Critical String from Non-Abelian Vortex in 4d

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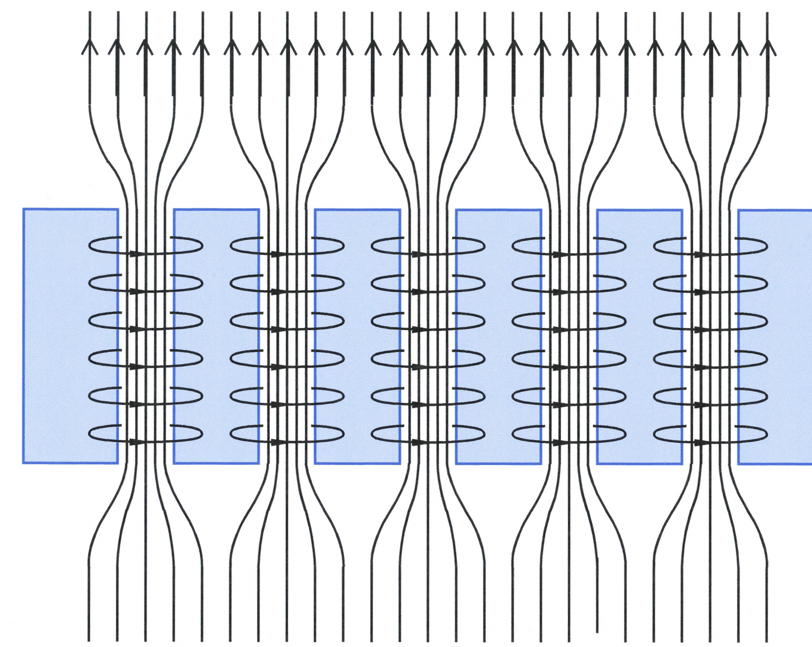
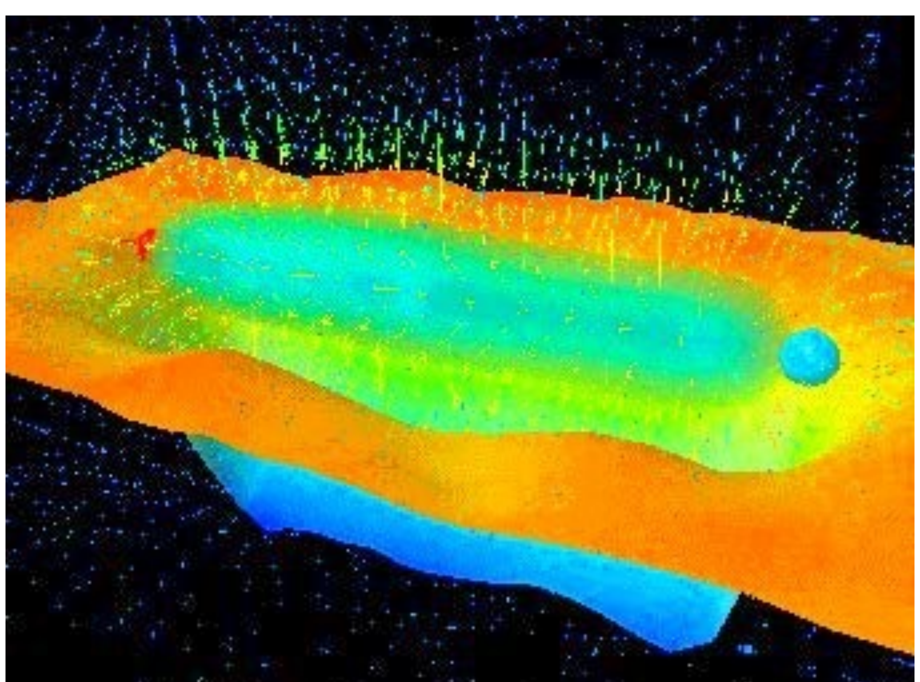
Effective strings

Abrikosov-Nilsen-Olisen (ANO) strings appear as flux tubes inside condensate of Cooper pairs in superconductors of second kind when superconductivity starts to break down. They cary Abelian magnetic flux



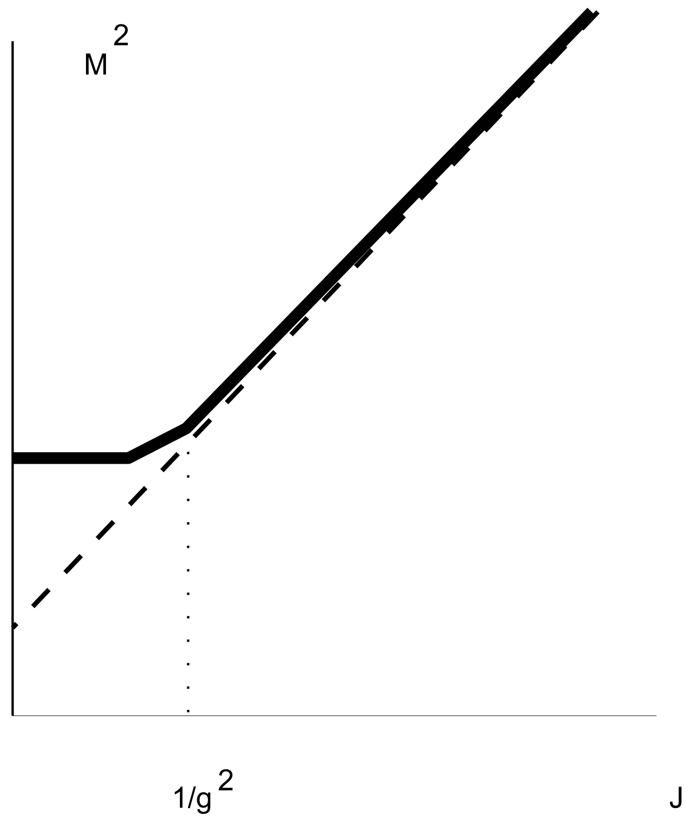


In QCD there are flux tubes stretched between quarks

Effective strings cont'd

These strings are bosonic so complete UV description of quantum spectrum of their excitations only exists in D=26

In noncritical regime quantum corrections completely change the dynamics and the object may not look like a string anymore (it crumples) and states do not obey Regge law for small spins



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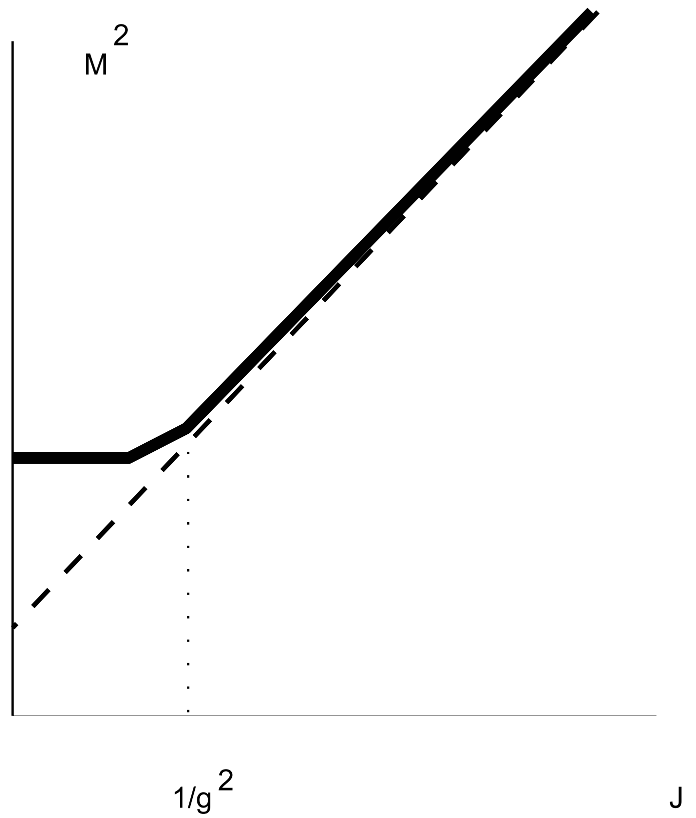

$$S_{\rm NG} = T\int d^2\sigma\left\{\sqrt{h} + O\left(\frac{\partial^n}{m^n}\right)\right\}$$

At weak coupling $m \sim g\sqrt{T}$ m-mass scale of bulk excitation

higher derivative terms become large

Effective String with SUSY

Thus if we want to find a good candidate for a fundamental string among effective strings criticality must be obeyed

When supersymmetry is present on the worldsheet of an effective string one has more control over the quantum corrections

In this talk we shall discuss strings which are formed as 1/2 BPS objects in four dimensional SQCD with 8 supercharges with gauge group U(N) and 2N (s)quarks

Worldsheet will have (2,2) superconformal symmetry so we should aim for a 10D description, but how?

‘ANO’ String

It is instructive to look at SQCD with N_f=N first

$U(N)$ **gauge theory with N hypers** $q \to UqV \qquad U \in U(N)_G, \quad V \in SU(N)_F$

$$S = \int d^4x \; \mathrm{Tr}\left(\frac{1}{2e^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2\right) + \sum_{i=1}^{N_f}|\mathcal{D}_\mu q_i|^2$$

$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{4}\,\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\,1_N)^2$$

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Vacuum

$$\phi = 0 \quad , \quad q^a{}_i = v \delta^a{}_i$$

breaks symmetry

(color-flavor locking)

$$U(N)_G \times SU(N)_F \to SU(N)_{diag} \times U(1)$$

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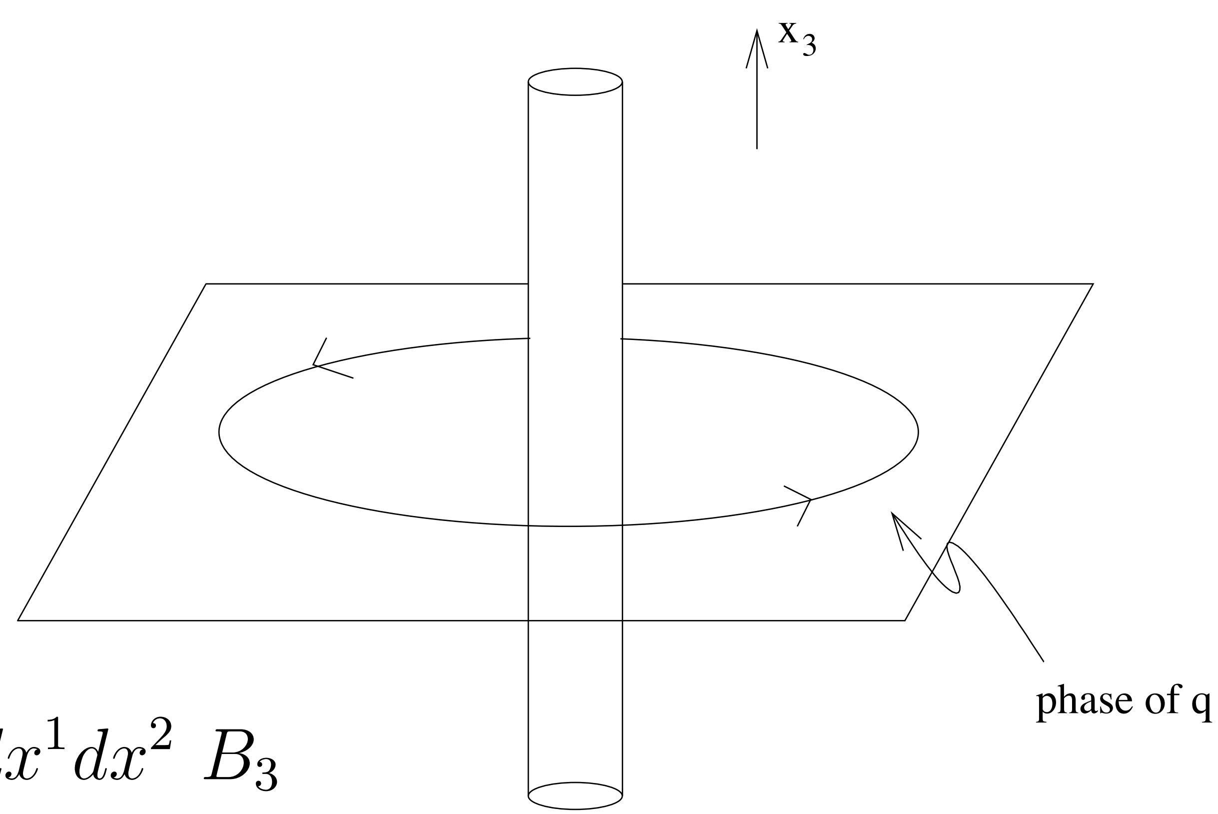
winding at infinity

$$q_N \sim q\,\mathrm{e}^{ik\theta}$$

$$A_\theta \sim \frac{k}{\rho}$$



$$2\pi k = \mathrm{Tr} \oint_{\mathbf{S}^1_\infty} i\partial_\theta q \ q^{-1} = \mathrm{Tr} \oint_{\mathbf{S}^1_\infty} A_\theta = \mathrm{Tr} \int dx^1 dx^2 \ B_3$$

Non-Abelian Vortices

ANO U(1) vortex has two collective coordinates-translations in x,y directions

U(N) vortex
has more moduli

$$A_z = \begin{pmatrix} A_z^\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \quad , \quad q = \begin{pmatrix} q^\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Moduli space
(k=1)

$$SU(N)_{\rm diag}/S[U(N-1)\times U(1)] \cong \mathbb{CP}^{N-1}$$

$$\mathcal{V}_{1,N} \cong \mathbf{C} \times \mathbb{CP}^{N-1}$$

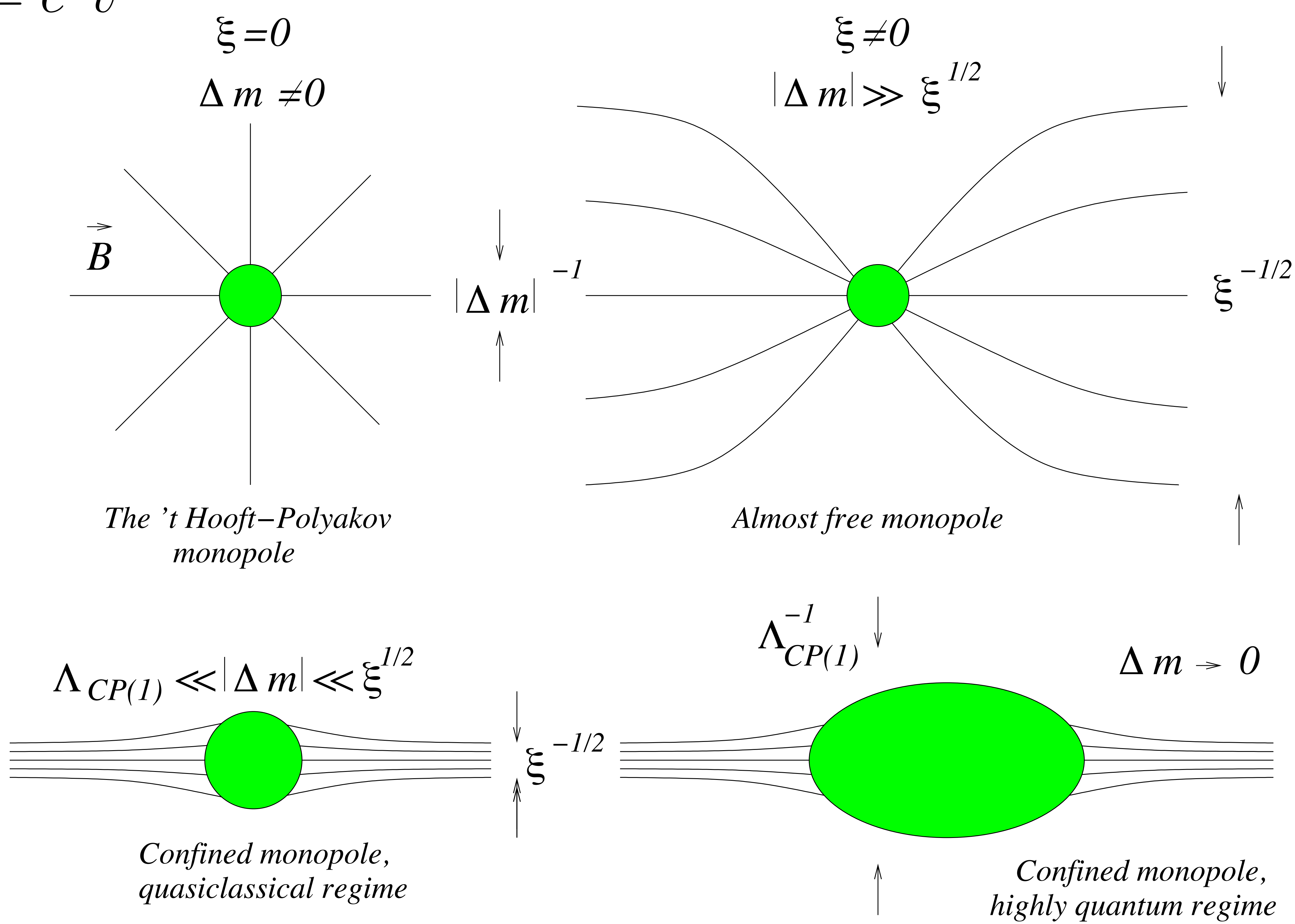
translational+
orientational

For higher k

$$\dim(\mathcal{V}_{k,N}) = 2kN$$

Confined monopoles [Shifman Yung]

$$\xi = e^2 v^2$$



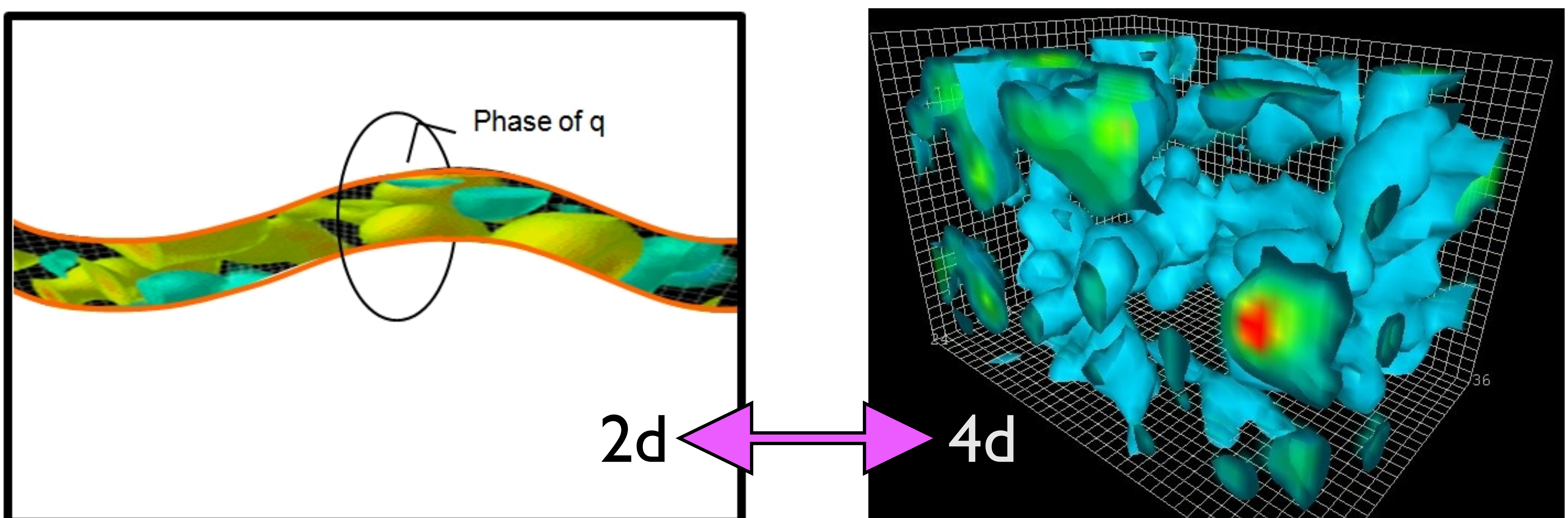
$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term β

4d/2d Duality

$N_f=N$ color-flavor locked phase $U(N)_G \times SU(N)_F \to SU(N) \times U(1)$

local vortex $$\frac{SU(N)}{SU(N-1)\times U(1)} = \mathbb{CP}^{N-1}$$

Duality between two strongly coupled theories



N_f=2N SQCD

Moduli space will still have the compact CP(N-1) part. But since it is not possible to Higgs all matter fields there will be noncompact moduli

Those ‘semilocal’ vortices are described by (2,2) sigma gauge linear sigma-model

$$
\begin{aligned}
S_1 &= \int d^2\sigma \sqrt{h} \left\{ h^{\alpha\beta} \left(\tilde{\nabla}_\alpha \bar{n}_P \, \nabla_\beta \, n^P + \nabla_\alpha \bar{\rho}_K \, \tilde{\nabla}_\beta \, \rho^K \right) \right. \\
&+ \left. \frac{e^2}{2} \left(|n^P|^2 - |\rho^K|^2 - \beta \right)^2 \right\} + \text{fermions} \, ,
\end{aligned}
$$

$$
\nabla_\alpha = \partial_\alpha - i A_\alpha \, , \qquad \tilde{\nabla}_\alpha = \partial_\alpha + i A_\alpha
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Its target manifold is a degree-N bundle over CP(N-1)

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Symmetry

$$n^P \leftrightarrow \rho^K$$

$$\beta \to \beta_D = -\beta$$

Its target manifold is a degree-N bundle over CP(N-1)

String action

In addition we have translational moduli

$$S_0 = \frac{T}{2} \int d^2\sigma \sqrt{h}\, h^{\alpha\beta} \partial_\alpha x^\mu \, \partial_\beta x_\mu + \text{fermions}$$

Full action is translational + orientational $\quad S = S_0 + S_1$

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To make a UV complete description the string needs to be infinitely thin $T \ll \rho_0^{-2}$ **[Polchinski Strominger]**

For our string $\frac{\beta}{T} \sim \rho_0^2$ so at weak coupling $\beta = \frac{4\pi}{g_{2d}} \gg 1$

At strong coupling $\beta = 0$

This maps to the fixed point of the S-duality of the 4d theory

$$\tau \to \tau_D = -\frac{1}{\tau}\,, \qquad \tau = i\frac{4\pi}{g^2} + \frac{\theta_{4D}}{2\pi}$$

Criticality

2d sigma model is conformal. The dimension of the full target space is 2(2N-1)+D

For N=2 and D=4 we get 10 dimensional target space

Checking Virasoro central charge $$c_{\mathrm{Vir}} = \frac{3}{2}(D + 2\hat{c}_0 - 10)$$

Here $\hat{c}_0 = 2N - 1\,,\, D = 4$

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The two conditions from Polchinski-Strominger are satisfied

The resulting target space is $\mathbb{R}^4 \times Y$

Y - resolved conifold with β

Type IIA on conifold

We obtained critical superstring in ten dimensions. Which type is it IIA or IIB?

Type IIA on conifold

We obtained critical superstring in ten dimensions. Which type is it IIA or IIB?

Our starting point — 4d SCQD is a vector-like theory which preserves parity. Therefore the string has to by of Type IIA

We can now unload the machinery of string compactifications on CY threefolds to study the effective 4d theory (a different 4d theory). We shall study how the 4d spectrum depends on β

Compactification

Parameter beta describes deformations of Kahler structure of the CY which are enumerated by cohomology $h^{1,1}$

For conifold $h^{1,1} = 1$

So if normalizable there should be a single vectormultiplet coming from such reduction

Since it lies in the same supergravity multiplet with graviton, existence of such state would imply presence of massless gravitons which is problematic [Winberg-Witten]

Fortunately this mode is non-normalizable

Deformed Conifold

Something interesting happens at beta=0. The conifold develops a singularity and we cannot use SUGRA

However, we can deform further past the singularity into a different topology — *deformed conifold*

$$w_\alpha = \frac{1}{2}\,\mathrm{Tr}\left[(\bar{\sigma}_\alpha)_{KP}\, n^P \rho^K\right] \qquad \sum_{\alpha=1}^{4} w_\alpha^2 = b$$

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In other words we opened a new mode parameterized by b

$$S(b) = T\int d^4x\, h_b |\partial_\mu b|^2,$$

The computation shows that b-mode is **log-normalizable** as well as the wave-functions of orientational moduli

Physics of b-mode

B-mode is related to the deformations of complex structure of the conifold described by Dolbeault cohomology $H^{2,1}$

For conifold $h^{2,1} = 1$

so there should be a one-(complex)dimensional branch parameterized by a VEV of some *hypermultiplet*

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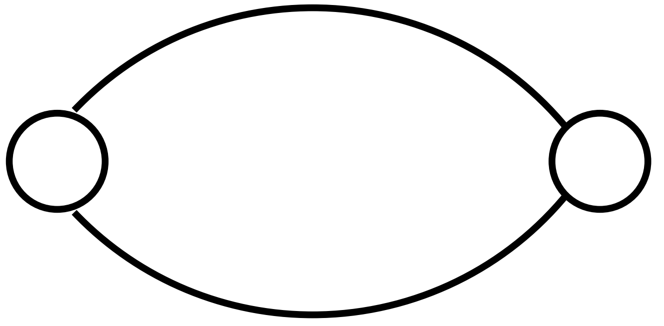
Global symmetry $SU(2) \times SU(2) \times U(1)$

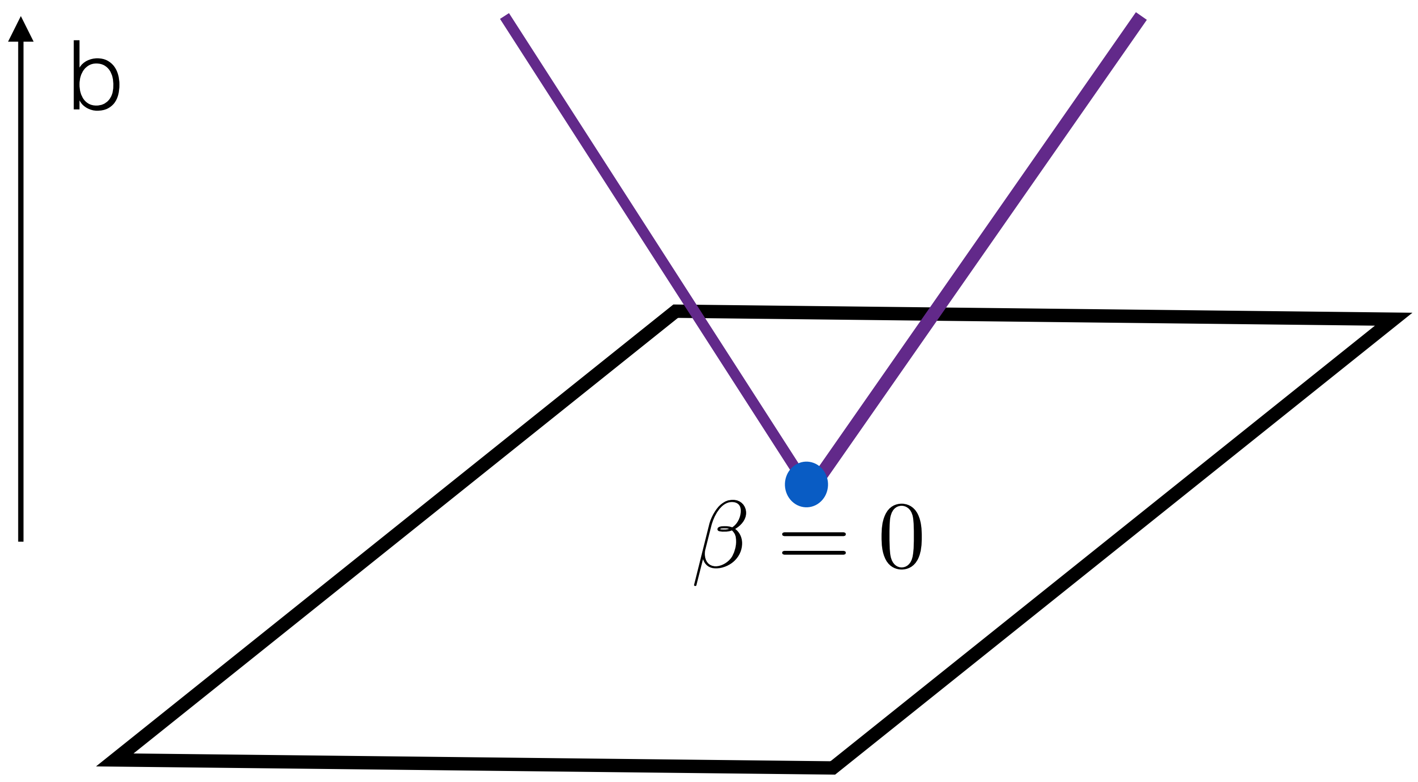
Since $w_\alpha = \frac{1}{2} \mathrm{Tr} \left[(\bar{\sigma}_\alpha)_{KP} \, n^P \rho^K \right]$ b-state transforms as (1,1,2)

monopole-monopole baryon



Recap

4d SQCD —> 2d sigma model —> Type IIA superstring —> effective 4d theory



Mon/Mon Baryon

=

n-rho Kink

=

Hyper