Lorentz Invariance Violation and Extra Dimensions

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In collaboration with M. Libanov, I. Gordeli

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Outline

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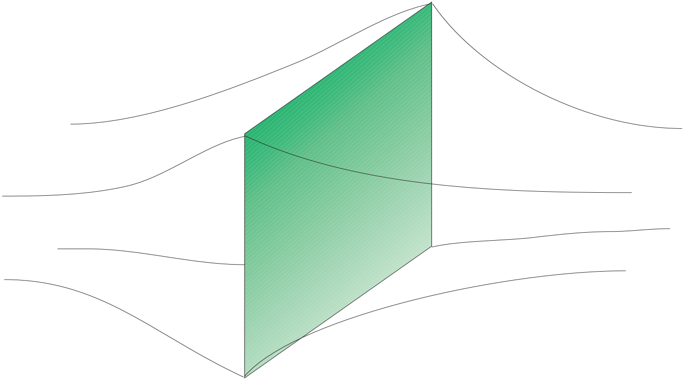
$$\int dt d^d x \sqrt{g} \left((\partial_0 \phi)^2 - (\Delta \phi)^2 - \epsilon (\partial_i \phi)^2 \right)$$

becomes Lorentz invariant in IR

Lorentz Invariant case

$\mathbb{Z}_2$ symmetric AdS_5 orbifold with brane of constant tension



$$\Lambda = -\frac{\sigma^2}{6} M^3, \quad k = \frac{\sigma}{6}$$

[Randall Sundrum]

Example of Macroscopic Solution

Metric [PK Libanov]

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$$T^A_B = u^A u_B(p + \rho) - p\delta^A_B + \Lambda, \quad u_A u^A = 1$$

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$$\rho = -\Lambda + 6k^2\zeta^2$$

$$w = -1 + \frac{3\zeta^2 - 2\zeta\xi - \xi^2}{\rho}$$

$$\omega = -1 + \frac{3\zeta(\zeta - \xi)}{\rho}$$

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But brane NEC implies $\zeta - \xi < 0$

Microscopic Solution

2+1 Lifshitz model [Kachru Liu Mulligan]

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Solution of Einstein equations $Z = \zeta/\xi$

$$\Lambda = -\frac{Z^2 + Z + 4}{2L^2}$$
$$A^2 = \frac{2Z(Z-1)}{L^2}$$
$$B^2 = \frac{4(Z-1)}{L^2}$$

To avoid tachyonic solutions $Z > 1$

Correspondence

[PK Gordeli]

	KL	KLM
Cosmological constant	$\Lambda = -\rho - 3k^2$	$\Lambda = -L^{-2}(Z^2 + Z + 4)$
First component	w	$-1 + \dfrac{A^2 + B^2}{2(-\Lambda - 3L^{-2})}$
Second component	ω	$-1 + \dfrac{B^2}{2(-\Lambda - 3L^{-2})}$
LIV parameter	$w - \omega$	A
Anisotropy	$p_1 - p_4$	Energy flux A^2 w.r.t z direction
Constraints	Reality of Fluxes	Null energy condition

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!! Recall that in these examples bulk NEC is inconsistent with brane NEC !!

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The statement does not depend on the volume $\int_{-\infty}^{+\infty} \sqrt{g}\, dz$ of the extra dimension

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  But one needs to have $\kappa/k \sim 1$ *which makes the model useless phenomenologically*

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- Obtain these solutions from String Theory
- So what is the dynamics in these backgrounds?

Spectra of Perturbations

Localization vs Delocalization

Metric

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Which means $a' < b'$ – localization and $a' > b'$ – delocalization

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potential

$$V = -\frac{1}{4} a'^2 + \frac{9}{4} b'^2 + p^2 \mathrm{e}^{2(b-a)} - \frac{1}{2}(a'' + 3b'')$$

(De)localization – Features

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- The sign of the delta-function term. The potential may have either delta-well or delta-peak depending on this sign.

(De)localization – Features

- If the momenta-dependent term increases as $y \to \infty$, then one has a discrete spectrum as in the box-type potential. The potential might have local minima and maxima but the behavior of this potential at infinity qualitatively defines the character of the spectrum. On the contrary, if the potential decays at infinity, then we have continuous spectrum of plane waves propagating along y-direction. Some combination of these two scenarios is possible when $V \to V_\infty = const$ as $z \to \infty$. Then those modes with $E^2 < V_\infty$ belong to discrete spectrum and modes with $E^2 > V_\infty$ contribute to continuous spectrum.

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- The sign of delta-function term affects zero mode existence. In a delta-well there might be a zero-mode and none in a delta-peak.

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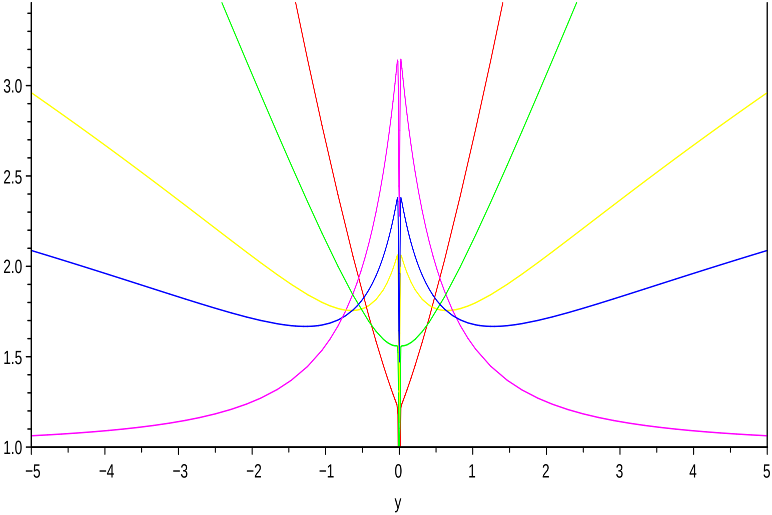
Potentials



Space of Metrics

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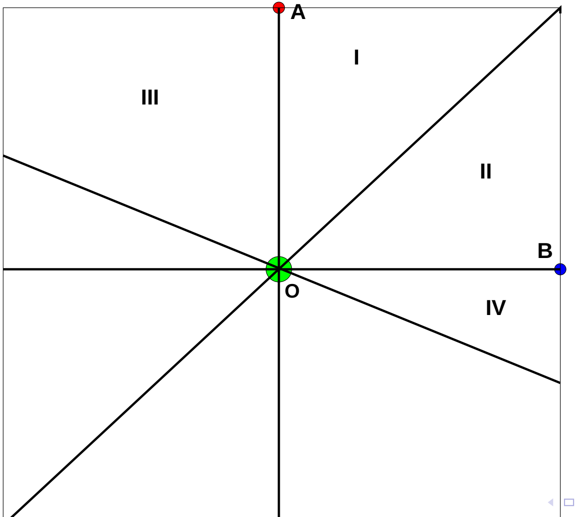

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Generic solution

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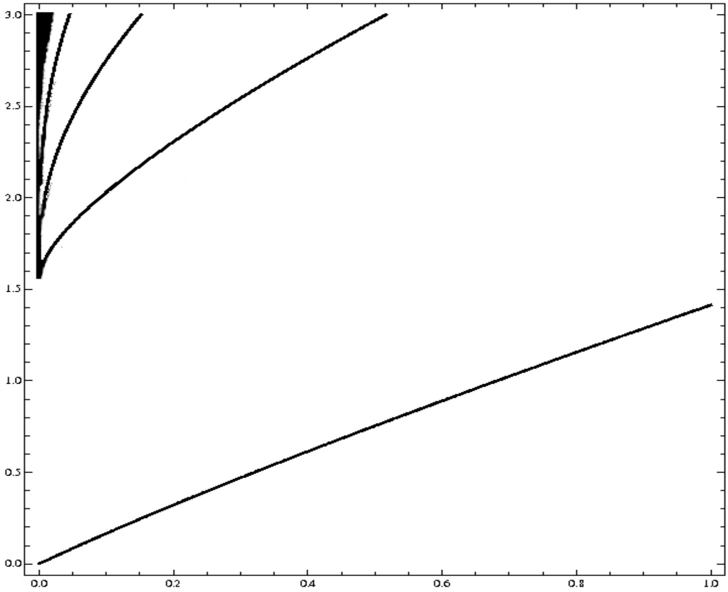
Matching

$$\frac{p}{k} \frac{\mathrm{K}_{\nu+1}\left(\frac{p}{k}\right)}{\mathrm{K}_\nu\left(\frac{p}{k}\right)} = \frac{3}{2} + \nu$$

Spectrum of Model A



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- large momenta – everything $E = p + \dots$

Static Potential

Brane-to-Bulk Propagator

$$\left[-\partial_z^2 - E^2 + p^2 e^{2k|z|} + \frac{9}{4}k^2 - 3k\delta(z)\right] \Delta_p(E, p, z) = \delta(z)$$

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Brane static potential

$$G(r) = \frac{k}{4\pi r}\left(1 + \frac{2}{\pi k r}\right)$$

Fermions

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Action

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In rescaled variables

$$\left(E\gamma^0 - \mathrm{e}^{k|z|}\gamma^i p_i - \gamma^5 \partial_z \right) \psi - m_\psi \psi = 0$$

Fermions

Action

$$S = \int dtd\mathbf{x} \int_{-\infty}^{+\infty} dz \sqrt{g} \left(i\bar{\Psi}\not{\nabla}\Psi + m_\psi \bar{\Psi}\Psi \right)$$

Dirac equation

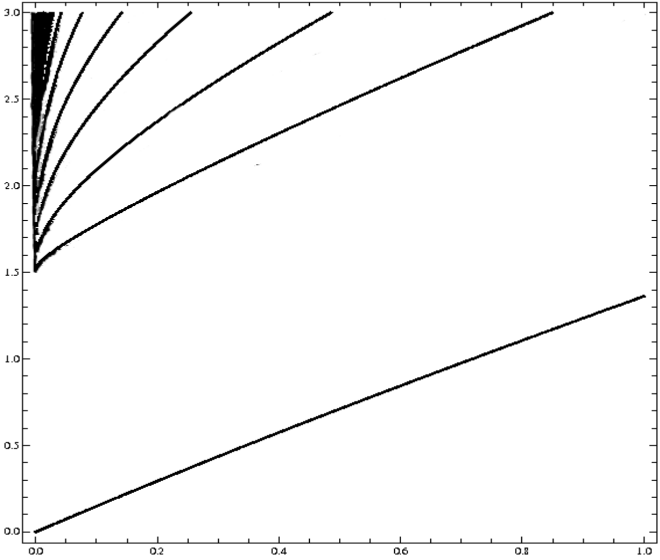
$$i\Gamma^A \nabla_A \Psi(x,z) + m_\psi \Psi(x,z) = 0, \quad m_\psi = m\,\mathrm{sign}(z)$$

In rescaled variables

$$\left(E\gamma^0 - \mathrm{e}^{k|z|}\gamma^i p_i - \gamma^5 \partial_z \right) \psi - m_\psi \psi = 0$$

Using special rep os γ-matrices

$$E\chi - ip\sigma_3 \phi \mathrm{e}^{k|z|} + \sigma_1 \phi' - m_\psi \phi = 0$$
$$E\phi + ip\sigma_3 \chi \mathrm{e}^{k|z|} - \sigma_1 \chi' - m_\psi \chi = 0$$



3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Dispersion Relations

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- At $0 < m < k/2$

$$E \simeq 2m \frac{\Gamma\left(\frac{1}{2}-\frac{m}{k}\right)}{\Gamma\left(\frac{1}{2}+\frac{m}{k}\right)} \left(\frac{p}{2k}\right)^{\frac{2m}{k}}$$

Higher modes

Higher modes at low momenta

$$E_n = m\sqrt{1 + \left(\frac{\pi n k}{k - 2\Psi\left(\frac{1}{2}\right) m + 2m \log \frac{p}{2k}}\right)^2}$$

The case $m = 0$ is very neat

$$E_n = -\frac{\pi k(2n+1)}{4 \log \frac{p}{2k}} + \mathcal{O}(p), \quad n \in \mathbb{N}$$

All modes at high momenta

$$E = p + \mathcal{O}(p^{1/3})$$

Chirality

Left and Right fermions $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$

$$\psi^{>}_{L,R} = \left(\frac{\xi_+(t\mathrm{e}^{k|z|})}{\xi_+(t)} \pm \gamma \frac{\xi_-(t\mathrm{e}^{k|z|})}{\xi_-(t)} \right) \frac{1}{2} \sum_{\alpha} C_\alpha (U_{\alpha,+} \pm U_{\alpha,-})$$

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Depending on α the modes have different helicities

Outlook

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