Non-Abelian Vortices and 4d/2d Correspondence

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1009.6207 1107.3779 work in progress

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Outline

- 4d/2d w/ 8 supercharges: what and why?

★ *Vortices in field theory vs. type IIA string theory*

★ *(2,2) GLSM, NLSM*

★ *The Dictionary of 4d/2d*

★ *Aspects of perturbation theory*

- Less Supersymmetry (4 supercharges)

★ *Heterotic deformation and Large-N solution - beyond BPS sector*

★ *NSVZ in (0,2) theories*

★ *Omega background* (bonus)

4d/2d

‘ANO’ String

$U(N)$ gauge theory with fundamental matter $q \to UqV \qquad U \in U(N)_G, \quad V \in SU(N)_F$

$N_f = N_c$

$$S = \int d^4x \ \mathrm{Tr}\left(\frac{1}{2e^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2\right) + \sum_{i=1}^{N_f}|\mathcal{D}_\mu q_i|^2$$
$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{4}\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\, 1_N)^2$$

Vacuum

$$\phi = 0 \quad , \quad q^a{}_i = v\delta^a{}_i$$

breaks symmetry

(color-flavor locking)

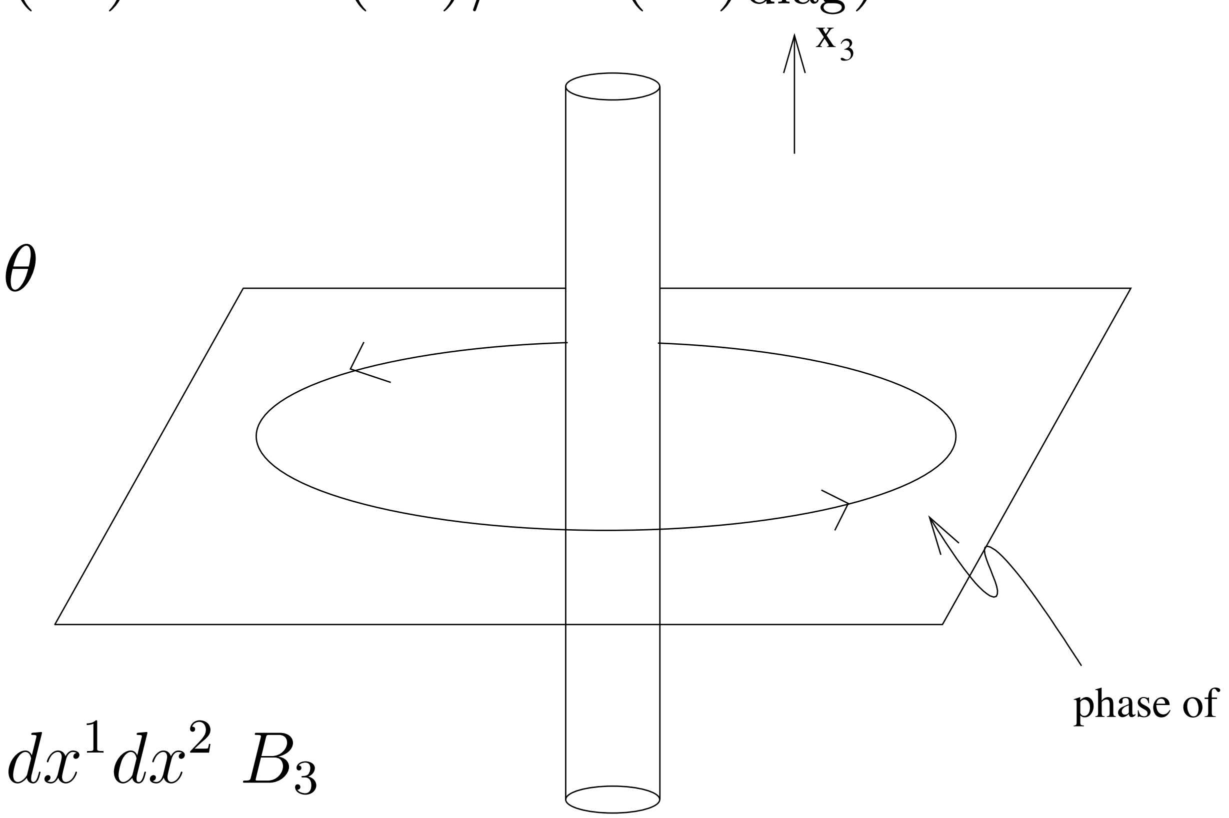
$$U(N)_G \times SU(N)_F \to SU(N)_{\rm diag}$$

Induces nontrivial topology on moduli space

$$\Pi_1\left(U(N) \times SU(N)/SU(N)_{\rm diag}\right) \cong \mathbf{Z}$$

To find a string need winding at infinity

$$q_N \sim q\,\mathrm{e}^{ik\theta}$$
$$A_\theta \sim \frac{k}{\rho}$$



$$2\pi k = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} i\partial_\theta q\ q^{-1} = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} A_\theta = \mathrm{Tr}\int dx^1 dx^2\ B_3$$

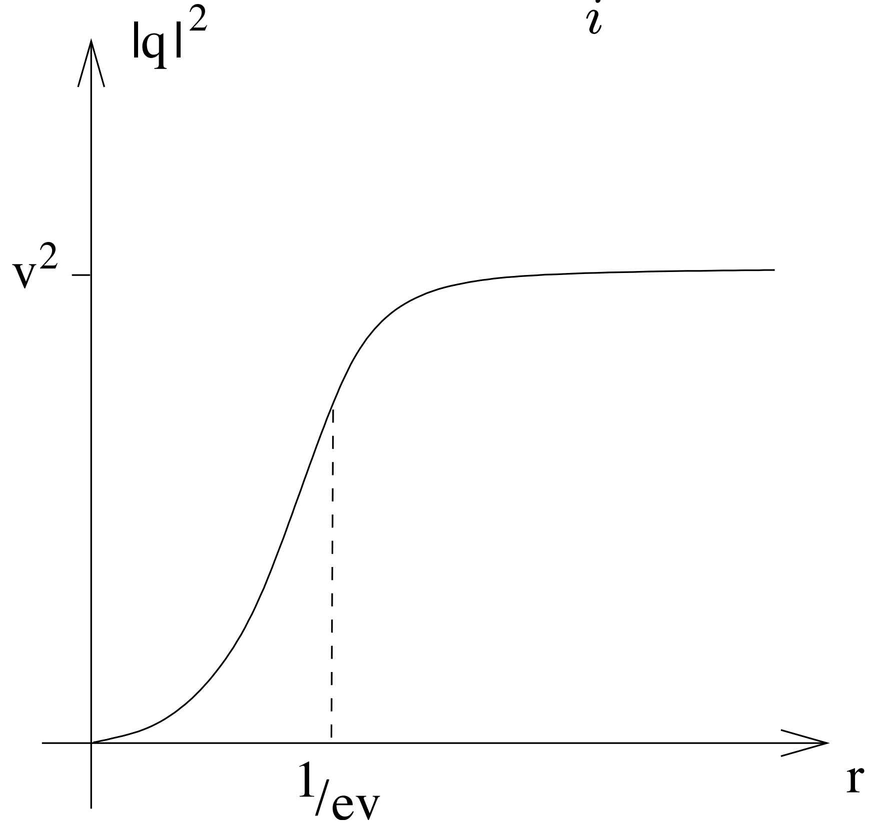
BPS equations for vortex

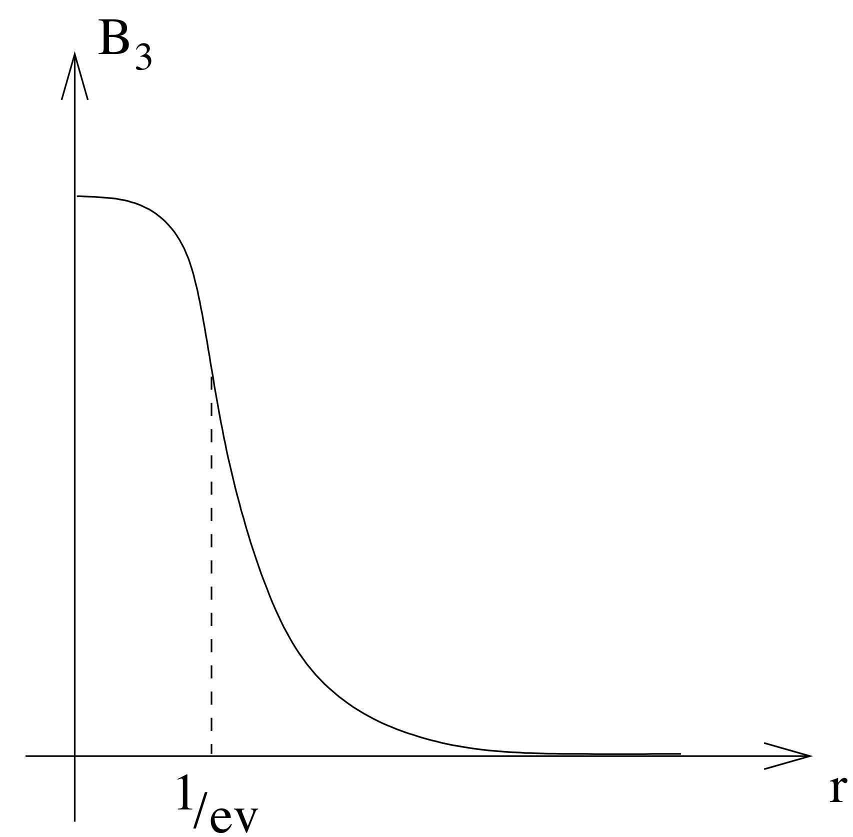
$$T_{\text{vortex}} = \int dx^1 dx^2 \ \text{Tr} \left(\frac{1}{e^2} B_3^2 + \frac{e^2}{4} (\sum_{i=1}^{N} q_i q_i^\dagger - v^2 \, 1_N)^2 \right) + \sum_{i=1}^{N} |\mathcal{D}_1 q_i|^2 + |\mathcal{D}_2 q_i|^2$$

$$= \int dx^1 dx^2 \ \frac{1}{e^2} \text{Tr} \left(B_3 \mp \frac{e^2}{2} (\sum_{i=1}^{N} q_i q_i^\dagger - v^2 \, 1_N) \right)^2 + \sum_{i=1}^{N} |\mathcal{D}_1 q_i \mp i \mathcal{D}_2 q_i|^2$$

$$\mp v^2 \int dx^1 dx^2 \ \text{Tr} \, B_3 \ \geq \mp v^2 \int d^2 x \text{Tr} B_3 \ = 2\pi v^2 |k| \qquad ($$

gives $B_3 = \frac{e^2}{2} (\sum_i q_i q_i^\dagger - v^2 \, 1_N)$ $\qquad (\mathcal{D}_x - i \mathcal{D}_y) q_i = 0$





Vortices

Simple vortex w/ N=1, k=1 (ANO) has two collective coordinates-translations in x,y directions

U(N) vortex has more moduli

$$A_z = \begin{pmatrix} A_z^\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \quad , \qquad q = \begin{pmatrix} q^\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Moduli space (k=1)

$$SU(N)_{\rm diag}/S[U(N-1)\times U(1)] \cong \mathbb{CP}^{N-1}$$

$$\mathcal{V}_{1,N} \cong \mathbf{C} \times \mathbb{CP}^{N-1}$$

For higher k

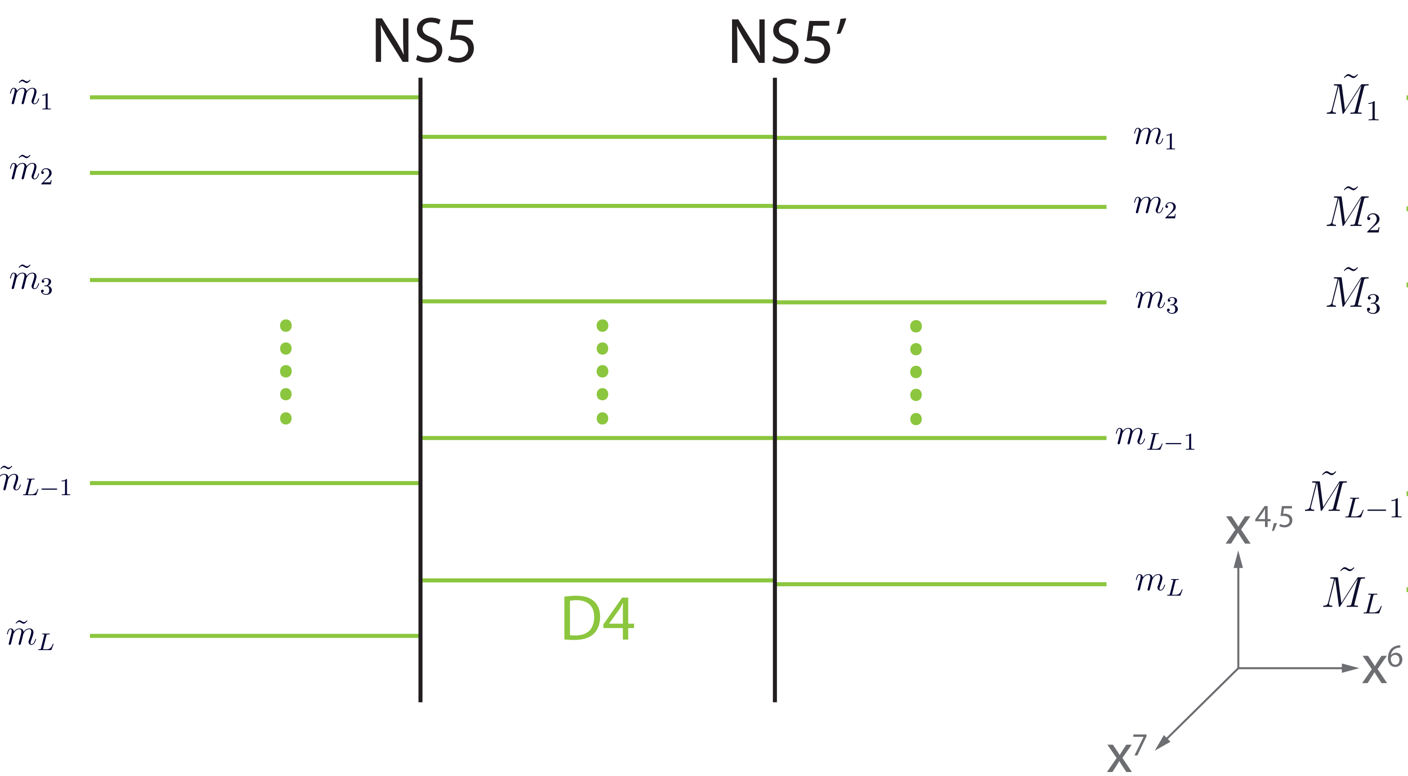
$$\dim(\mathcal{V}_{k,N}) = 2kN$$

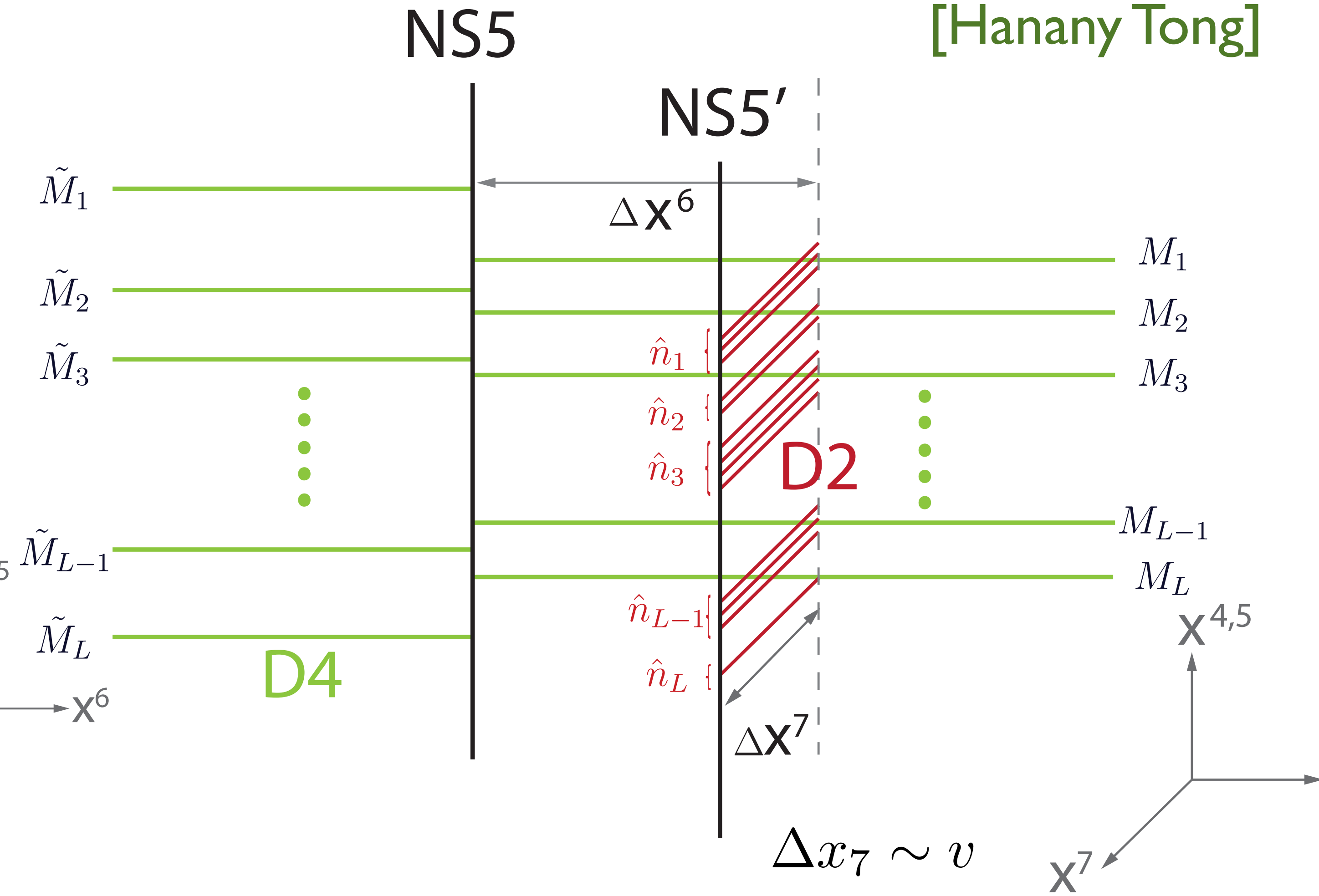
Again:

$T \geq 2\pi v^2 |k|$ bound saturates for BPS states

Hanany-Witten construction

[Witten]
[Hanany Tong]





SQCD $N_f = 2N_c$

2d FI parameter $r = \dfrac{\Delta x^6}{2\pi g_s l_s} = \dfrac{4\pi}{e^2}$

Higgs branch root

$$\sigma = X^4 + iX^5 \quad , \quad Z = X^1 + iX^2$$

Color-flavor locked phase of SQCD

$$V = \frac{1}{g^2}\mathrm{Tr}\,|[\sigma,\sigma^\dagger]|^2 + \mathrm{Tr}\,|[\sigma,Z]|^2 + \mathrm{Tr}\,|[\sigma,Z^\dagger]|^2 + \sum_{a=1}^{N}\psi_a^\dagger\sigma^\dagger\sigma\psi_a$$
$$+\frac{g^2}{2}\mathrm{Tr}\left(\sum_a \psi_a\psi_a^\dagger + [Z,Z^\dagger] - r\,1_k\right)^2$$

@Dorey

4d / 2d duality

[Dorey Hollowood, Tong]

$\mathcal{N}=2 \quad SU(N)$ SQCD	(2,2) $U(1)$ GLSM e
$N_f = N + \tilde{N}$ fund hypers w/ masses	N chiral +1 $\tilde{N}$ chiral -1 w/ *twisted* masses
$m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$	$m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$
$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$	$\tau = ir + \frac{\theta}{2\pi}$
on baryonic Higgs branch	vortex moduli space
BPS dyons (Seiberg-Witten)	kinks interpolating between different vacua

BPS spectra (as functions of masses, Lambda) are the same

Goal: understand it from field theory constructions

$U(N_c)$ $\mathcal{N}=2$ $d=4$ SQCD w/ N_f quarks

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\beta}\} = 2\delta^{IJ} P_{\alpha\dot\beta} + 2\delta^{IJ} Z_{\alpha\dot\beta}$$

$$\{Q^I_\alpha, Q^J_\beta\} = 2Z^{IJ}_{\alpha\beta}$$

strings

monopoles domain walls

$$\mathcal{L} = \mathrm{Im}\left[\tau \int d^4\theta \,\mathrm{Tr}\left(Q^{i\,\dagger} e^V Q_i + \tilde{Q}^{i\,\dagger} e^V \tilde{Q}_i + \Phi^\dagger e^V \Phi\right)\right]$$
$$+\mathrm{Im}\left[\tau \int d^2\theta \left(\mathrm{Tr} W^{\alpha\,2} + m^i_j \tilde{Q}_i Q^j + Q_i \Phi \tilde{Q}^i\right)\right]$$

bosonic part

FI term

$$S = \int d^4x \,\mathrm{Tr}\left\{\frac{1}{2g^2} F^2_{\mu\nu} + \frac{1}{g^2}|D_\mu \Phi|^2 + |\nabla_\mu Q|^2 + \frac{g^2}{4}(Q\bar{Q} - \xi)^2 + |\Phi Q + QM|^2\right\}$$

BPS conditions

$$B_3 - g^2(Q\bar{Q} - \xi^2) = 0$$
$$\nabla_3 Q = 0$$

String tension

$$T = \xi \int d^2x \,\mathrm{Tr} F_{12} = 2\pi\xi n$$

Non-Abelian String

[Auzzi, Bolognesi, Evslin, Konishi, Yung]

[Shifman Yung]

$$\varphi = U \begin{pmatrix} \phi_2(r) & 0 & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \ldots & \phi_2(r) & 0 \\ 0 & 0 & \ldots & \phi_1(r) \end{pmatrix} U^{-1} ,$$

Take Abelian string solution

Make global rotation

$$A_i^{\mathrm{SU}(N)} = \frac{1}{N} U \begin{pmatrix} 1 & \ldots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \ldots & 1 & 0 \\ 0 & 0 & \ldots & -(N-1) \end{pmatrix} U^{-1} \, (\partial_i \alpha) \, f_{NA}(r)$$

$$A_i^{\mathrm{U}(1)} = -\frac{1}{N} \, (\partial_i \alpha) \, f(r) \, , \qquad A_0^{\mathrm{U}(1)} = A_0^{\mathrm{SU}(N)} = 0 \, ,$$

Matrix U parameterizes orientational modes

Gauge group is broken to $\mathbb{Z}_N$

All bulk degrees of freedom massive $M^2 = e^2 v^2$

Theory is fully Higgsed

Vortex moduli space

Nf=Nc color-flavor locked phase
single SUSY vacuum

$$U(N_c) \times SU(N_f) \to SU(N)$$

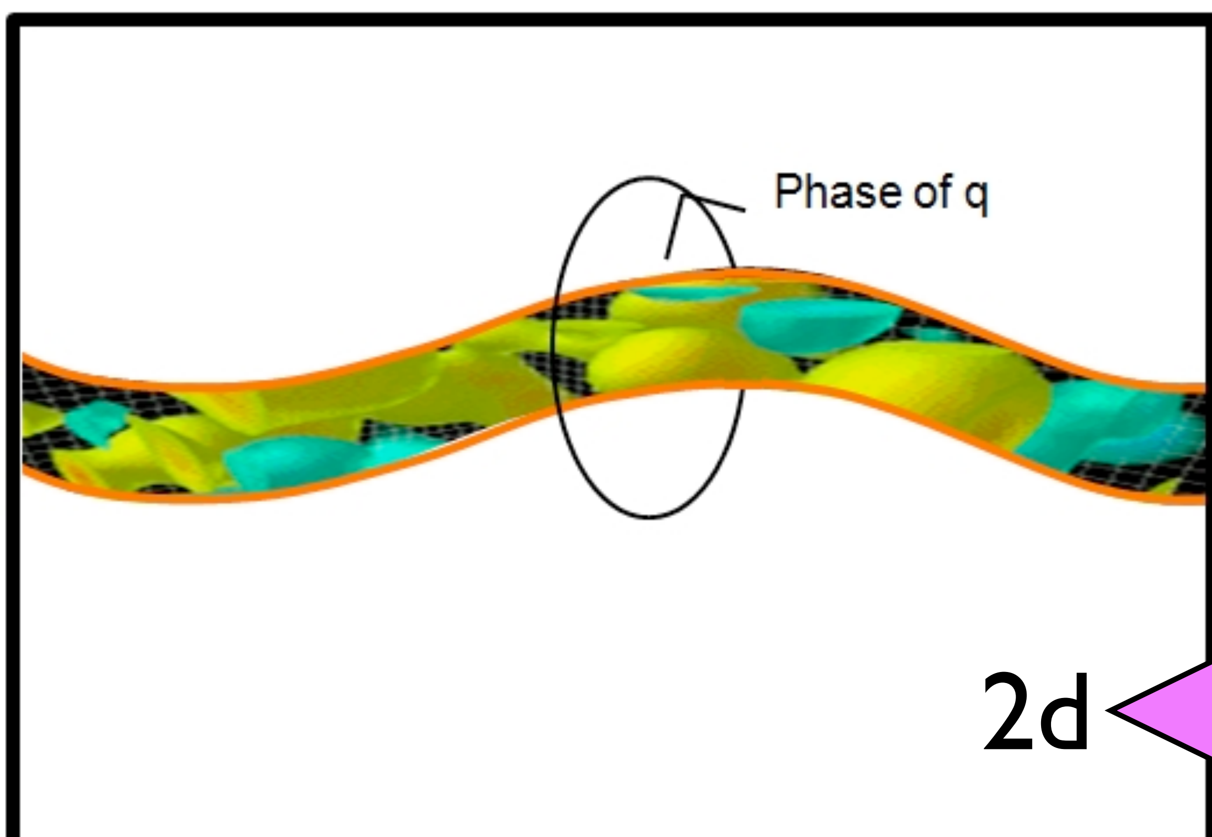
Nf=Nc local vortex

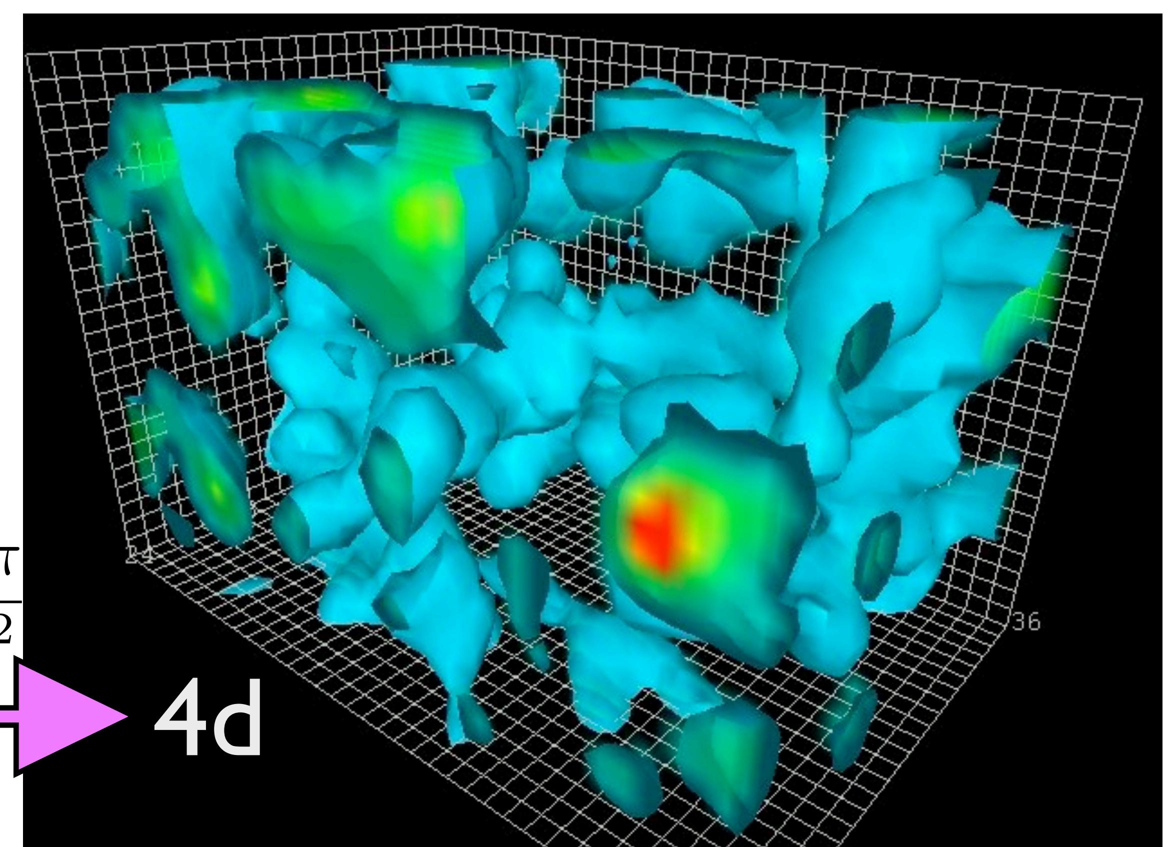
$$\frac{SU(N)}{SU(N-1) \times U(1)} = \mathbb{CP}^{N-1}$$

Nf>Nc semilocal
(+size moduli)

$$\pi_2(\mathcal{M}_{vac}) = \pi_2\left(\frac{SU(N+\tilde{N})}{SU(N) \times SU(\tilde{N}) \times U(1)}\right) = \mathbb{Z}$$

Duality between two strongly coupled theories





2d ⟷ 4d

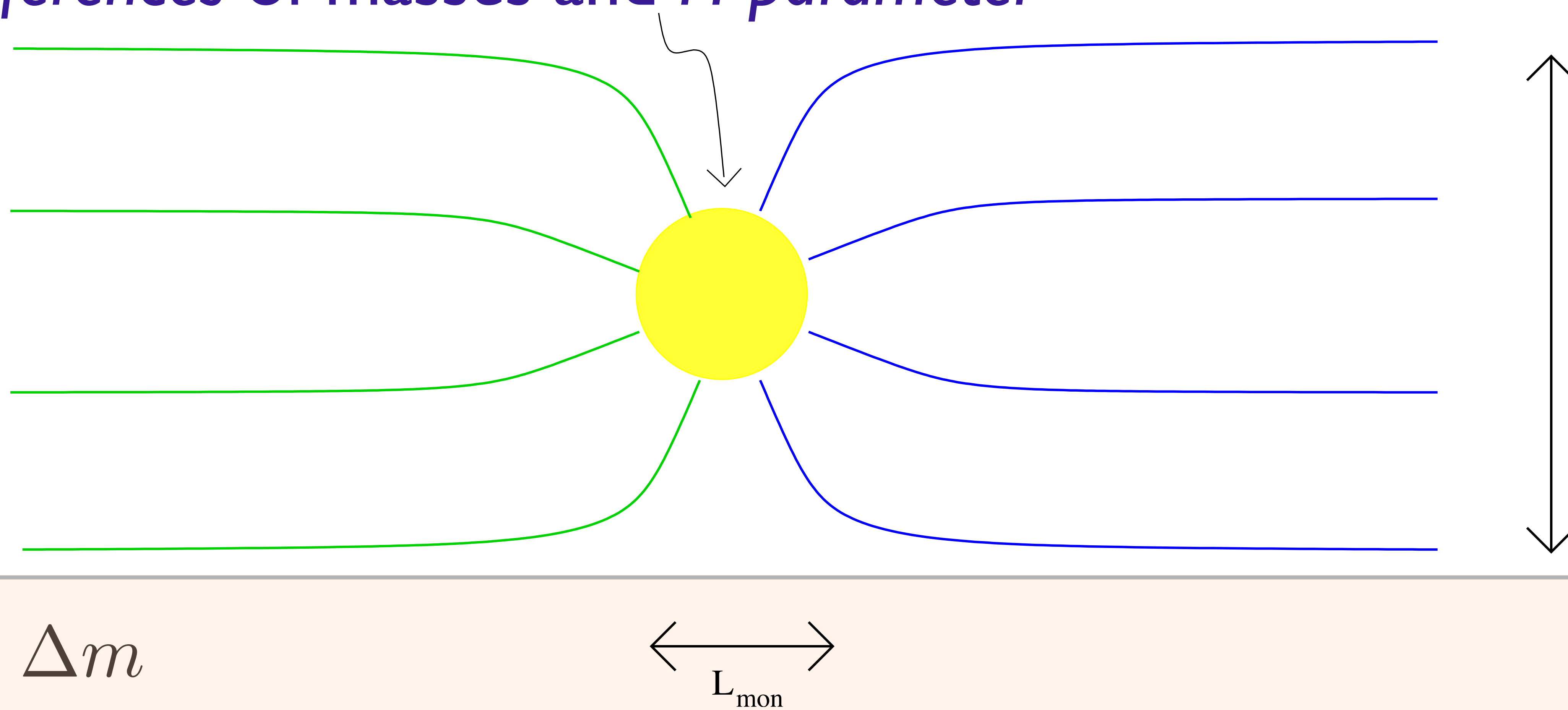
$$r = \frac{4\pi}{g^2}$$

Monopoles in Higgs Phase

[Shifman, Yung]
[Tong]

Add masses. New vacuum $\phi = \mathrm{diag}(m_i) \quad , \quad q^a{}_i = v\delta^a{}_i \quad , \quad \tilde{q}^a{}_i = 0$

Pattern of symmetry breaking depends on the relationship between the *differences* of masses and *FI parameter*



$$ev \gg \Delta m$$

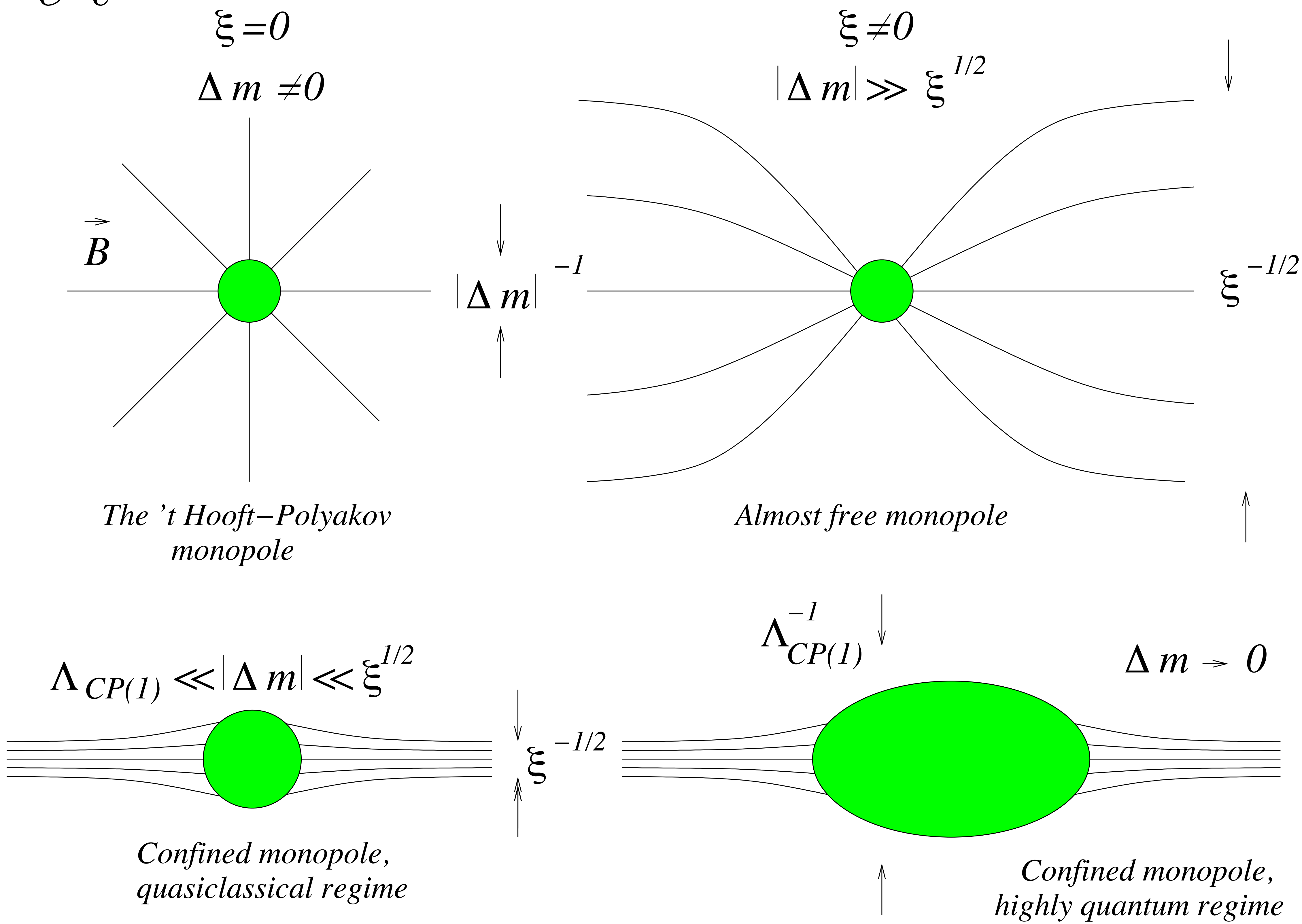
$$U(N)_G \times SU(N)_F \xrightarrow{v} SU(N)_{\mathrm{diag}} \xrightarrow{m} U(1)^{N-1}_{\mathrm{diag}}$$

$$ev \ll \Delta m$$

$$U(N)_G \times SU(N)_F \xrightarrow{m} U(1)^N_G \times U(1)^{N-1}_F \xrightarrow{v} U(1)^{N-1}_{\mathrm{diag}}$$

Confined monopoles

$\xi = e^2 v^2$



$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term r

BPS dyons in 4d N=2

$$Z = \sum_{a=1}^{N_c} \phi_a (j_a + \tau h_a) + \sum_{i=1}^{N_f} m_i s_i$$

Central charge

$$Z = \sum_{i=1}^{N_c} m_i (S_i + \tau h_i)$$

At baryonic root of Higgs branch

$$F(t,u) = \left(t - \prod_{i=1}^{N_c} (u - m_i) \right) (u - \Lambda^{N_c})$$

SW curve degenerates
has Nc branching pts

$$Z = \sum_{i=1}^{N_c} (m_i S_i + m_{Di} h_i)$$

All quantum corrections in mD

$$m_{Dl} - m_{Dk} = \frac{1}{2\pi} N_c (e_l - e_k) + \frac{1}{2\pi} \sum_{i=1}^{N_c} m_i \log \left(\frac{e_l - m_i}{e_k - m_i} \right)$$

(2,2) 2d GLSM

[Witten]

Consider U(1) gauge theory

$$\mathcal{L}_{\text{vortex}} = \frac{1}{2g^2}\left(F_{01}^2 + |\partial\sigma|^2\right) + \sum_{i=1}^{N_c}\left(|\mathcal{D}\psi_i|^2 + |\sigma - m_i|^2|\psi_i|^2\right) + \frac{g^2}{2}\Big(\sum_{i=1}^{N_c}|\psi_i|^2 - r\Big)^2$$

$$\text{Vacuum } i: \quad \sigma = m_i \quad , \quad |\psi_j|^2 = r\delta_{ij}$$

for vortex embedded into i's U(1) subgroup

FI term runs

$$r(\mu) = r_0 - \frac{N_c}{2\pi}\log\left(\frac{M_{UV}}{\mu}\right) \Rightarrow \Lambda = \mu\exp\left(-\frac{2\pi r(\mu)}{N_c}\right)$$

Effective twisted superpotential

$$\mathcal{W}(\Sigma) = \frac{i}{2}\tau\Sigma - \frac{1}{4\pi}\sum_{i=1}^{N_c}(\Sigma - m_i)\log\left(\frac{2}{\mu}(\Sigma - m_i)\right)$$

Vacua $\exp\dfrac{\partial\widetilde{\mathcal{W}}}{\partial\sigma} = 1$

Central charge $Z = -i\sum_{i=1}^{N_c}(m_i S_i + m_{D\,i}T_i)$

$$m_{D\,i} = -2i\mathcal{W}(e_i) = \frac{1}{2\pi i}N_c e_i + \frac{1}{2\pi i}\sum_{j=1}^{N_c} m_j\log\left(\frac{e_i - m_j}{\Lambda}\right)$$

Hanany-Tong model as U(1) GLSM

$$\mathcal{L} = \int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, \mathrm{e}^{\mathcal{V}} \Phi_i + \sum_{i=1}^{\tilde{N}} \widetilde{\Phi}_i^\dagger \, \mathrm{e}^{-\mathcal{V}} \widetilde{\Phi}_i - r\mathcal{V} + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

$$V = \theta^+\bar{\theta}^+(A_0 + A_3) + \theta^-\bar{\theta}^-(A_0 - A_3) - \theta^-\bar{\theta}^+\sigma - \theta^-\bar{\theta}^+\bar{\sigma} + \bar{\theta}^2\theta\lambda + \theta^2\bar{\theta}\bar{\lambda} + \bar{\theta}\theta\bar{\theta}\theta D$$

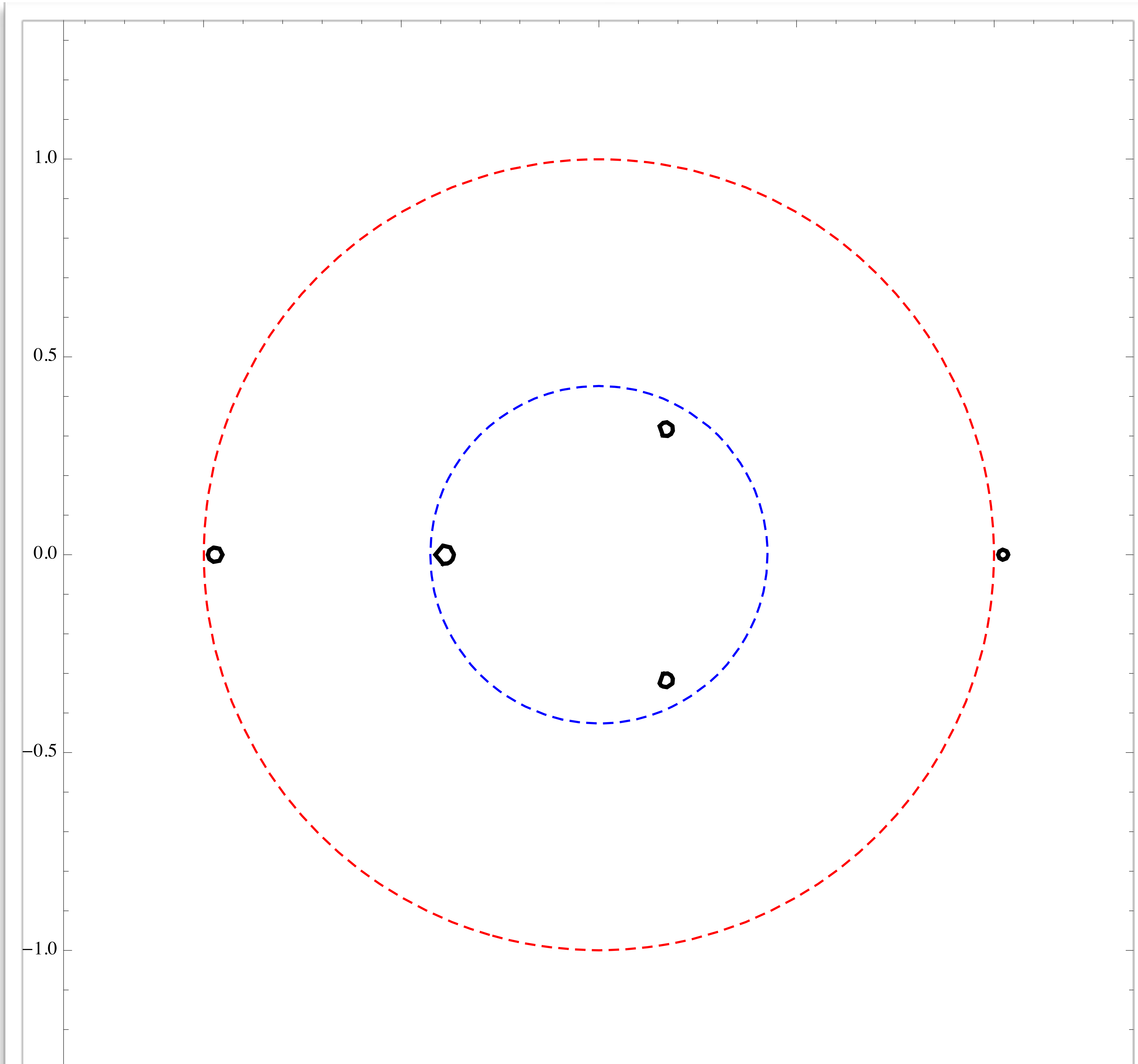
One loop twisted effective superpotential is exact in (2,2)

$$\begin{aligned} \widetilde{W}_{\text{eff}} &= -\frac{1}{2\pi} \sum_{i=1}^{N} (\sqrt{2}\sigma + m_i) \left(\log \frac{\sqrt{2}\sigma + m_i}{\Lambda} - 1 \right) + \\ &+ \frac{1}{2\pi} \sum_{j=1}^{\widetilde{N}} (\sqrt{2}\sigma + \widetilde{m}_j) \left(\log \frac{\sqrt{2}\sigma + \widetilde{m}_j}{\Lambda} - 1 \right) . \end{aligned}$$

gives vacua of the theory and its BPS spectrum !!

[PK Monin Vinci]

N=5 Nf=8



Derivation of 2d theory from 4d theory

From GLSM

$$\mathcal{L} = \int d^4\theta \left(\left(|X_1|^2 + |X_2|^2\right) e^V - rV + \frac{1}{e^2}|\Sigma|^2 \right)$$

Take limit $e \to \infty$ solve for V

Kahler potential $K = r \log(1 + |X|^2)$ $X = X_2/X_1$

For HT model

$$\mathcal{L}_{\rm HT} = \int d^4\theta \, \left(|\mathcal{N}_i|^2 {\rm e}^V + |\mathcal{Z}_j|^2 {\rm e}^{-V} - rV\right)$$

Limit $e \to \infty$ defines vacuum manifold

$$\mathcal{O}(-1)^{\tilde{N}} \longrightarrow \mathbb{CP}^{N-1}$$

Kahler potential

$$K_{\rm HT} = \sqrt{r^2 + 4r|\zeta|^2} - r \log\left(r + \sqrt{r^2 + 4r|\zeta|^2}\right) + r \log(1 + |\Phi_i|^2)$$

$$|\zeta|^2 \equiv |\mathfrak{z}_j|^2 (1 + |\Phi_i|^2) \qquad \mathfrak{z}_j = r^{-1/2} \mathcal{N}_N \mathcal{Z}_j \,, \quad j = 1, \ldots, \widetilde{N}$$

Let’s see what is the metric on the vortex sigma model

From 4d theory

[Shifman Vinci Yung]

$$S = \int d^4x \, \mathrm{Tr} \left\{ \frac{1}{g^2} \left(F_{12} + \frac{g^2}{2}(Q\bar{Q} - \xi) \right)^2 + \right.$$
$$+ \; |\nabla_1 Q + i\nabla_2 Q|^2 + |\Phi Q + QM|^2 + \xi \, F_{12} +$$
$$\left. + \; \frac{1}{g^2}(F_{ik})^2 + (\nabla_k Q)^* (\nabla_k Q) + \frac{1}{g^2}(F_{kl})^2 \right\},$$

String tension

$$T = \xi \int d^2x \, \mathrm{Tr} \, F_{12} = 2\pi\xi \, n$$

Ansatz

$$Q_0 = \left(\begin{array}{ccc|c} \phi_1(r) & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \phi_2(r) & \phi_3(r) \end{array} \right) \qquad A_{0,i} = \epsilon_{ij} \frac{x_j}{r^2} f(r) \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}$$

After making color-flavor rotation

$$Q = \Big(\phi_1(r) - n\, n^* (\phi_1(r) - \phi_2(r)) \,|\, n\, \phi_3(r) \Big)$$

$$A_i = n\, n^* \, \epsilon_{ij} \frac{x_j}{r^2} f(r) \,,$$

Bogomol'ny equations [Shifman Vinci Yung]

$$\nabla_1 Q + i\nabla_2 Q = 0\,,$$

$$F_{12} + \frac{g^2}{2}(Q\bar{Q} - \xi) = 0\,.$$

Setting

$$\phi_1(r) = \sqrt{\xi}, \quad \phi_3 = \frac{\rho}{r}\,\phi_2$$

size modulus

can solve the rest of equations analytically provided that

$$\frac{1}{g\sqrt{\xi}|\rho|} << 1$$

e.g. gauge field

$$A_k = -i\Big(\partial_k n\, n^* - n\,\partial_k n^* - 2n\,n^*(n^*\partial_k n)\Big)\omega(r)$$

$$-i\,n\,n^*\Big(\rho^*\partial_k\rho - \rho\partial_k\rho^* + 2|\rho|^2(n^*\partial_k n)\Big)\gamma(r)$$

determine $\gamma(r)$ and $\omega(r)$

after some work [Shifman Vinci Yung] we get...

Effective action on semilocal vortex

Radial integral diverges due to power like behavior

$$\mathcal{L}_{\rm eff} = \pi\xi\left(\ln\frac{L^2}{|\rho|^2}\right)\left|\partial_k(\rho\, n)\right|^2 - \pi\xi|\partial_k\rho + \rho\,(n^*\partial_k n)|^2$$
$$+ \frac{2\pi}{g^2}\left[\partial_k n^*\partial_k n + (\partial_k n^* n)^2\right].$$

already includes subleading corrections

for large L can insert Log under derivative

$L \sim |\Delta m|^{-1}$

In addition we have size moduli

$$z = \rho\left[2\pi\xi\,\ln\frac{L}{|\rho|}\right]^{1/2}$$

Arrive to a new model (ZN) with Kahler potential

$$K_{zn} = r|\zeta|^2 + r\log(1+|\Phi_i|^2)$$

$$|\zeta|^2 \equiv |\mathfrak{z}_j|^2(1+|\Phi_i|^2)$$

$$\Phi_i = \frac{\mathcal{N}_i}{\mathcal{N}_N}, \quad i = 1,\ldots,N-1,$$

$$\mathfrak{z}_j = r^{-1/2}\mathcal{N}_N\mathcal{Z}_j, \quad j = 1,\ldots,\widetilde{N}.$$

for one extra flavor reduces to blow up of $\mathbb{C}^N$

ZN model vs HT model

$$K_{\rm HT} = \sqrt{r^2 + 4r|\zeta|^2} - r\log\left(r + \sqrt{r^2+4r|\zeta|^2}\right) + r\log(1+|\Phi_i|^2)$$

$$\zeta \to 0$$

$$K_{zn} = r|\zeta|^2 + r\log(1+|\Phi_i|^2)$$

$$K_{\rm HT} = K_{zn} + \mathcal{O}(|\zeta|^2)$$

IR physics of ZN and HT models is the same

BPS spectra are the same, but otherwise **different**

Perturbation theory

Perturbation theory

Gel-Mann-Low function

$$\beta_{i\bar{\jmath}} = a^{(1)} R^{(1)}_{i\bar{\jmath}} + \frac{1}{2r} a^{(2)} R^{(2)}_{i\bar{\jmath}} + \ldots$$

$$R^{(1)}_{i\bar{\jmath}} = R_{i\bar{\jmath}}\,,$$

$$R^{(2)}_{i\bar{\jmath}} = R_{i\bar{k}l\bar{m}} R^{\bar{k}}{}_{\bar{\jmath}}{}^{l\bar{m}}$$

Kaehler metric

$$g_{i\bar{\jmath}} = \partial_i \bar{\partial}_{\bar{\jmath}} K(z_i, \bar{z}_i)$$

Ricci tensor

$$R_{i\bar{\jmath}} = -\partial_i \bar{\partial}_{\bar{\jmath}} \log\det(g_{i\bar{\jmath}})$$

for Hanany-Tong model N=2, Nf=3

$$-\log\det(g^{(\mathrm{HT})}_{i\bar{\jmath}}) = \log(1 + |\Phi_i|^2) - \log\left(1 + \frac{r}{\sqrt{r^2 + 4r|\zeta|^2}}\right)$$

FI term renormalization (GLSM)

$$r_{\text{ren}}(\mu) = r_0 - \frac{N-\widetilde{N}}{2\pi}\log\frac{M}{\mu} \qquad r_{\text{ren}} = 0 \Longrightarrow r_0 = \frac{N-\widetilde{N}}{2\pi}\log\frac{M}{\Lambda}$$

$$c_1(M_{\text{HT}})\Big|_{\mathbb{CP}^{N-1}} = (N-\widetilde{N})\,[\omega_{\mathbb{CP}^{N-1}}]$$

Kaehler class is renormalized only at one loop, hence the result above should be the full answer for the coupling renormalization

If so what does the extra term in the last formula on the previous slide mean?

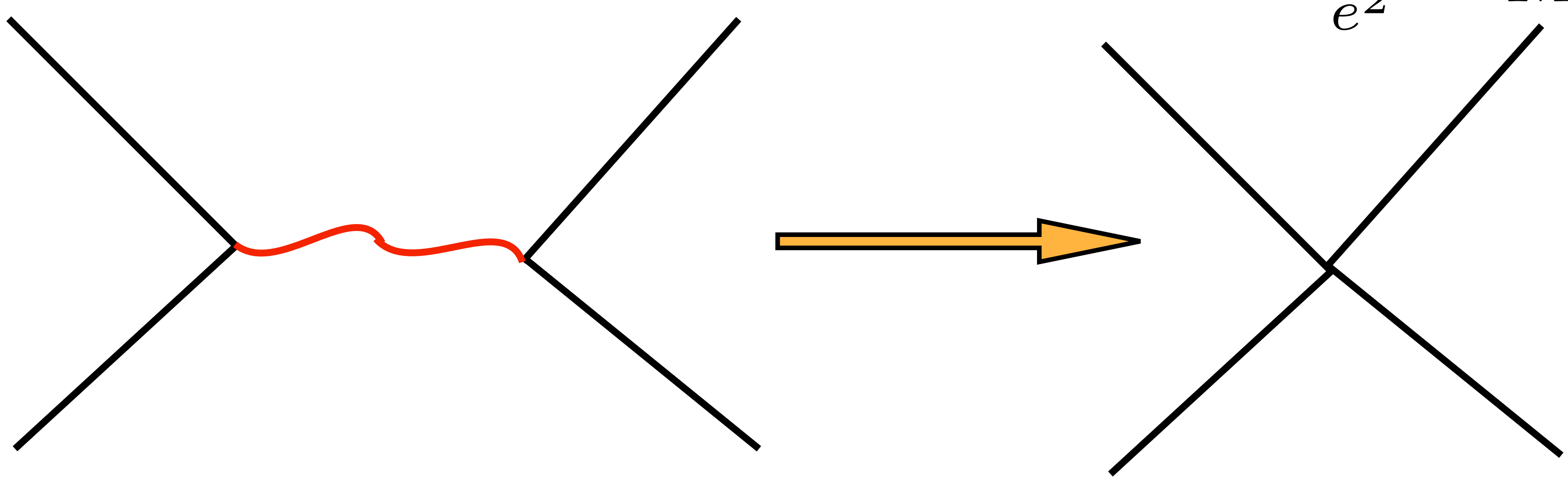
To understand why we need to compare renormalization schemes used in both calculations

GLSM vs NLSM

$$\int d^2x \int d^4\theta \left(|\Phi|^2 e^V - rV + \frac{1}{e^2}|\Sigma|^2 \right)$$

V-massive vector field w/ propagator

$$\frac{1}{\frac{p^2}{e^2} - M^2} \xrightarrow{p \ll e} \frac{1}{-M^2}$$



Integrating out V

$$-\log\det(g_{i\bar{j}}) = (N - \widetilde{N})\log(1 + |\Phi_i|^2) - (N-1)|\zeta|^2 + \mathcal{O}(|\zeta|^4)\,.$$

Dimensional regularization (GLSM perturbation theory) mixes up UV and IR divergencies. Need to single out the UV piece out, IR contribution is not seen in the GLSM limit

Less SUSY I
Heterotic deformation

(0,2) Theory

[Gorsky Shifman Yung]
[Distler Kachru]
[Edalati Tong][Shifman Yung]

In 4d introduce masses $\int d^2\theta\, \mu^2 (\Phi^a)^2$

breaks $\mathcal{N}=2$ to $\mathcal{N}=1$

On the flux tube
$(2,2) \longmapsto (0,2)$

obtain heterotic sigma model

$$\mathcal{L} = \int d^4\theta \left(\Phi_i^\dagger e^V \Phi^i - rV - \mathcal{B}V \right)$$

Note: Cannot be (1,1) since then it's automatically (2,2)

B-right handed superfield

can be treated as model w/ field dependent FI term

$$K = (r+\mathcal{B}) \log(1+|\phi^i|^2)$$

CP(N-1) model

[PK Monin]

$$\mathcal{L}_{\mathbb{CP}^N} = \int d^2\theta \Big[\tfrac{1}{2}\varepsilon_{\beta\alpha}(\mathcal{D}_\alpha + i\mathcal{A}_\alpha)\mathcal{N}_i^\dagger(\mathcal{D}_\beta - i\mathcal{A}_\beta)\mathcal{N}_i + i\mathcal{S}(\mathcal{N}_i^\dagger\mathcal{N}_i - r_0)$$
$$+ \tfrac{1}{4}\varepsilon_{\beta\alpha}\mathcal{D}_\alpha\mathcal{B}^\dagger\,\mathcal{D}_\beta\mathcal{B} + \left(i\,\omega\,\mathcal{B}(\mathcal{S} - \tfrac{i}{2}\overline{\mathcal{D}}\gamma^5\mathcal{A}) + \text{H.c.}\right)\Big]\,,$$

Isovector $\mathcal{N}^i = n^i + \bar\theta\xi^i + \tfrac{1}{2}\bar\theta\theta F^i,$

Spinor $\mathcal{A}_\alpha = -i(\gamma^\mu\theta)_\alpha A_\mu + \sqrt{2}(\gamma^5\theta)_\alpha\sigma_2 + \sqrt{2}\bar\theta\theta\, v_\alpha,$

Constraint $\mathcal{S} = \sqrt{2}\sigma_1 + \sqrt{2}\bar\theta u + \tfrac{1}{2}\bar\theta\theta D$

complex fields $\sigma = \sigma_1 + i\sigma_2\,,\quad \lambda_\alpha = u_\alpha + iv_\alpha$

if negatively charged fields are included

$$\begin{aligned}
\mathcal{L}^{\rm w}_{\mathbb{CP}^N} &= |\nabla_\mu n_i|^2 + |\nabla_\mu\rho_i|^2 + i\bar\xi_L^i\nabla_R\xi_L^i + i\bar\xi_R^i\nabla_L\xi_R^i + i\bar\eta_L^i\nabla_R\eta_L^i + i\bar\eta_R^i\nabla_L\eta_R^i \\
&- 2|\sigma|^2|n_i|^2 - 2|\sigma|^2|\rho_i|^2 - D\left(|n_i|^2 - |\rho_i|^2 - r_0\right) - 4|\omega|^2|\sigma|^2 \\
&+ \left[i\sqrt{2}\bar n_i\left(\lambda_L\xi_R^i - \lambda_R\xi_L^i\right) - i\sqrt{2}\sigma\bar\xi_R^i\xi_L^i + \text{H.c.}\right] \\
&+ \left[-i\sqrt{2}\bar\rho_i\left(\bar\lambda_L\eta_R^i - \bar\lambda_R\eta_L^i\right) + i\sqrt{2}\bar\sigma\bar\eta_R^i\eta_L^i + \text{H.c.}\right] \\
&+ \tfrac{i}{2}\bar\zeta_R\partial_L\zeta_R - \left[i\sqrt{2}\omega\lambda_L\zeta_R + \text{H.c.}\right],
\end{aligned}$$

(0,2) deformation of HT [PK Monin Vinci]

$$\int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, e^V \, \Phi_i + \sum_{i=1}^{N_c-N_f} \tilde{\Phi}_i^\dagger \, e^{-V} \, \tilde{\Phi}_i - (r + \mathcal{B})V + \frac{1}{2e^2}\Sigma^\dagger \Sigma \right]$$

$$\Phi^i = n^i + \bar\theta \xi^i + \theta \bar\xi^i + \bar\theta\theta F^i \,, \quad i = 1, \ldots, N_c$$
$$\widetilde{\Phi}^j = \rho^j + \bar\theta \eta^j + \theta \bar\eta^j + \bar\theta\theta \tilde{F}^j \,, \quad j = 1, \ldots, \tilde{N}$$

$$\Sigma = \sigma + i\theta^+ \bar\lambda_+ - i\bar\theta^- \lambda_- + \theta^+ \bar\theta^- (D - iF_{01})$$

$$\mathcal{B} = \omega(\bar\theta \zeta_R + \bar\theta\theta \bar{\mathcal{F}} \mathcal{F})$$

deformation adds

$$\mathcal{L}^{het} = \mathcal{L} + \bar\zeta_R \partial_L \zeta_R - |\omega|^2 |\sigma|^2 - [i\omega\lambda_L\zeta_R + \text{H.c.}]$$

Not enough SUSY

non-pert. corrections out of control

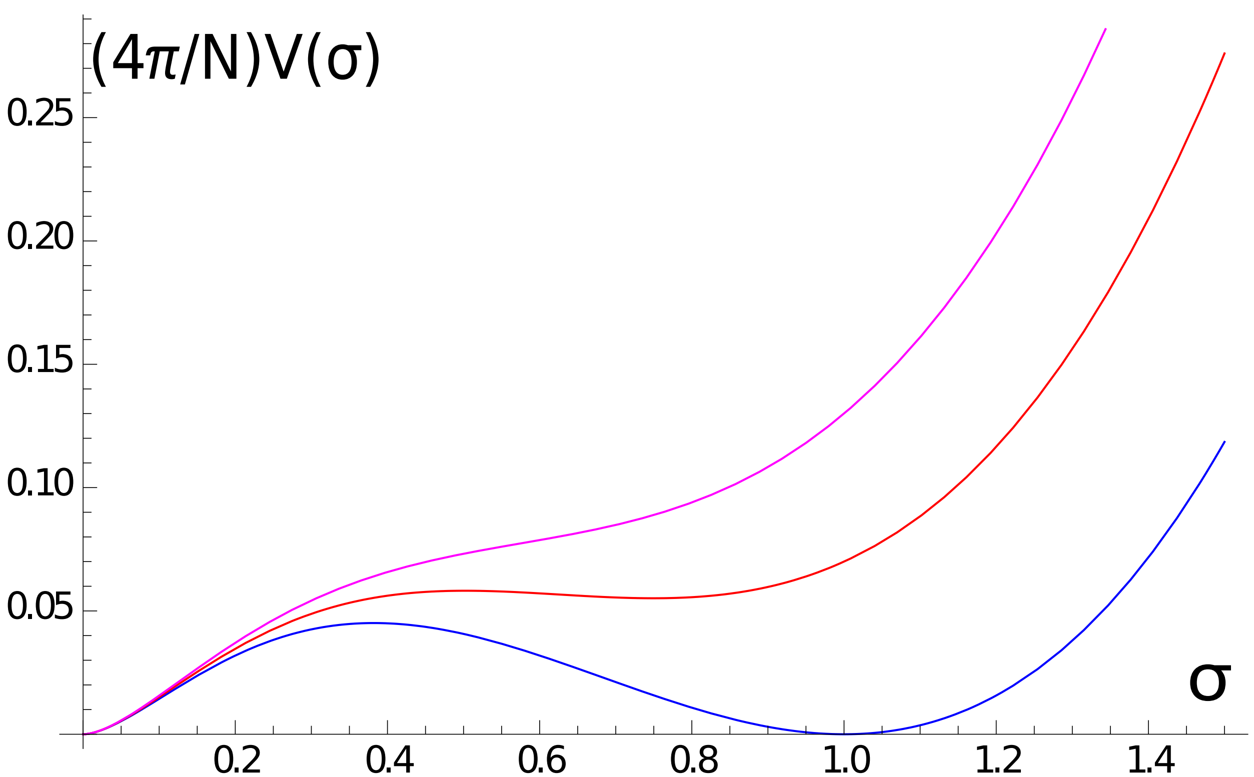
Have to dwell on large-N approach

Large-N solution of (0,2)

$$V_{1-loop} = \frac{1}{4\pi}\sum_{i=1}^{N-1}\left(-\left(D+|\sigma-m_i|^2\right)\log\frac{|\sigma-m_i|^2+D}{\Lambda^2}+|\sigma-m_i|^2\log\frac{|\sigma-m_i|^2}{\Lambda^2}\right)$$
$$-\frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1}\left(-\left(D-|\sigma-\mu_j|^2\right)\log\frac{|\sigma-\mu_j|^2-D}{\Lambda^2}-|\sigma-\mu_j|^2\log\frac{|\sigma-\mu_j|^2}{\Lambda^2}\right)$$
$$+\frac{N-\tilde{N}}{4\pi}D\,.$$

$$V_{eff} = V_{1-loop} + \left(|\sigma-m_0|^2+D\right)|n_0|^2 + \left(|\sigma-\mu_0|^2-D\right)|\rho_0|^2 + \frac{uN}{4\pi}|\sigma|^2$$

for zero masses



Symmetric masses

$$m_k = m\, e^{2\pi i \frac{k}{N}}\,, \quad k = 0, \ldots, N-1\,,$$
$$\mu_l = \mu\, e^{2\pi i \frac{l}{\tilde{N}}}\,, \quad l = 0, \ldots, \tilde{N}-1\,.$$

Vacuum equations

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \qquad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0\,,$$

$$\frac{1}{4\pi}\sum_{i=1}^{N-1} \log \frac{|\sigma - m_i|^2 + D}{\Lambda^2} - \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} \log \frac{|\sigma - \mu_j|^2 - D}{\Lambda^2} = |n_0|^2 - |\rho_0|^2\,,$$

$$\frac{1}{4\pi}\sum_{i=1}^{N-1} (\sigma - m_i) \log \frac{|\sigma - m_i|^2 + D}{|\sigma - m_i|^2} + \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} (\sigma - \mu_j) \log \frac{|\sigma - \mu_j|^2 - D}{|\sigma - \mu_j|^2} =$$

$$= (\sigma - m_0)\,|n_0|^2 + (\sigma - \mu_0)\,|\rho_0|^2 + \frac{uN}{4\pi}\,\sigma\,.$$

Solution of (2,2) model

Phase transitions -- artifact of large-N

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0$$

Higgs in n (Hn)

$$\rho_0 = 0 \quad D = -|\sigma - m|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{m}{\Lambda}\,, & \mu < m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m\,. \end{cases}$$

Higgs in rho (Hρ)

$$n_0 = 0 \quad D = |\sigma - \mu|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu < m \end{cases}$$

Coulomb (C)

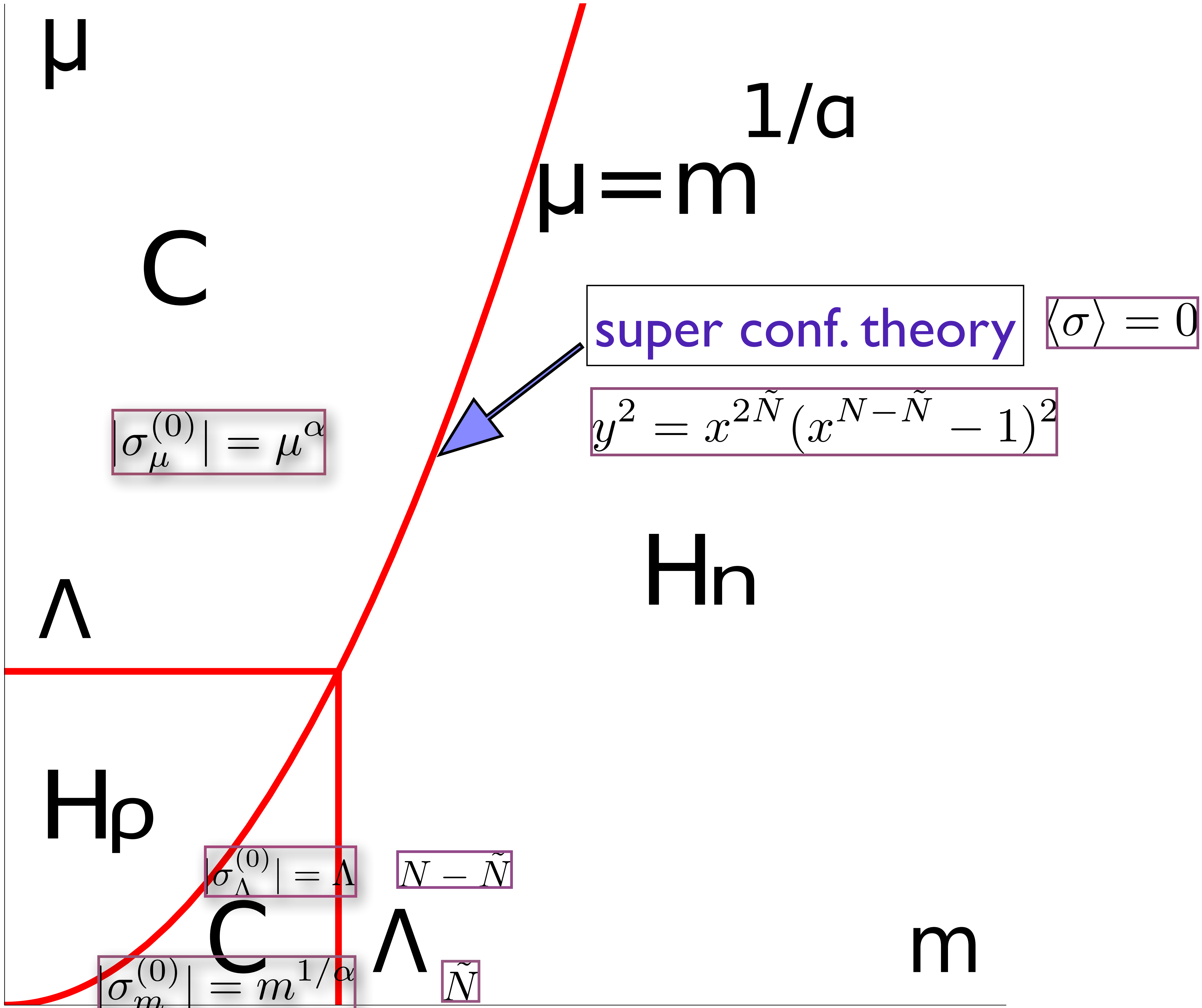
$$n_0 = \rho_0 = 0$$

renormalized FI term vanishes in C phase

in (2,2) from exact superpotential

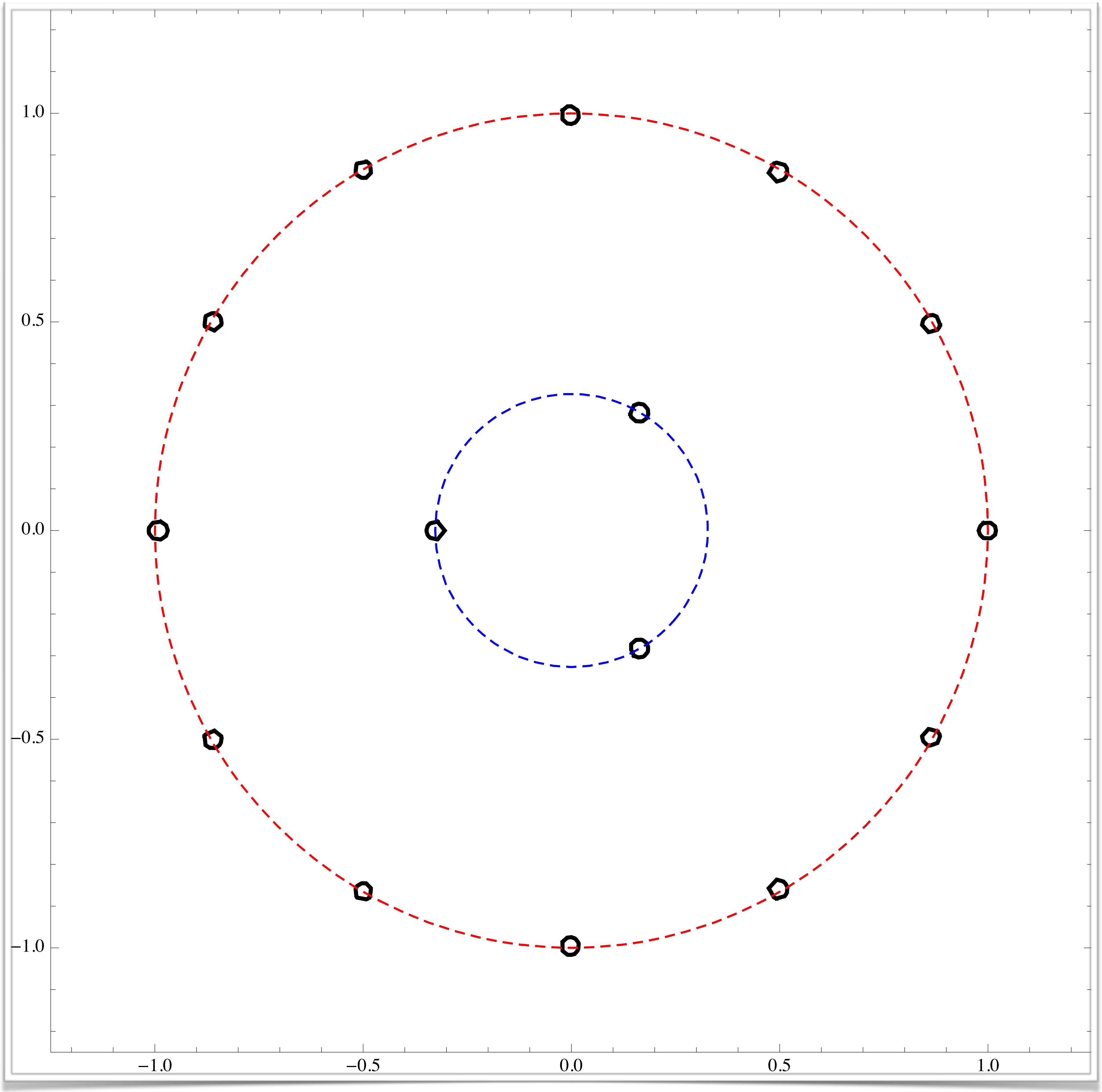
$$\frac{\prod_i (\sigma - m_i)}{\prod_i (\sigma - \mu_j)} = \Lambda^{N-\tilde{N}} \qquad \sigma = 0$$

is one of the solutions...

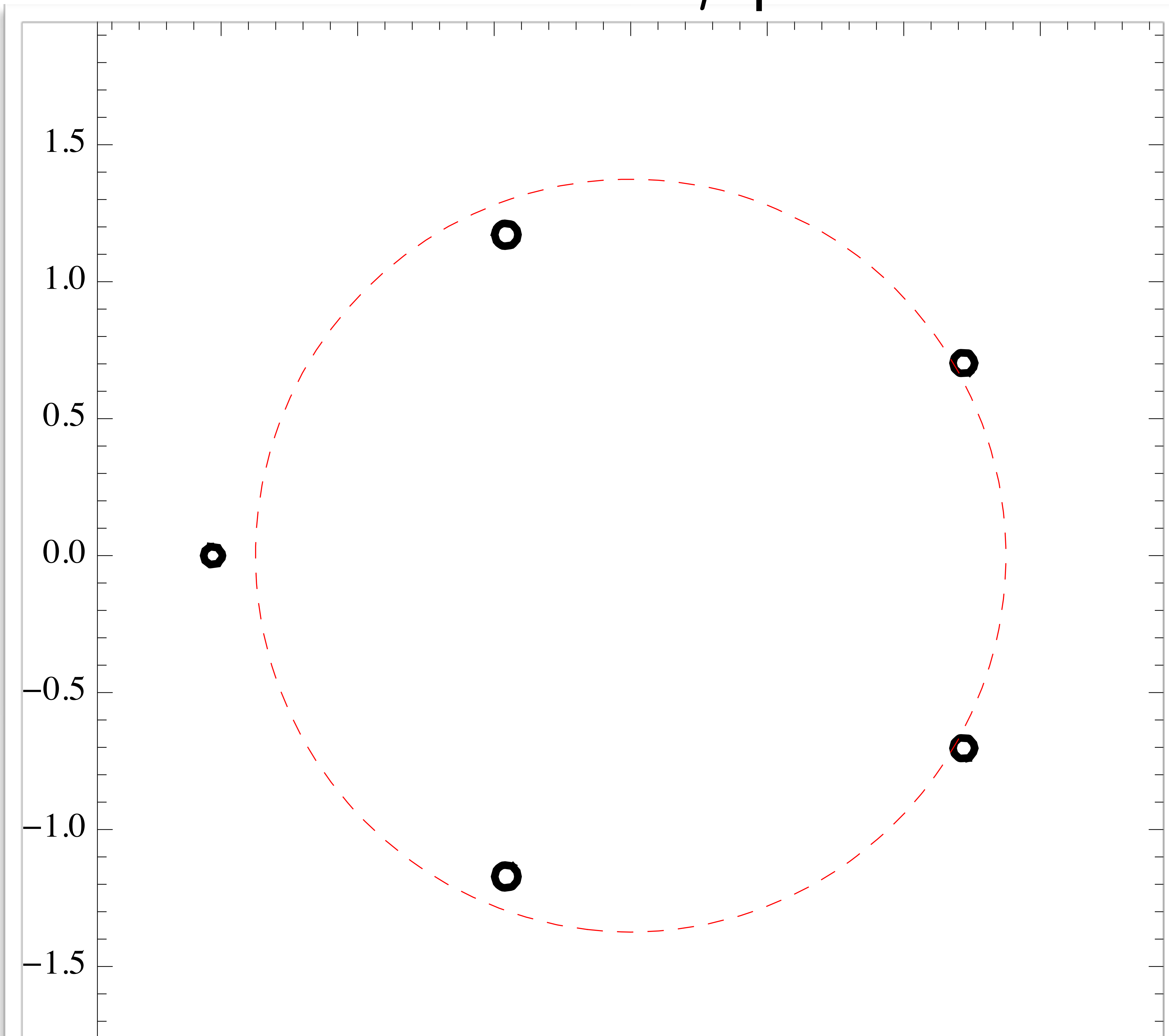


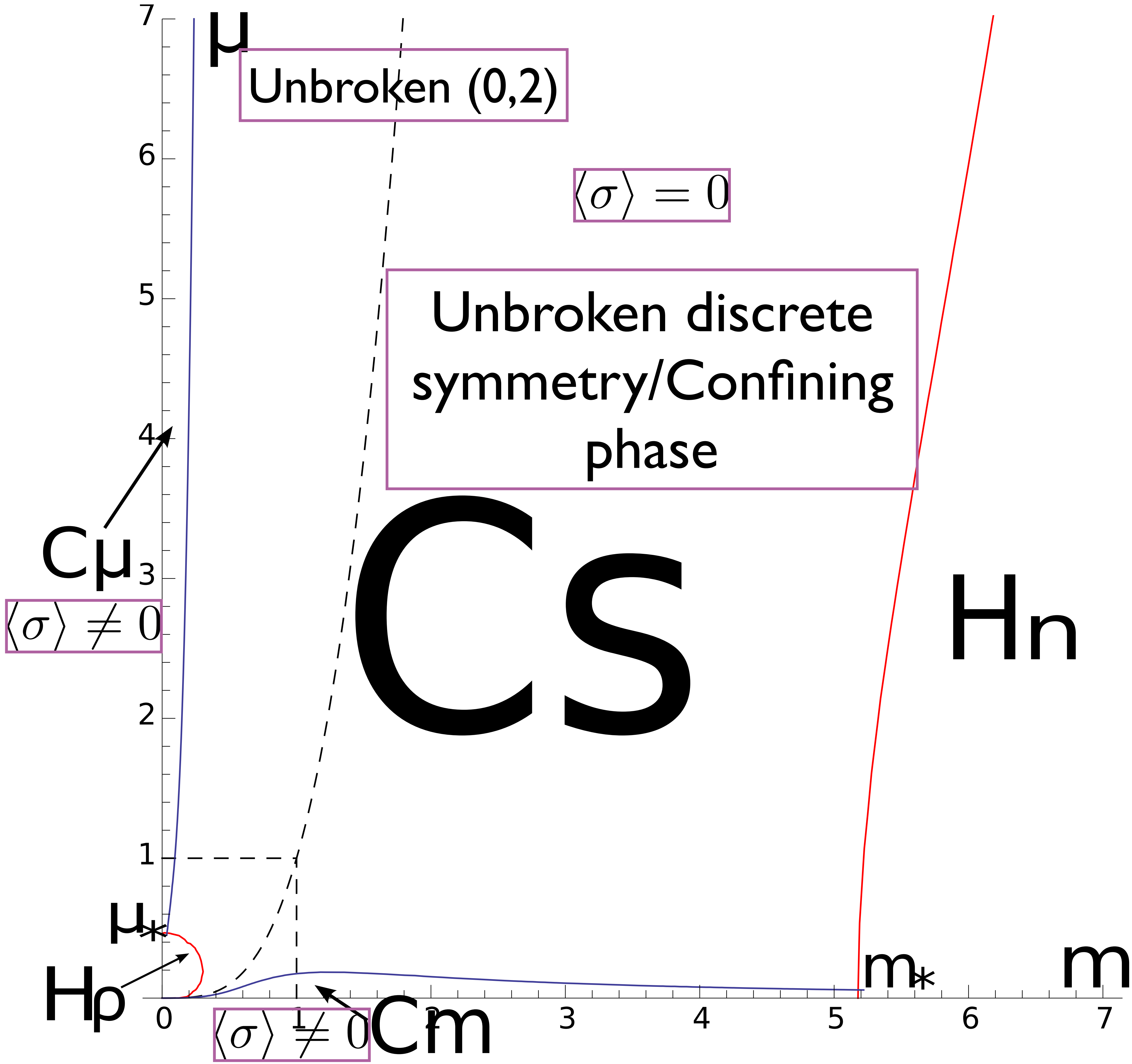
μ
1/a
μ=m
C
super conf. theory
$\langle \sigma \rangle = 0$
$y^2 = x^{2\tilde{N}}(x^{N-\tilde{N}} - 1)^2$
$|\sigma_\mu^{(0)}| = \mu^\alpha$
Hn
Λ
Hp
$|\sigma_\Lambda^{(0)}| = \Lambda$
$N - \tilde{N}$
C
Λ
$\tilde{N}$
m
$|\sigma_m^{(0)}| = m^{1/\alpha}$

N=15 Nf=18



Nf=5 Cμ phase





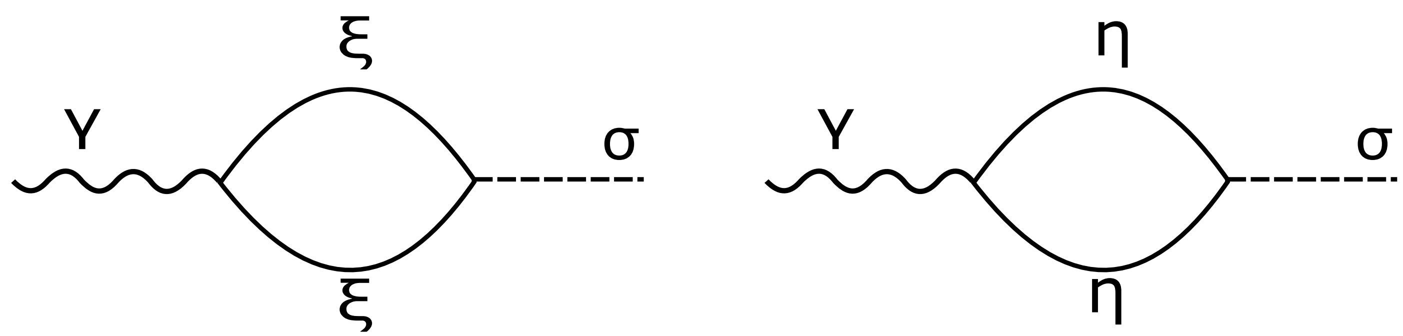
μ
Unbroken (0,2)
$\langle\sigma\rangle = 0$
Unbroken discrete symmetry/Confining phase
Cs
Hn
Cμ
$\langle\sigma\rangle \neq 0$
μ_*
Hp
m_*
m
$\langle\sigma\rangle \neq 0$
Cm
0
1
2
3
4
5
6
7

Spectrum

[Bolokhov Shifman Yung]
[PK Monin Vinci]

$$\mathcal{L} = -\frac{1}{4e_\gamma^2}F_{\mu\nu}^2 + \frac{1}{e_{\sigma\,1}^2}(\partial_\mu \mathfrak{Re}\,\sigma)^2 + \frac{1}{e_{\sigma\,2}^2}(\partial_\mu \mathfrak{Im}\,\sigma)^2 + i\mathfrak{Im}(\bar{b}\,\delta\sigma)\epsilon_{\mu\nu}F^{\mu\nu} - V_{\rm eff}(\sigma) + \text{Fermions}$$

Anomaly



$$b = \frac{N}{4\pi}\left(\frac{1}{N}\sum_{i=1}^{N-1}\frac{1}{\bar\sigma_0 - \bar m_i} - \alpha\frac{1}{\tilde N}\sum_{i=1}^{\tilde N-1}\frac{1}{\bar\sigma_0 - \bar\mu_i}\right)$$

$$m_\gamma = e_{\sigma\,2}e_\gamma|b|$$

Photon becomes massless in Cs phase!!

Confinement!

Note that Lambda vacua disappear at large deformations

Need to sit in zero-vacua

e.g. in Cm phase

$$m_\gamma = \sqrt{6}\,\Lambda\left(\frac{\Lambda}{m}\right)^{1/\alpha}\left(\left(\frac{m}{\Lambda}\right)^{2/\alpha} - \left(\frac{\mu}{\Lambda}\right)^2 e^{u/\alpha}\right)\mathrm{e}^{-\frac{u}{2\alpha}}$$

Massless goldstino in fermionic sector

NSVZ in (0,2) sigma model

$\mathbb{P}^N$ sigma models exhibit instanton solutions [Cui Shifman]

Let us now remove half of the fermions

An instanton has four bosonic zero modes but only two fermionic ones

$$A_{\rm inst} = \frac{y}{z - z_0}\,, \qquad A^\dagger_{\rm inst} = \frac{\bar{y}(1 + 4i\theta^\dagger\beta^\dagger)}{\bar{z}_{\rm ch} - \bar{z}_0 - 4i\theta^\dagger\alpha}$$

One loop corrections in the instanton background do not cancel completely

$$d\mu \;=\; \left(\frac{M^2}{g^2}\right)^{n_b}\left(\frac{g^2}{M}\right)^{n_f}(M)^{-1}e^{-\frac{4\pi}{g^2}}\;{\rm d}\log(y){\rm d}\log(\bar{y})\;dz_0 d\bar{z}_0\;d\alpha d\beta^\dagger$$

One loop modification

Exact beta function

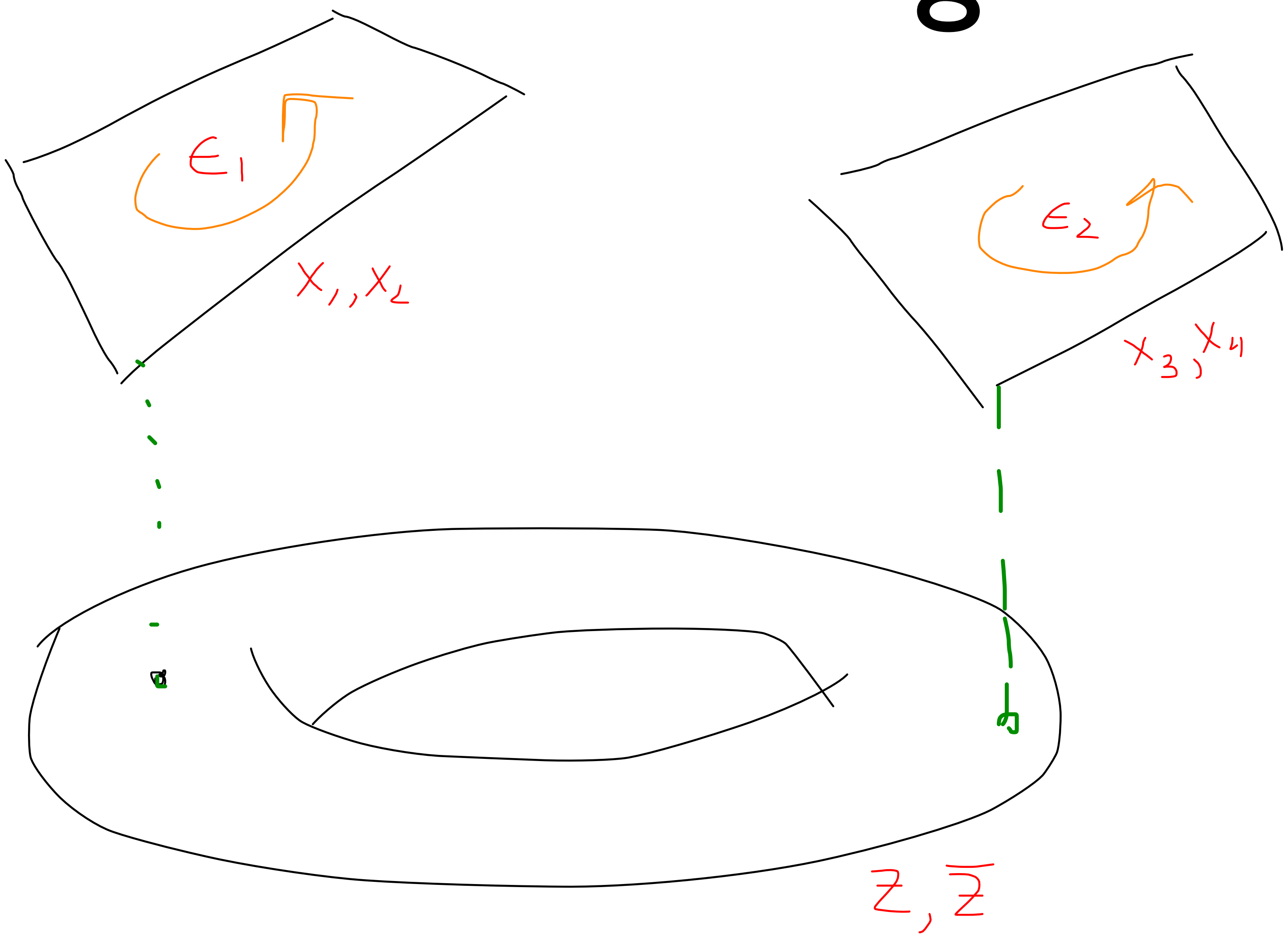
$$\beta(g^2) = -\frac{g^4}{2\pi}\frac{1}{1-\frac{g^2}{4\pi}}$$

What does it mean for 4d/2d?

Less SUSY II
Omega background

Omega background

[Nekrasov et al]



Rotational symmetry broken to maximal torus

$$SO(4) \rightarrow SO(2) \times SO(2)$$

6d Metric

$$G_{AB} dx^A dx^B = A dz d\bar{z} + (dx^m + \Omega^m dz + \bar{\Omega}^m d\bar{z})^2$$

We will be interested in Nekrasov-Shatashvili limit

$$\Omega^m = (-i\epsilon x^2, i\epsilon x^1, 0, 0)$$

$$\epsilon_2 \rightarrow 0$$

4d/2d in Omega background

[Dorey Hollowood Lee]

N=2 SQCD in Omega background
in NS limit with Nf=2Nc

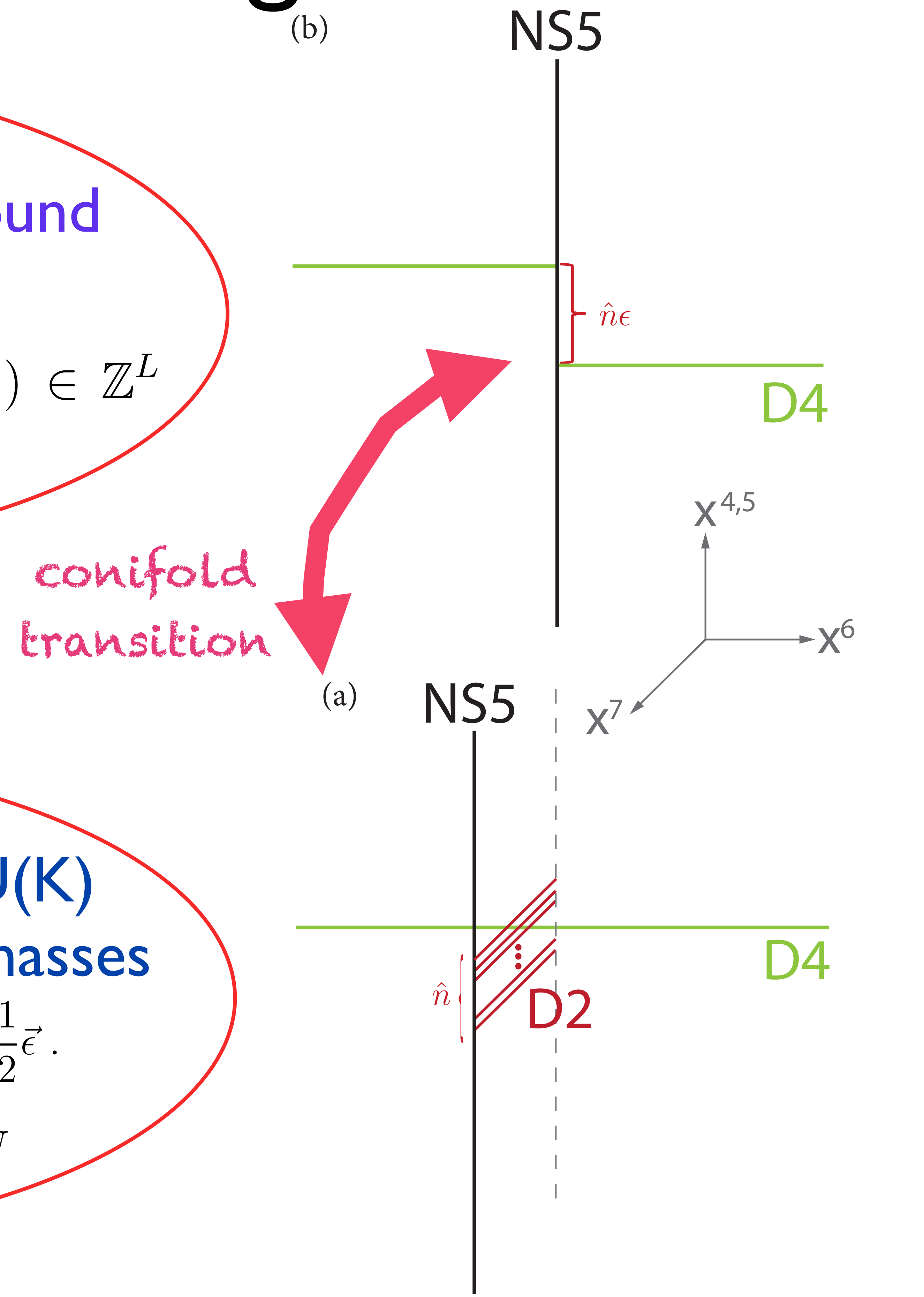
$$\vec{a} = \vec{m}_F - \vec{n}\epsilon \qquad \vec{n} = (n_1, \ldots, n_L) \in \mathbb{Z}^L$$

$$\mathcal{W}^{(I)} \overset{\text{on-shell}}{\equiv} \mathcal{W}^{(II)}$$

(2,2) GLSM w/ gauge group U(K)
massive adjoint and twisted masses

$$\vec{M}_F = \vec{m}_F - \frac{3}{2}\vec{\epsilon}\,, \qquad \vec{M}_{AF} = \vec{m}_{AF} + \frac{1}{2}\vec{\epsilon}\,.$$

$$M_{adj} = \epsilon \qquad K = \sum_{i=1}^{N} n_i - N$$



Vortices in Omega background

[PK Gorsky Chen] in progress

SUSY transform pure SYM

$$\delta\Lambda^I_\alpha = \zeta^I_\beta((\sigma^{mn})^\beta_\alpha F_{mn} + i[\phi,\bar\phi]\delta^\beta_\alpha + \nabla_m(\bar\Omega^m\phi - \Omega^m\bar\phi)\delta^\beta_\alpha) + \bar\zeta^I_{\dot\beta}(\sigma^m)^{\dot\beta}_\alpha(\nabla_m\phi - F_{mn}\Omega^n)$$

String central charge current

$$\zeta_3 = \tfrac{1}{2}\partial_m\left((\phi^a\bar\Omega^m - \bar\phi^a\Omega^m)B^a_3\right)\sigma^3_{\alpha\dot\alpha}\delta^{IJ} = \tfrac{i}{2}B^a_3\partial_\varphi(\phi^a\bar\epsilon - \bar\phi^a\epsilon)\sigma^3_{\alpha\dot\alpha}\delta^{IJ}$$

yields for a string of tension ~ epsilon

$$\mathcal{L} = \tfrac{1}{2}|B^a_z + \phi\tau^a\bar\phi - i\nabla_m(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a)|^2 + \tfrac{1}{2}|\mathcal{D}_1\phi^a + i\mathcal{D}_2\phi^a - (\Omega_2 - i\Omega_1)B^a_z|^2 + \partial_m(B^a_z(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a)) \geq \partial_m(B^a_z(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a))\,.$$

Symmetry breaking pattern

$$SU(2)_c \times SU(2)_R \times SU(2)_{\mathcal{R}} \to U(1)_c \times SU(2)_{R+\mathcal{R}}$$

Searching for the field theoretical explanation of the new duality

Conclusions and open questions

- Study BPS (and beyond) spectrum of SQCD can effectively be done using 2d NLSM (and GLSM)
- Rich variety of phases in (0,2) model at strong coupling
- Other heterotic deformations

$$\bar{D}\Phi_+ \sim \bar{D}\Phi_-$$

- Generalization of the 4d/2d duality to theories in Omega background
- Connections to integrable systems in 2d...
- Relationship w/ another 4d/2d duality [Vafa et al]
- Holography for Non-Abelian vortices