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- Diversifies inflation scenarios [Rubakov et al]
- Conformal behavior of field theories with anisotropic scaling in IR

Lifshitz field theory

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Leads to $c = (bk)^{z-1}$

Gravity description

Metric

$$ds^2 = L^2 \left(-\frac{dt^2}{r^{2z}} + \frac{d\mathbf{x}^2}{r^2} + \frac{dr^2}{r^2} \right)$$

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Identify the RG scale

$$bk \sim 1/r\,,$$

Hence the speed of light $c = (bk)^{z-1} = r^{1-z}$ tends to zero in the IR if $z > 1$.

Macroscopic Solution

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$$ds^2 = L^2\left(-\frac{dt^2}{r^{2\xi}} + \frac{d\mathbf{x}^2}{r^{2\zeta}} + \frac{dr^2}{r^2}\right)$$

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Matter

$$\begin{aligned} T_0^0 &= (1+\omega)\rho u_0 u^0 - p_d + 1\,, \\ T_1^1 = \cdots = T_d^d &= (1+w)\rho u_1 u^1 - p_1\,, \\ T_{d+1}^{d+1} &= (1+\omega)\rho u_{d+1} u^{d+1} - p_{d+1}\,, \end{aligned}$$

Completed by equations of state

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$$\begin{aligned} \rho &= -\Lambda - \tfrac{1}{2}(d-1)(d-2)k^2\zeta^2 \,, \\ w &= -1 + k^2 \frac{(\xi + (d-2)\zeta)(\xi - \zeta)}{\rho} \,, \\ \omega &= -1 + k^2 \frac{(d-2)\zeta(\xi - \zeta)}{\rho} \,. \end{aligned}$$

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NEC $\omega > -1$ and $w > -1$ iff $\xi > \zeta$.

Microscopic Solution

2+1 Lifshitz model [Kachru Liu Mulligan 2008]

$$S = -\int \frac{1}{e^2} F_{(2)} \wedge *F_{(2)} + F_{(3)} \wedge *F_{(3)} - c \int F_{(2)} \wedge B_{(2)} \,,$$

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Solution of Einstein equations $2\xi\zeta = (cL)^2, \quad Z = \zeta/\xi$

$$\Lambda = -\frac{Z^2 + Z + 4}{2L^2}$$

$$A^2 = \frac{2Z(Z-1)}{L^2}$$

$$B^2 = \frac{4(Z-1)}{L^2}$$

To avoid tachyonic solutions $Z > 1$

Correspondence

[PK Gordeli]

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Cosmological constant	$\Lambda = -\rho - 3k^2$	$\Lambda = -L^{-2}(Z^2 + Z + 4)$
First component	w	$-1 + \dfrac{A^2 + B^2}{2(-\Lambda - 3L^{-2})}$
Second component	ω	$-1 + \dfrac{B^2}{2(-\Lambda - 3L^{-2})}$
LIV parameter	$w - \omega$	A
Anisotropy	$p_1 - p_4$	Energy flux A^2 w.r.t z direction
Constraints	Reality of Fluxes	Null energy condition

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(For those who like RS-type models) !! In these examples bulk NEC is inconsistent with brane NEC !!

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Near the boundary $u = 0$ a solution

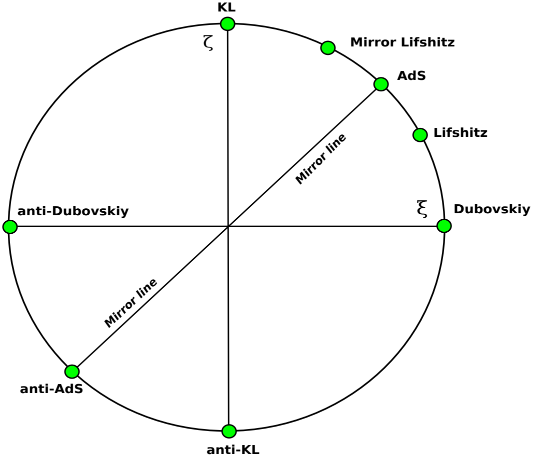
$$\phi(t, \mathbf{x}, u) = r^{\Delta_+}\phi_+(t, \mathbf{x}) + r^{\Delta_-}\phi_-(t, \mathbf{x})\,,$$

where $\Delta_\pm$ are solutions of

$$\Delta(\Delta - \xi - 2\zeta) = 0\,.$$

Known Solutions

Green functions can be explicitly calculated in the following cases (ξ, ζ): AdS model $(1,1)$, Lifshitz model $(2,1)$, Dubovsky model ($AdS_2 \times \mathbb{R}^2$) $(1,0)$, Mirror Lifshitz model $(1,2)$, KL model (mirror Dubovsky model) $(0,1)$.



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  [KLM 2008] checked it for generic critical exponents, let's see what happens in degenerate cases.

The Green function

$$G(E,\mathbf{p},u)=\frac{u}{\epsilon}\frac{\mathrm{K}_\nu(|\mathbf{p}|u)}{\mathrm{K}_\nu(|\mathbf{p}|\epsilon)}\,,$$

where $\nu=\sqrt{1-E^2}$. Near the boundary $u=0$ the Green function can be expanded

$$G(E,p,u)=\left(\frac{u}{\epsilon}\right)^{1-\nu}\left(1+\left(\frac{|\mathbf{p}|u}{2}\right)^{2\nu}\frac{\Gamma(-\nu)}{\Gamma(\nu)}+\dots\right).$$

The boundary correlator in the momentum space is given by

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$$\langle \mathcal{O}(E, \mathbf{p}) \mathcal{O}(-E, -\mathbf{p}) \rangle = 2^{1-2\nu} \epsilon^{2\nu-2} \frac{\Gamma(-\nu)}{\Gamma(\nu)} |\mathbf{p}|^{2\nu} .$$

Performing Fourier transformation in spatial directions

$$\frac{1}{(2\pi)^{3/2}} \int \mathrm{e}^{i\mathbf{p}\mathbf{x}} \langle \mathcal{O}(E, \mathbf{p}) \mathcal{O}(0, \mathbf{0}) \rangle d^3p \sim \frac{1}{|\mathbf{x}|^{2\Delta}} ,$$

which properly reproduces the scaling behavior of $\mathcal{O}$.

Holography. Degenerate cases

However, there is a special configuration of scaling parameters for which the above constraint needs to be modified. Indeed, if $\xi = 0, \zeta = 1$ or $\xi = 1, \zeta = 0$ the equation on Δ will have the following form

$$\Delta(\Delta - 2) = E^2\,, \quad \text{or} \quad \Delta(\Delta - 1) = -p^2\,.$$

Solutions of these equations are

$$\Delta_\pm = 1 \pm \sqrt{1 - E^2}\,,$$

for the $AdS_2 \times \mathbb{R}^2$ model and

$$\Delta_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + 4p^2}\,,$$

for the Dubovsky model. We see here that scaling dimension becomes energy(momenta)-dependent. One can now observe that in the above two critical cases scaling dimensions have similar form to those in Lifshitz theory [KLM] but *in the massive case.* It appears that an *energy scale gets generated* when we go from noncritical cases to critical cases.

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Where does the Lifshitz background flow?

If flows into an AdS [Kachru, Liu, Mulligan].

Asymptotic IR behavior

In deep IR dual theory flows into conformal regime. One can also think of the following gravitational interpretation of this phenomena. It appears that many solutions in backgrounds of Lifshitz-type have matter distributions which are localized near the UV boundary. In these solutions the invariant energy density

$$\sqrt{-g(r)}\rho(r) \to 0 \quad \text{as} \quad r \to +\infty$$

vanishes as we approach the IR boundary leaving only cosmological constant Λ. Asymptotic behavior of the solution with negative Λ as a source at $r \to \infty$ is of AdS type.

Holographic flow

Backreaction on the metric

$$ds^2 = L^2 \left(-f^2(r)\frac{dt^2}{r^{2z}} + g^2(r)\frac{d\mathbf{x}^2}{r^2} + \frac{dr^2}{r^2} \right)$$

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Derive the functions f, g from Einstein equations. For nonzero z there two fixed points: UV fixed point–Lifshitz, IR fixed point – AdS space.

Holographic flow

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A deformation of the flow can also be done (Einstein-Proca theory) [Cheng, Hartnoll, Keeler].

Lifshitz from String Theory

Top-down approach
Naive way to proceed – try to construct a type IIA (IIB) solution by deforming existing ones (It works for Schrödinger backgrounds) fails – NO GO theorems. However, one may construct "spatial" Lifshitz [Takayanagi]

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Three different constructions [Hartnoll, Polchinski, Silverstein, Tong]

- from Landscape dual pairs
  compactifications of F theory on an elliptic fibration over a six-manifold of the form $Y_5 \times S^1$
- from brane polarization
- Baryon-induced
  Lifshitz in the IR, AdS_4 in the UV, Fermi sea in between.

All constructions are nonSUSY

Wilson loops

A new parameter z has appeared – interesting to study dependence of nonlocal observables

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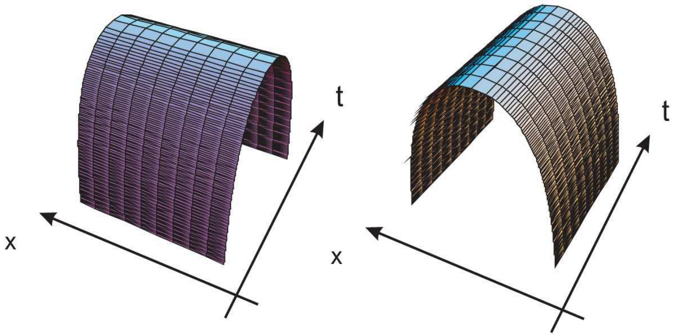


$$S = \int \sqrt{\det_{\alpha,\beta} \partial_\alpha X^A \partial_\beta X^B g_{AB}} d\sigma d\tau,$$

Wilson loops

For the surfaces we consider here the embedding function has the following form

$$X^A = \begin{pmatrix} \tau & \sigma & r(\tau,\sigma) \end{pmatrix} ,$$

where worldsheet coordinates τ, σ are trivially mapped onto (t, x) plane at some constant value of r, e.g. onto the UV boundary. The action reads

$$S = \int d\sigma d\tau \sqrt{r^{2\xi+2\zeta} + r^{2(\xi-1)} r'^2 + r^{2(\zeta-1)} \dot{r}^2} .$$

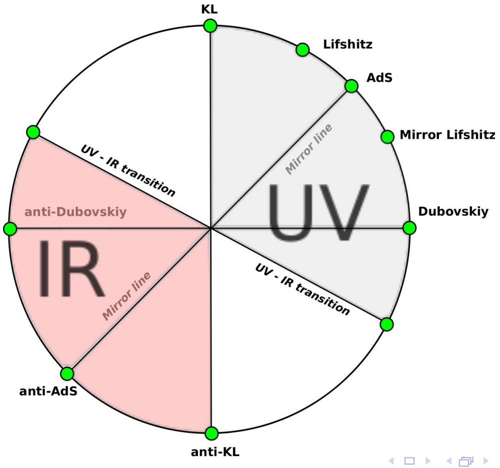
This action remains the same if we interchange

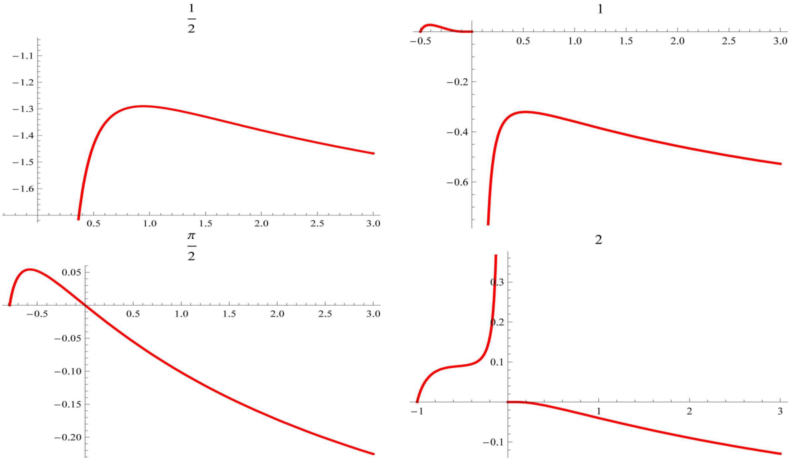
$$\xi \longleftrightarrow \zeta, \quad t \longleftrightarrow x$$

simultaneously. However, a different functional can be obtained if only one of the above transformations is performed. We shall refer to either of those transformations as mirror transformations and shall use them to obtain mirror solutions.

Rectangular Wilson loops

$$S_{\mathrm{reg}}^{\mathrm{UV}}(\xi,\zeta) = \frac{4T}{R^{\xi/\zeta}} \frac{\pi^{\frac{\xi+\zeta}{2\zeta}}}{2\xi+2\zeta} \frac{\Gamma\left(-\frac{\xi}{2\xi+2\zeta}\right)}{\Gamma\left(\frac{\zeta}{2\xi+2\zeta}\right)} \left[\frac{\Gamma\left(\frac{\xi+2\zeta}{2\xi+2\zeta}\right)}{\zeta\Gamma\left(\frac{\zeta}{2\xi+2\zeta}\right)}\right]^{\frac{\xi}{\zeta}}$$





$\frac{1}{2}$
−1.1
−1.2
−1.3
−1.4
−1.5
−1.6
0.5
1.0
1.5
2.0
2.5
3.0
1
−0.5
0.5
1.0
1.5
2.0
2.5
3.0
−0.2
−0.4
−0.6
$\frac{\pi}{2}$
0.05
−0.5
0.5
1.0
1.5
2.0
2.5
3.0
−0.05
−0.10
−0.15
−0.20
2
0.3
0.2
0.1
−1
1
2
3
−0.1

A note

Two critical exponents ξ and ζ. Absorb on of the exponents by redefining $w = r^{\zeta}$. The metric will be modified as follows

$$ds^2 = L^2 \left(-\frac{dt^2}{w^{2\xi/\zeta}} + \frac{d\mathbf{x}^2}{w^2} + \frac{1}{\zeta^2}\frac{dw^2}{w^2} \right) \, .$$

We now need to rescale

$$L \to \zeta L \quad t \to \zeta^{-1} t \, , \quad \mathbf{x} \to \zeta^{-1}\mathbf{x} \, ,$$

and the metric takes the standard Lifshitz form

$$ds^2 = L^2 \left(-\frac{dt^2}{w^{2z}} + \frac{d\mathbf{x}^2}{w^2} + \frac{dw^2}{w^2} \right) \, ,$$

where $z = \xi/\zeta$.

Conclusions

- Further (supersymmetric) embedding of Lifshitz into string theory
- Construction of holography dual gauge theories
- What properties does a theory (either gauge or string side) have to have in order to have a holography dual description?
- CMT applications