# Strings Monopoles Domain Walls in Omega Background and Integrability

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In progress with K. Bulycheva, H. Chen and A.Gorsky 1206.xxxx

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# Some interesting facts about N=2 physics

A full partition function of N=2 d=4 theory can be computed by localization [Nekrasov]

Recently a solid connection to non-SUSY CFTs was outlined [Alday, Gaiotto, Tachikawa]

and connection to 2d sigma models

[Dorey, Hollowood, Tong] [Shifman, Yung] [Gaiotto, Moore, Neitzke]...

*This talk: interplay between the last two points*

# Outline

- 4d/2d w/ 8 supercharges
  - ★ *Vortices in field theory and type IIA string theory*
  - ★ *(2,2) GLSM, NLSM*
  - ★ *The Dictionary of 4d/2d*
- AGT duality vs 4d/2d correspondence
  - ★ Omega Background
  - ★ Liouville theory at large central charge
  - ★ 4d/2d in NS limit and duality
- Zoo of BPS solitons in Omega deformed theory
  - ★ *Monopoles, strings, domain walls*

# 4d/2d

# 4d / 2d duality

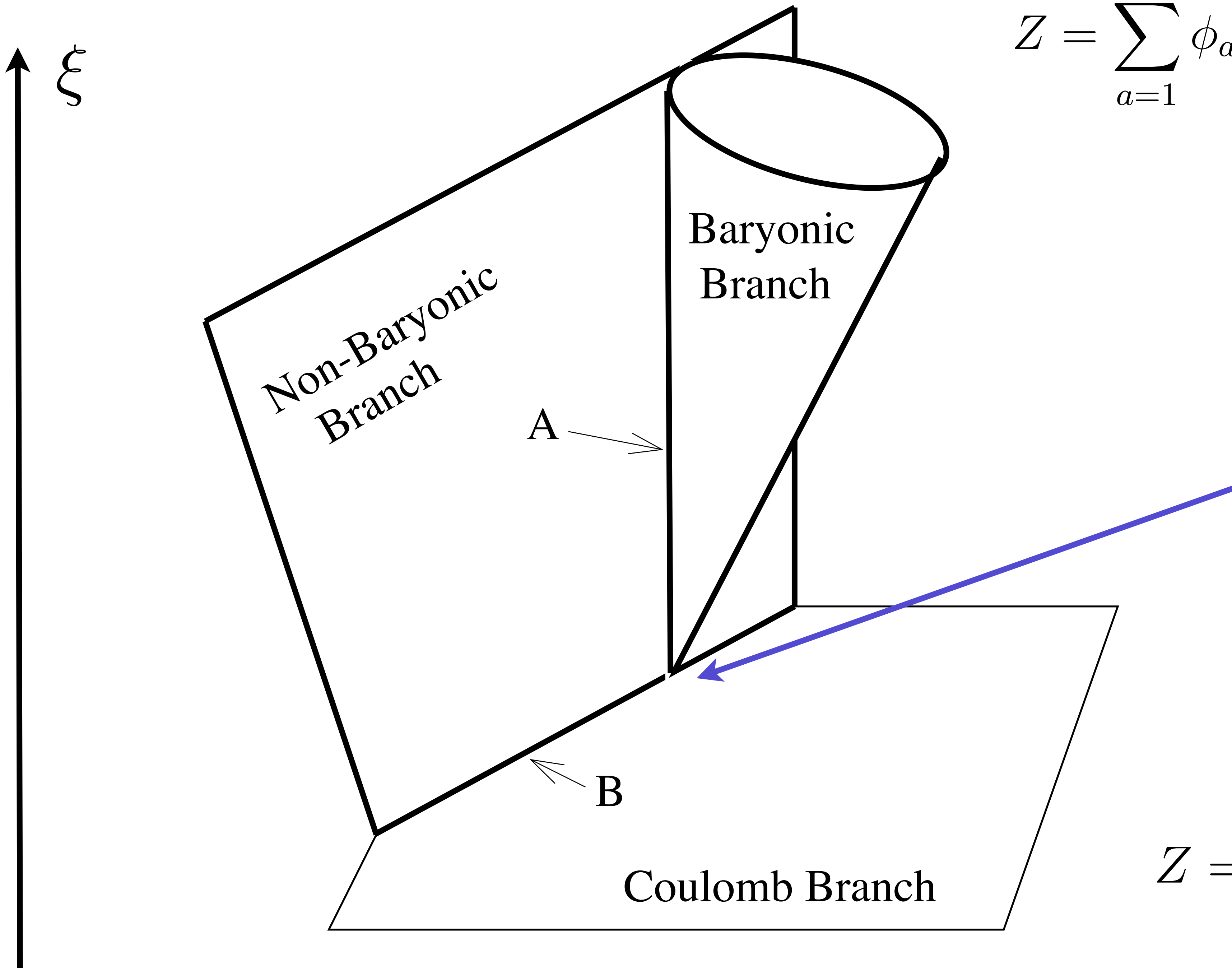
[Dorey Hollowood, Tong]

| | |
|---|---|
| $\mathcal{N}=2 \quad SU(N)$ SQCD | (2,2) $U(1)$ GLSM e |
| $N_f = N + \tilde{N}$ fund hypers | $N$ chiral +1 $\quad \tilde{N}$ chiral -1 |
| w/ masses | w/ *twisted* masses |
| $m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$ | $m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$ |
| $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$ | $\tau = ir + \frac{\theta}{2\pi}$ |
| on baryonic Higgs branch | |
| BPS dyons (Seiberg-Witten) | kinks interpolating between different vacua |

BPS spectra (as functions of masses, Lambda) are the same

# Coulomb vs Higgs branches

$$Z = \sum_{a=1}^{N_c} \phi_a (j_a + \tau h_a) + \sum_{i=1}^{N_f} m_i s_i$$

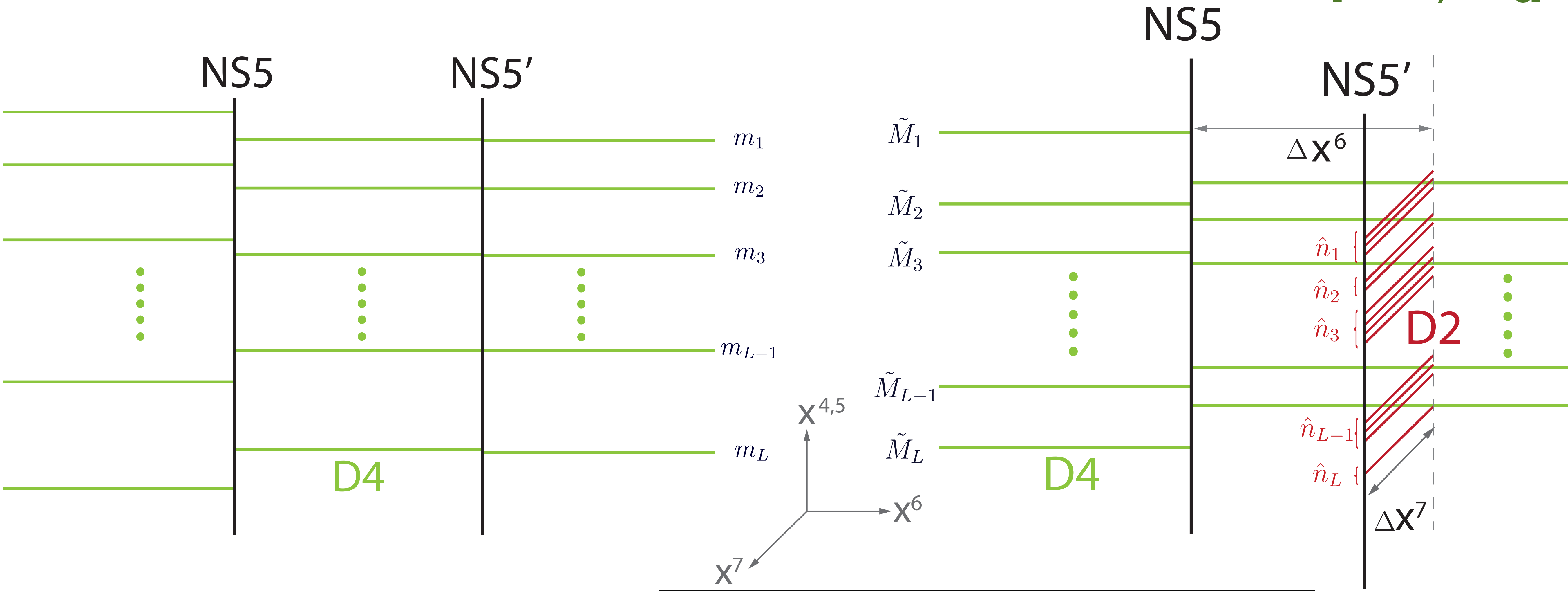



$$\phi_a = m_a$$

$$Z = \sum_{i=1}^{N_c} (m_i S_i + m_{Di} h_i)$$

# Hanany-Witten construction

[Witten]
[Hanany Tong]




SQCD $\quad N_f = 2N_c$

Higgs branch root

Color-flavor locked phase of SQCD

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| NS5 | x | x | x | x | x | x | | | | |
| D4 | x | x | x | x | | | x | | | |
| D2 | x | | | x | | | | x | | |

2d FI parameter $\quad r = \dfrac{\Delta x^6}{2\pi g_s l_s} = \dfrac{4\pi}{e^2}$

$$V_{2d}(\sigma, Z)$$

$$\sigma = X^4 + iX^5 \quad , \quad Z = X^1 + iX^2$$

@Dorey

# Understanding 2d theory: 'ANO' String

$U(N)$ gauge theory with fundamental matter $q \to UqV \qquad U \in U(N)_G, \quad V \in SU(N)_F$

$N_f = N_c$

$$S = \int d^4x \; \mathrm{Tr}\left(\frac{1}{2e^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2\right) + \sum_{i=1}^{N_f} |\mathcal{D}_\mu q_i|^2$$
$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{4}\,\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\, 1_N)^2$$

Vacuum

$$\phi = 0 \quad , \quad q^a{}_i = v\delta^a{}_i$$

breaks symmetry

(color-flavor locking)

$$U(N)_G \times SU(N)_F \to SU(N)_{\rm diag}$$

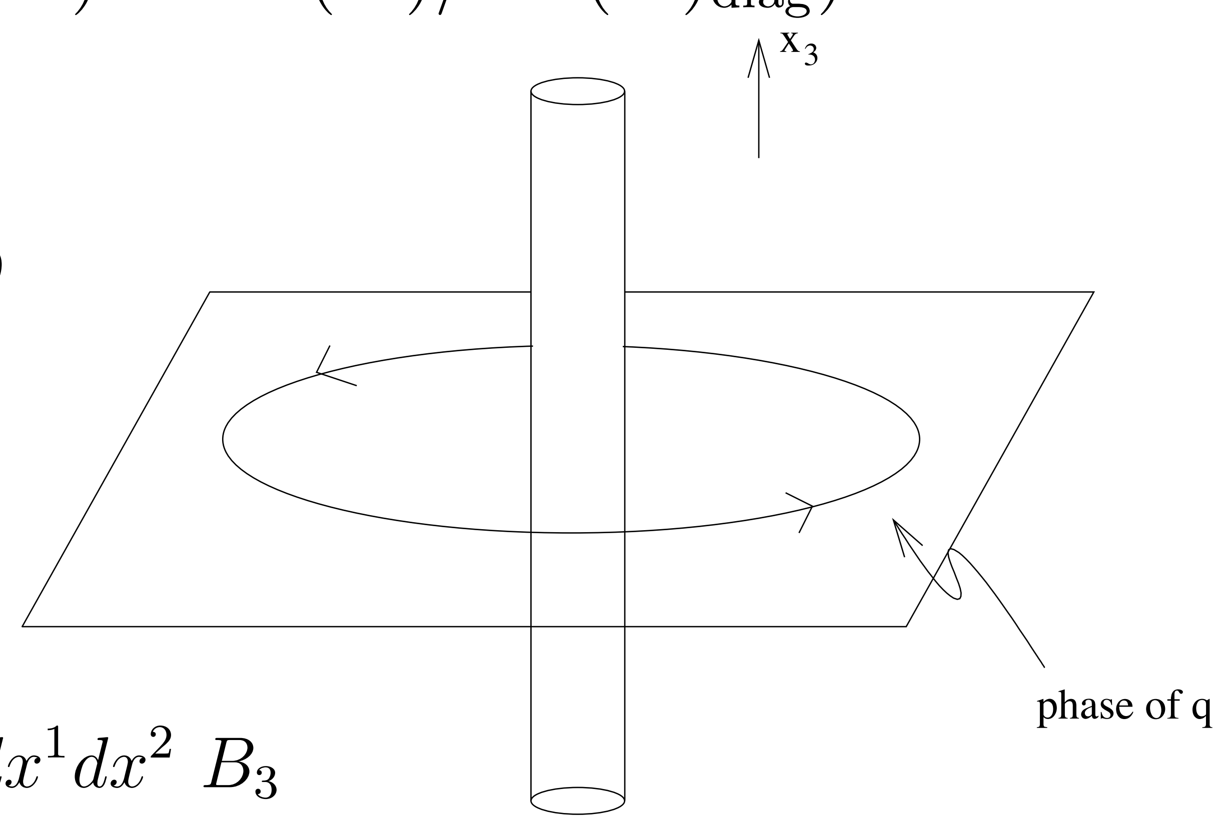
Induces nontrivial topology on moduli space

$$\Pi_1\left(U(N) \times SU(N)/SU(N)_{\rm diag}\right) \cong \mathbf{Z}$$

To find a string need winding at infinity

$$q_N \sim q\,\mathrm{e}^{ik\theta}$$

$$A_\theta \sim \frac{k}{\rho}$$




$$2\pi k = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} i\partial_\theta q\; q^{-1} = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} A_\theta = \mathrm{Tr}\int dx^1 dx^2\; B_3$$

# BPS equations for vortex

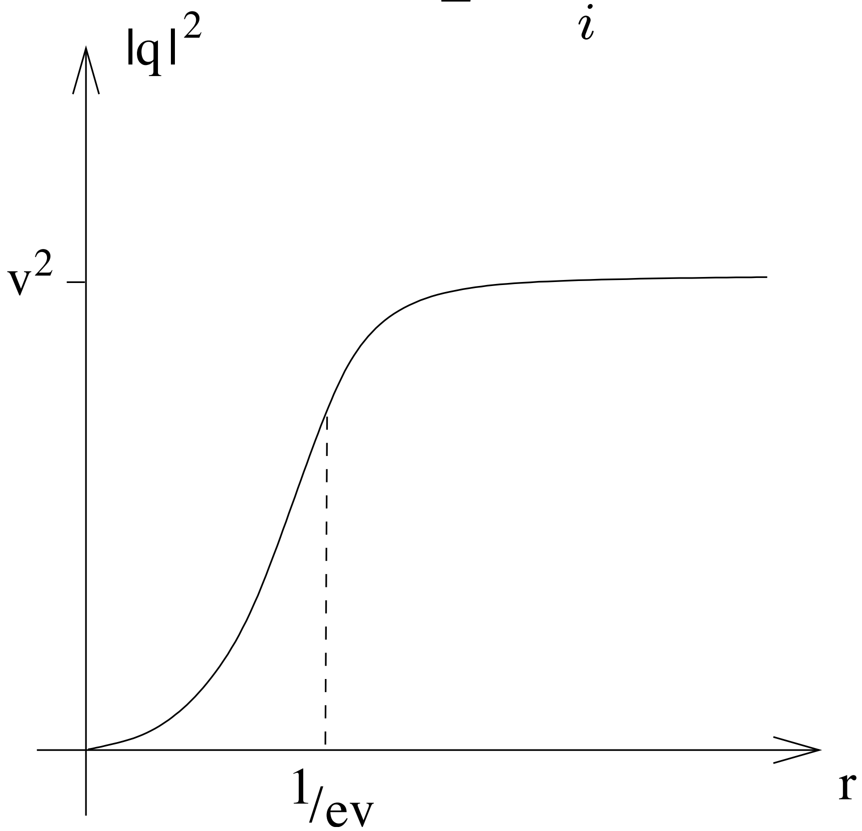
$$T_{\rm vortex} = \int dx^1 dx^2 \ {\rm Tr} \left( \frac{1}{e^2} B_3^2 + \frac{e^2}{4} (\sum_{i=1}^N q_i q_i^\dagger - v^2 \, 1_N)^2 \right) + \sum_{i=1}^N |\mathcal{D}_1 q_i|^2 + |\mathcal{D}_2 q_i|^2$$

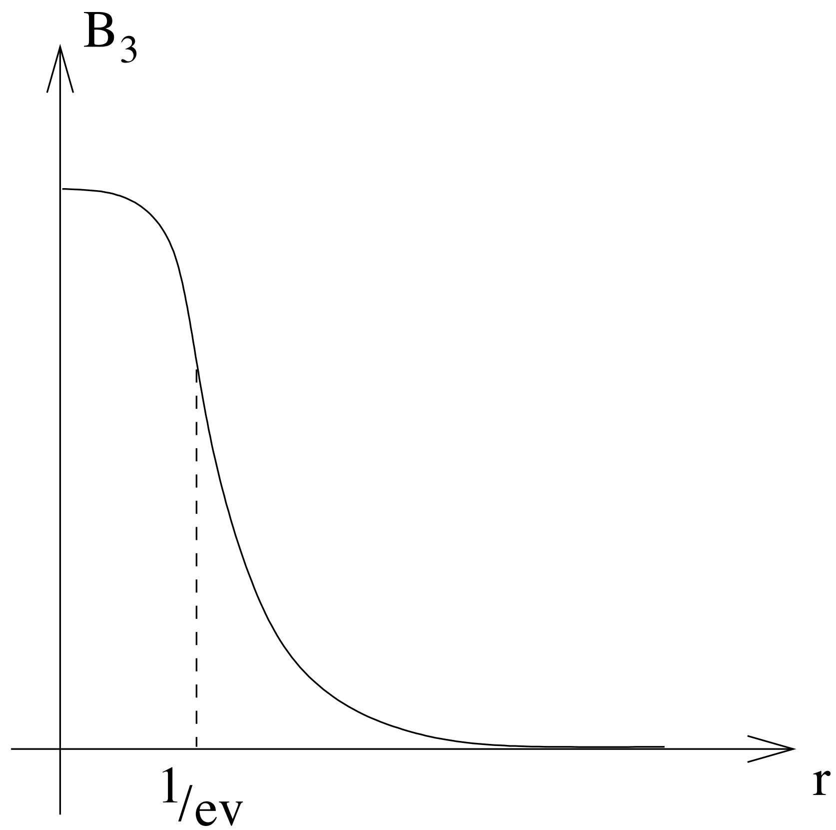
$$= \int dx^1 dx^2 \ \frac{1}{e^2} {\rm Tr} \left( B_3 \mp \frac{e^2}{2} (\sum_{i=1}^N q_i q_i^\dagger - v^2 \, 1_N) \right)^2 + \sum_{i=1}^N |\mathcal{D}_1 q_i \mp i \mathcal{D}_2 q_i|^2$$

$$\mp v^2 \int dx^1 dx^2 \ {\rm Tr}\, B_3 \quad \geq \mp v^2 \int d^2x {\rm Tr} B_3 \quad = 2\pi v^2 |k|$$

(

**gives** $$B_3 = \frac{e^2}{2} (\sum_i q_i q_i^\dagger - v^2 \, 1_N) \qquad (\mathcal{D}_x - i \mathcal{D}_y) q_i = 0$$



# Vortices

Simple vortex w/ N=1, k=1 (ANO) has two collective coordinates-translations in x,y directions

U(N) vortex has more moduli

$$A_z = \begin{pmatrix} A_z^\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \quad , \quad q = \begin{pmatrix} q^\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Moduli space (k=1)

$$SU(N)_{\text{diag}}/S[U(N-1)\times U(1)] \cong \mathbb{CP}^{N-1}$$

$$\mathcal{V}_{1,N} \cong \mathbf{C} \times \mathbb{CP}^{N-1}$$

For higher k

$$\dim(\mathcal{V}_{k,N}) = 2kN$$

*Again:*

$T \geq 2\pi v^2 |k|$ bound saturates for BPS states

# Non-Abelian String

[Auzzi, Bolognesi, Evslin, Konishi, Yung]

[Shifman Yung]

$$q = U \begin{pmatrix} q_1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 \\ 0 & 0 & \dots & q_2 \end{pmatrix} U^{-1}$$

Take Abelian string solution
Make global rotation

$$A_i^{SU(N)} = \frac{1}{N} U \begin{pmatrix} 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -N+1 \end{pmatrix} U^{-1} (\partial_i \alpha) f(\rho)$$

Matrix U parameterizes orientational modes

Gauge group is broken to $\mathbb{Z}_N$

All bulk degrees of freedom massive $M^2 \sim \xi$

Theory is fully Higgsed

# Vortex moduli space

Nf=Nc color-flavor locked phase
single SUSY vacuum

$$U(N_c) \times SU(N_f) \to SU(N)$$

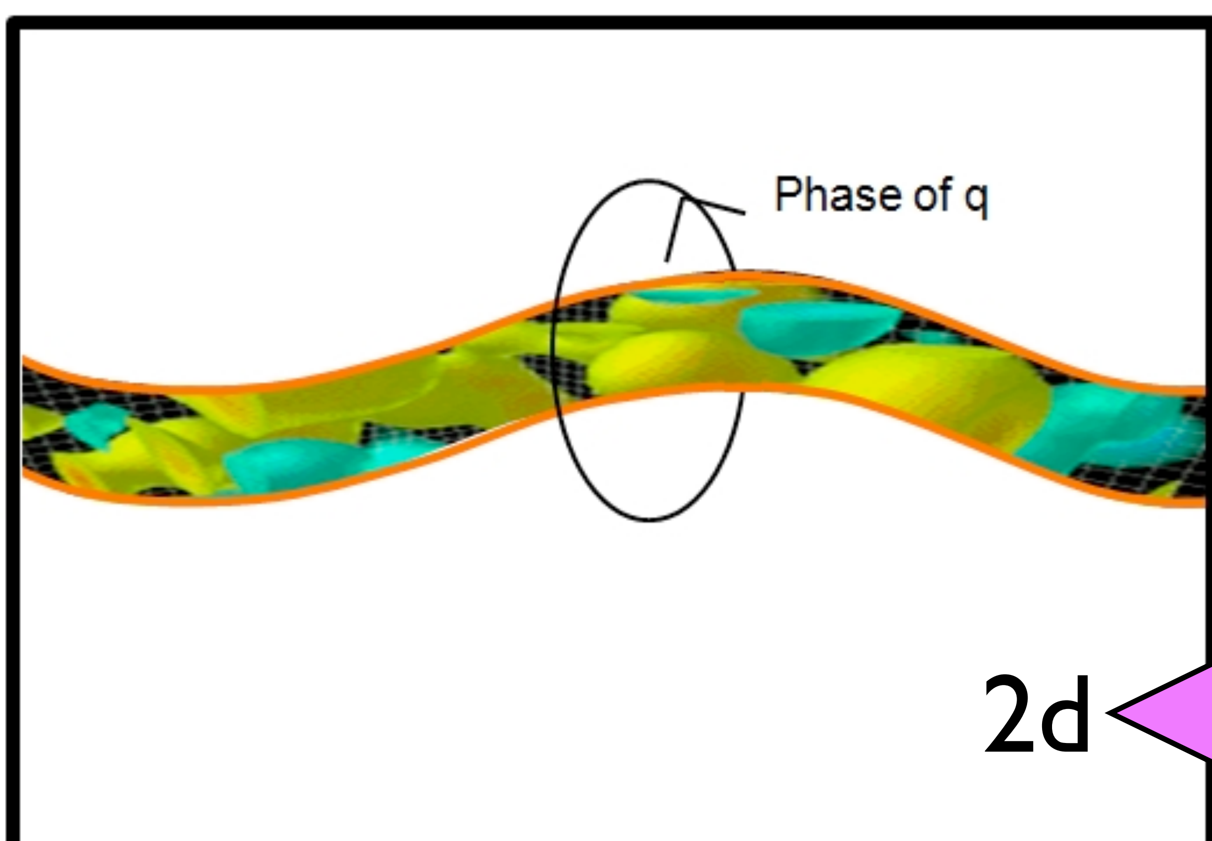
Nf=Nc local vortex

$$\frac{SU(N)}{SU(N-1) \times U(1)} = \mathbb{CP}^{N-1}$$

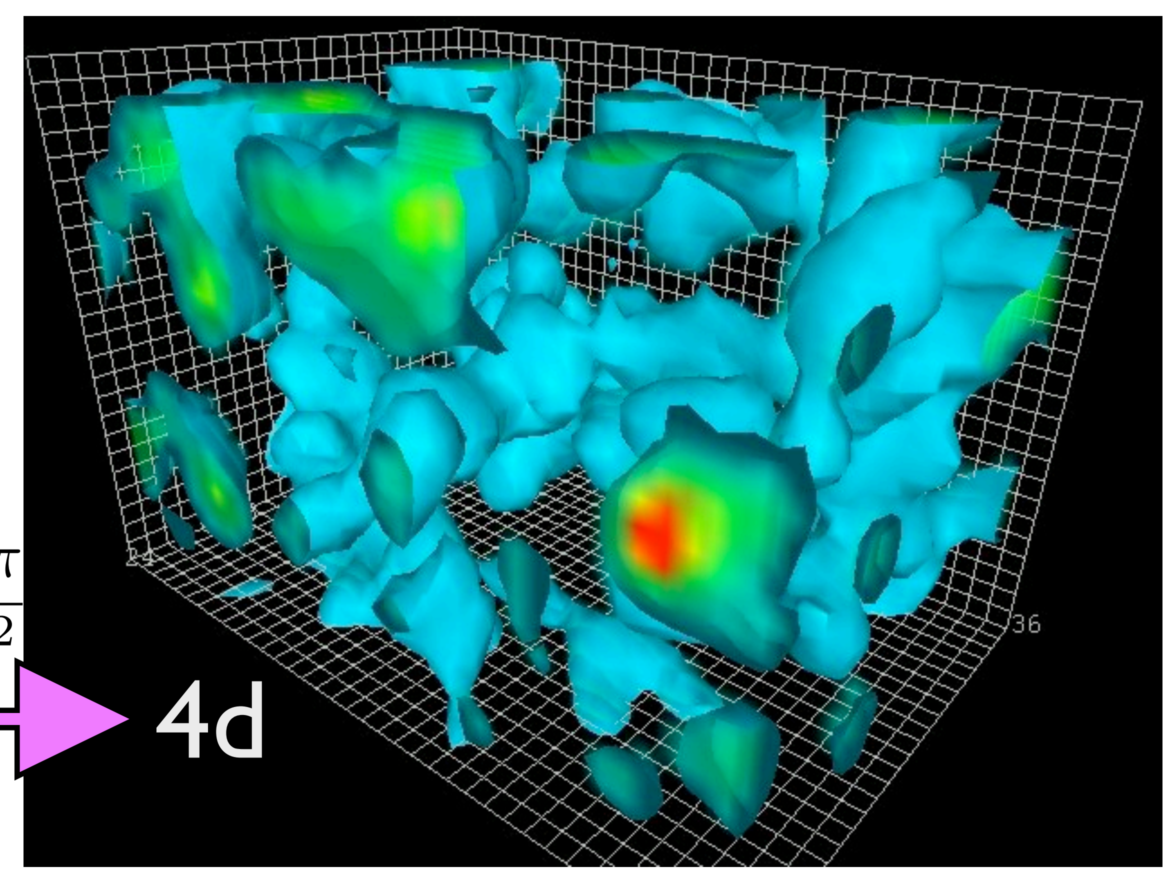
Nf>Nc semilocal
(+size moduli)

$$\pi_2(\mathcal{M}_{vac}) = \pi_2\left(\frac{SU(N+\tilde{N})}{SU(N) \times SU(\tilde{N}) \times U(1)}\right) = \mathbb{Z}$$

*Duality between two strongly coupled theories*

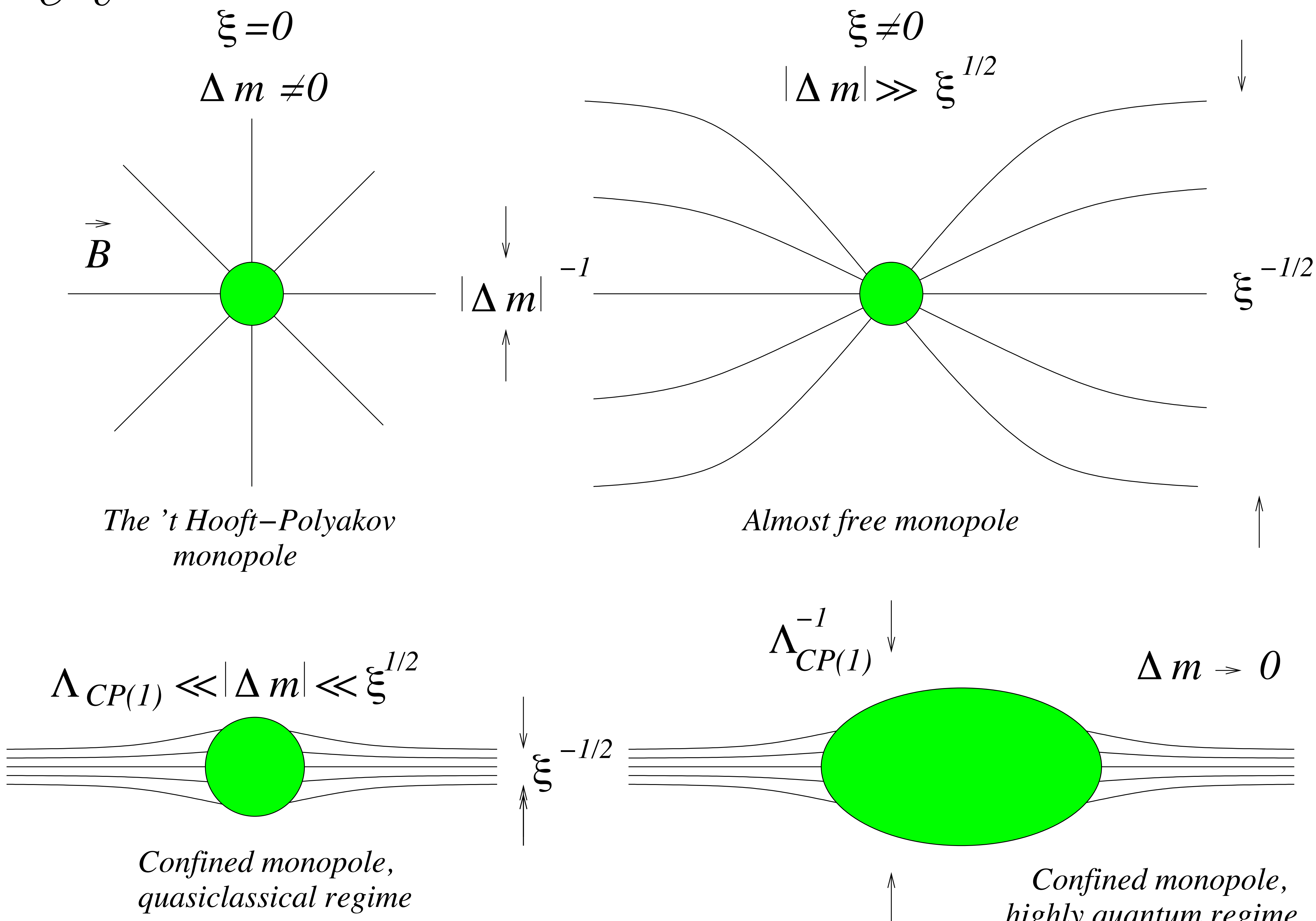



$$r = \frac{4\pi}{g^2}$$



# Confined monopoles

$$\xi = e^2 v^2$$




$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term $r$

# Hanany-Tong model as U(1) GLSM

$$\mathcal{L} = \int d^4\theta \left[ \sum_{i=1}^{N_c} \Phi_i^\dagger \,\mathrm{e}^{\mathcal{V}} \Phi_i + \sum_{i=1}^{\tilde{N}} \widetilde{\Phi}_i^\dagger \,\mathrm{e}^{-\mathcal{V}} \widetilde{\Phi}_i - r\mathcal{V} + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

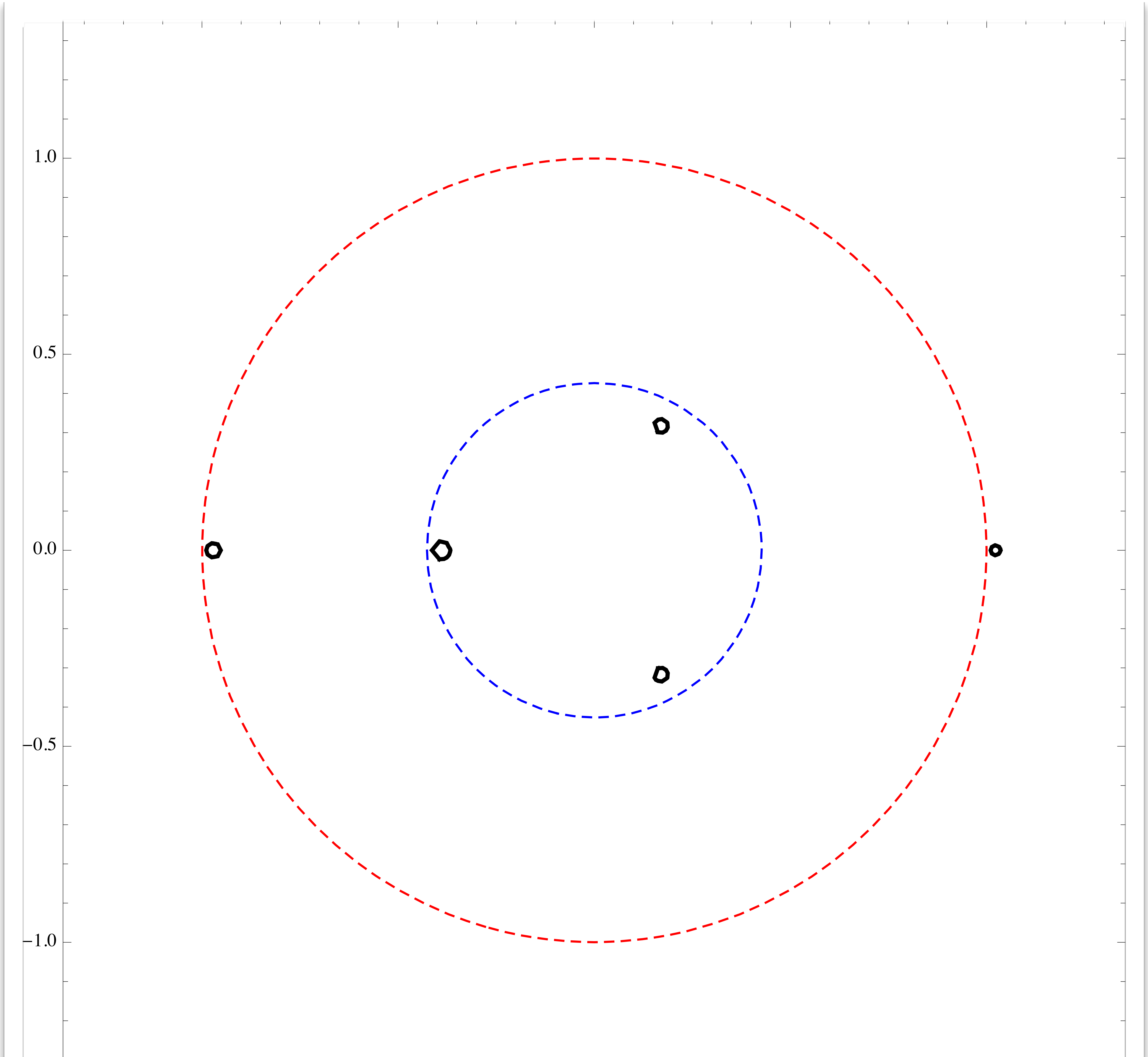
$$V = \theta^+ \bar{\theta}^+ (A_0 + A_3) + \theta^- \bar{\theta}^- (A_0 - A_3) - \theta^- \bar{\theta}^+ \sigma - \theta^- \bar{\theta}^+ \bar{\sigma} + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta} \bar{\lambda} + \bar{\theta}\theta\bar{\theta}\theta D$$

*One loop twisted effective superpotential is exact in (2,2)*

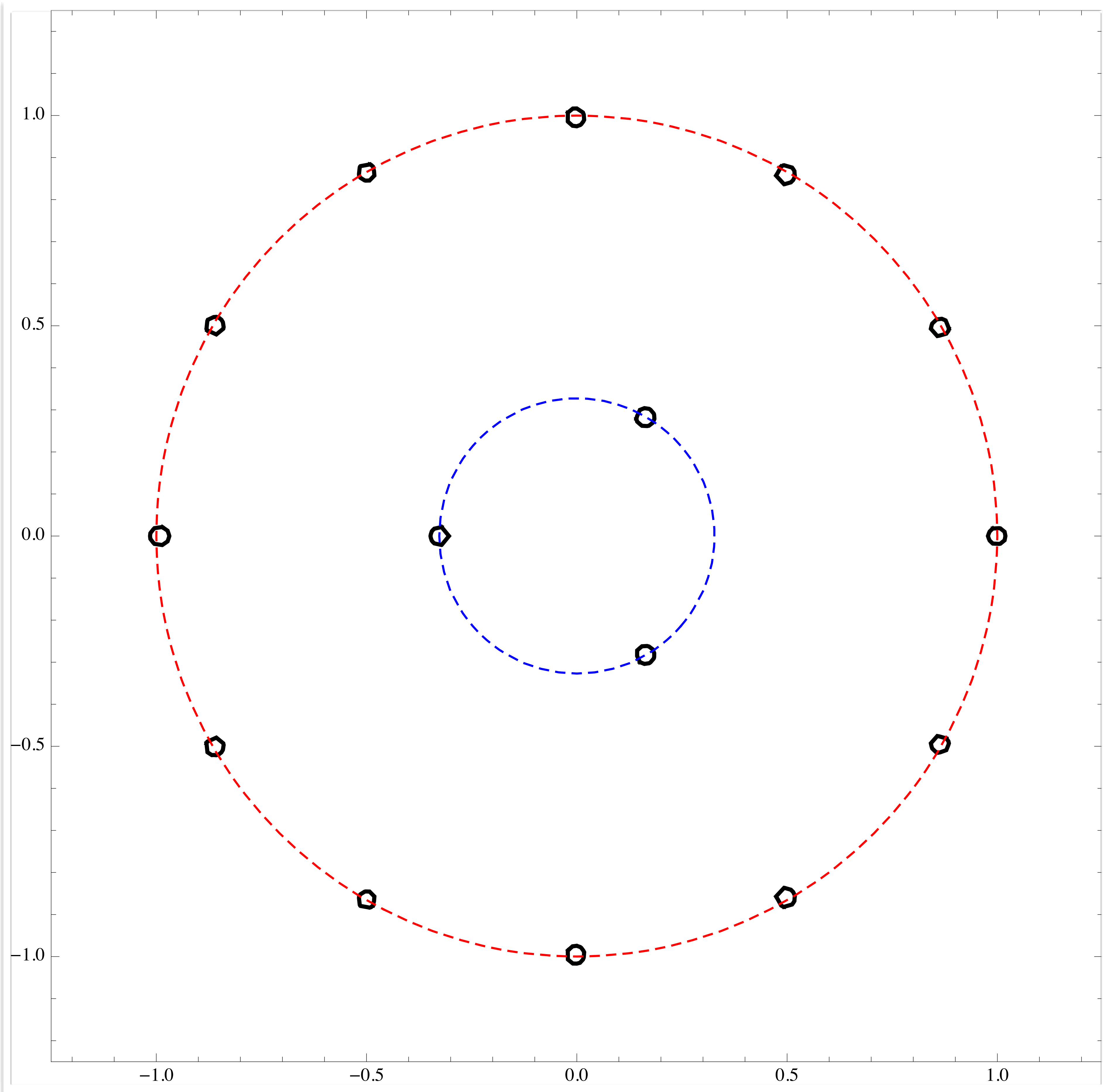
$$\begin{aligned}\widetilde{W}_{\text{eff}} &= -\frac{1}{2\pi} \sum_{i=1}^{N} (\sqrt{2}\sigma + m_i) \left( \log \frac{\sqrt{2}\sigma + m_i}{\Lambda} - 1 \right) + \\ &+ \frac{1}{2\pi} \sum_{j=1}^{\widetilde{N}} (\sqrt{2}\sigma + \widetilde{m}_j) \left( \log \frac{\sqrt{2}\sigma + \widetilde{m}_j}{\Lambda} - 1 \right) .\end{aligned}$$

gives vacua of the theory and its BPS spectrum

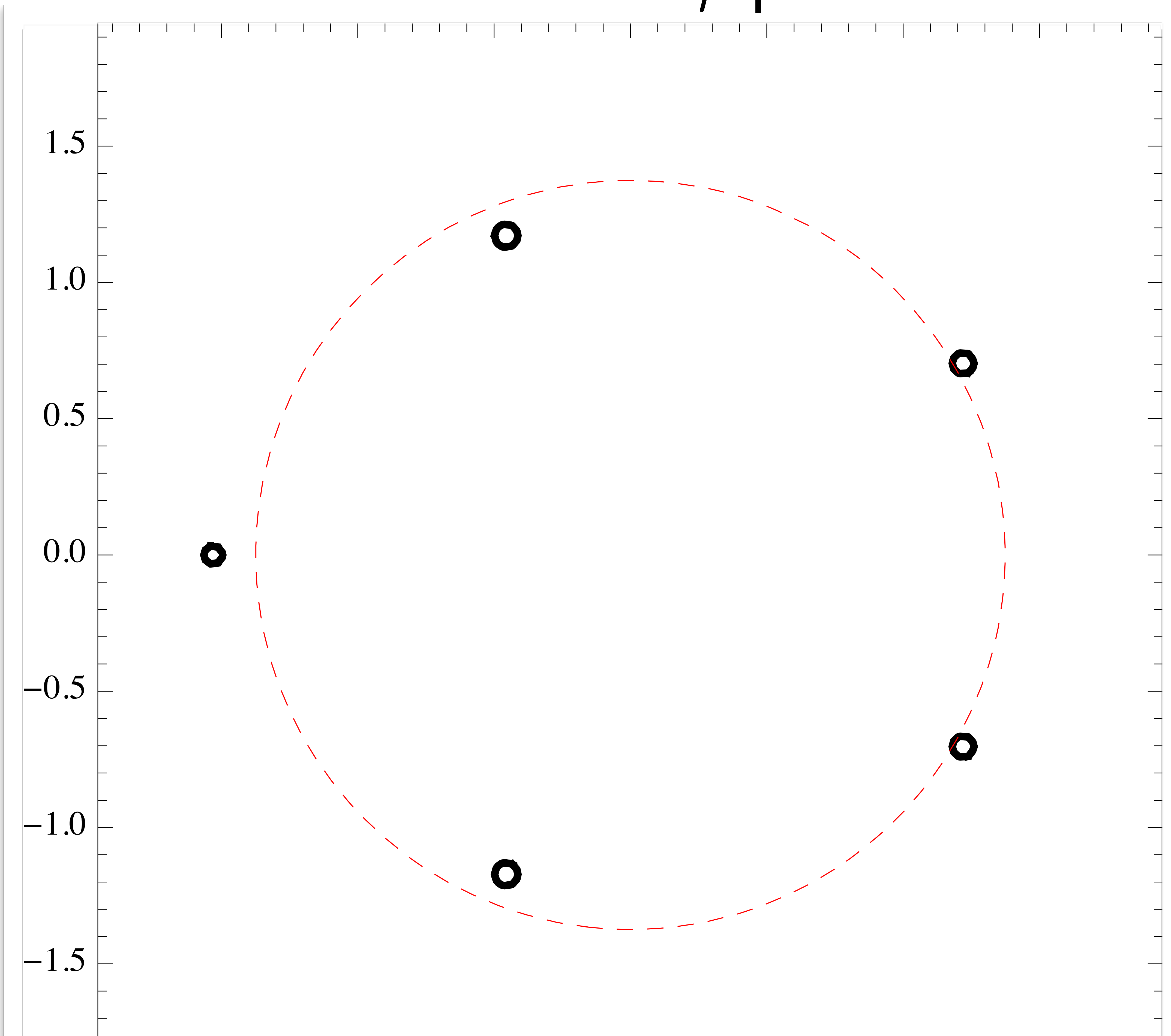
# N=5 Nf=8

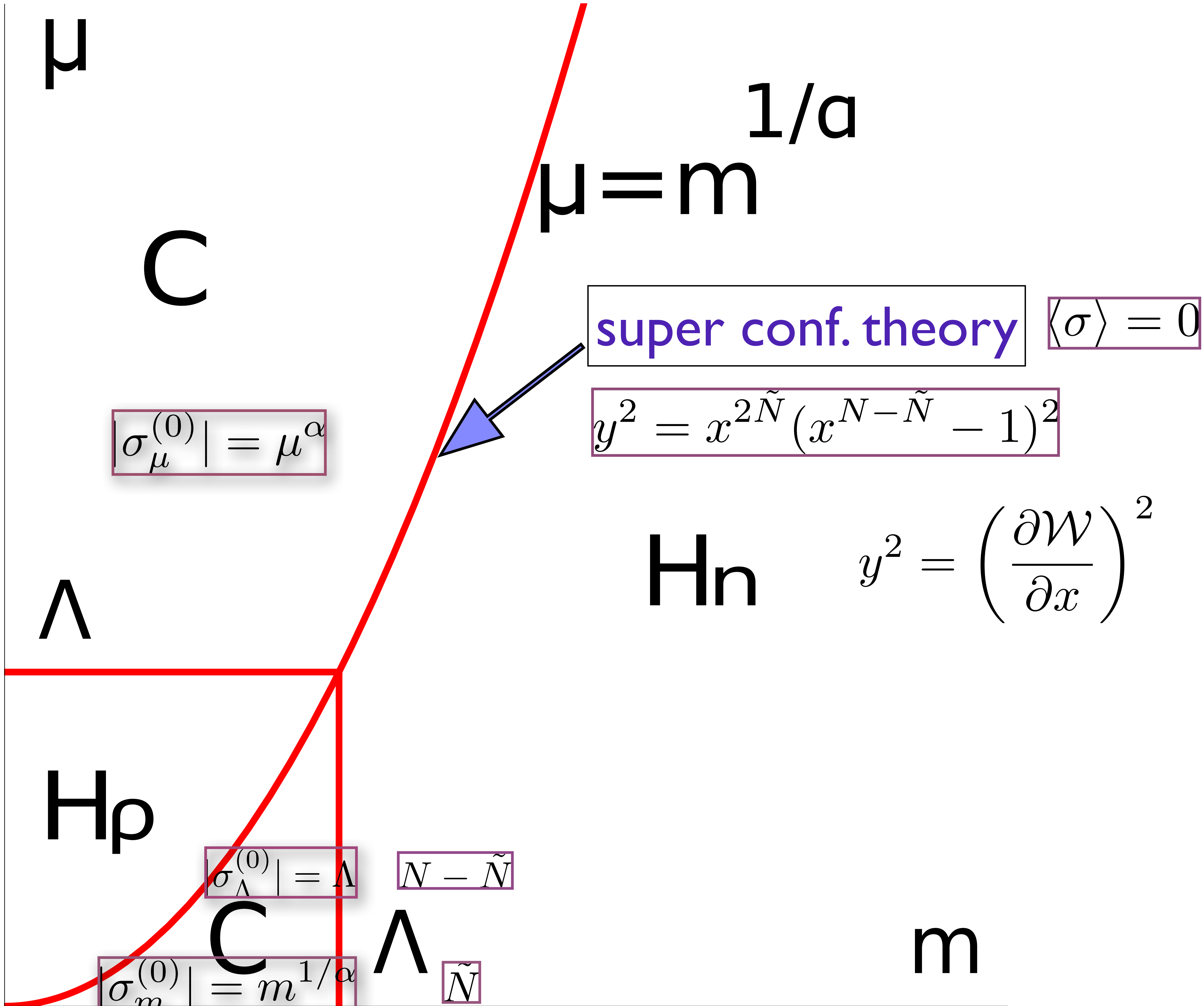


# N=15 Nf=18



# Nf=5 Cμ phase

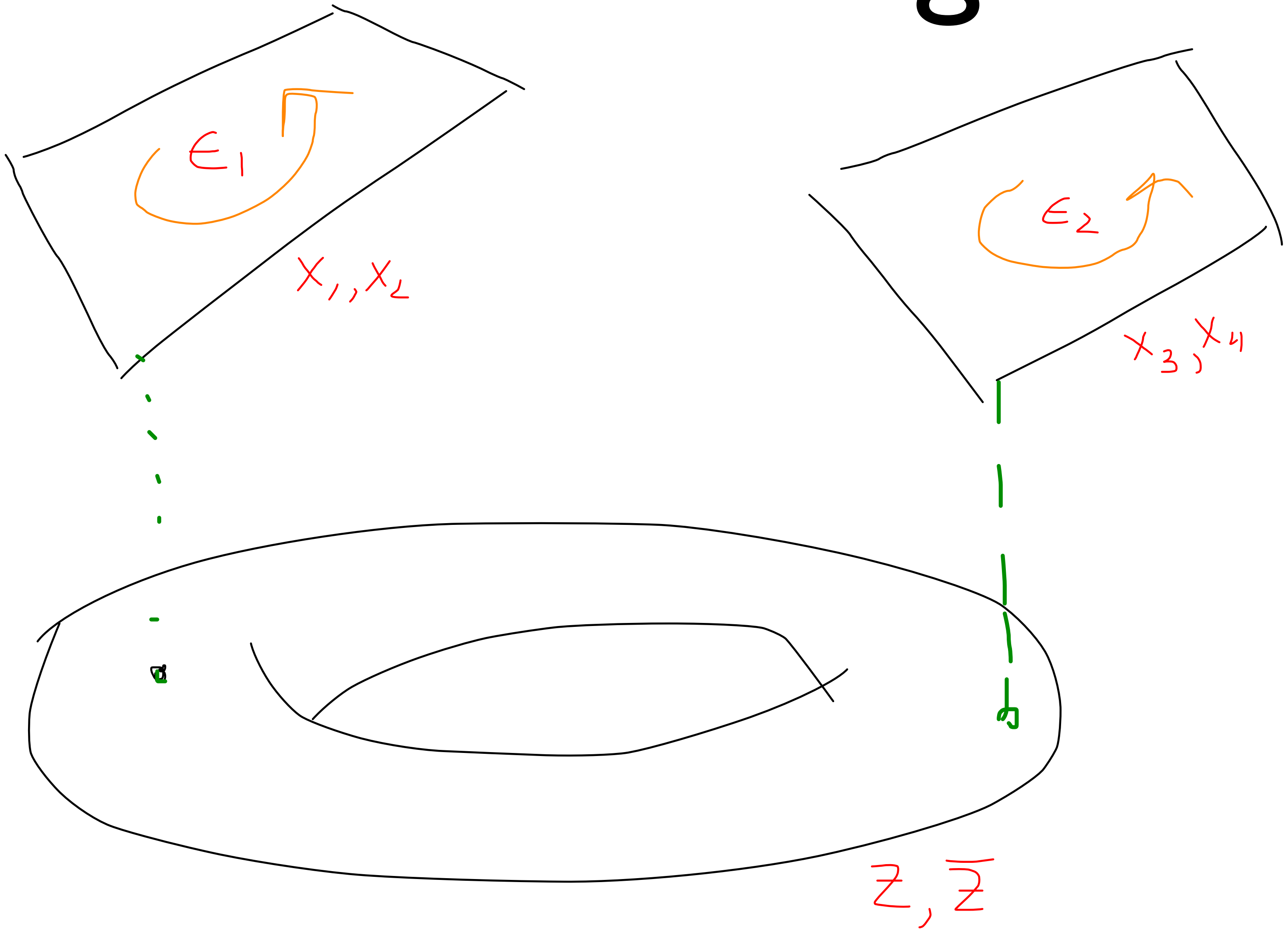



μ
1/a
μ=m
C
super conf. theory
$\langle\sigma\rangle = 0$
$y^2 = x^{2\tilde{N}}(x^{N-\tilde{N}} - 1)^2$
$|\sigma_\mu^{(0)}| = \mu^\alpha$
Hn
$y^2 = \left(\frac{\partial W}{\partial x}\right)^2$
Λ
Hp
$|\sigma_\Lambda^{(0)}| = \Lambda$
$N - \tilde{N}$
C
Λ
$\tilde{N}$
m
$|\sigma_m^{(0)}| = m^{1/\alpha}$

# AGT in NS limit

# Omega background

[Nekrasov et al]




Rotational symmetry broken to maximal torus

$$SO(4) \to SO(2) \times SO(2)$$

6d Metric

$$G_{AB}dx^A dx^B = A dz d\bar{z} + (dx^m + \Omega^m dz + \bar{\Omega}^m d\bar{z})^2$$

We will be interested in Nekrasov-Shatashvili limit

$$\Omega^m = (-i\epsilon x^2, i\epsilon x^1, 0, 0) = i\epsilon \partial_\varphi \qquad \epsilon_2 \to 0$$

# The AGT duality

$3g-3+n$ Coulomb branch

| Liouville theory on 2-sphere with 4 punctures at $\infty, 1, q, 0$ | 4d U(2) SQCD w/ 4 flavors with masses $m_1, m_2, m_3, m_4$ |
|---|---|

central charge

$$c = 1 + 6Q^2\,, \qquad Q = b + \frac{1}{b}$$

conformal dimensions of chiral operators

$$\Delta_1 = \alpha_0(Q-\alpha_0)\,, \quad \Delta_2 = \mu_0(Q-\mu_0)\,, \quad \Delta_3 = \mu_1(Q-\mu_1)\,, \quad \Delta_4 = \alpha_1(Q-\alpha_1)$$

$$\alpha_0 = \tfrac{1}{2}Q + \widetilde{\mu}_0\,, \quad \alpha = \tfrac{1}{2}Q + a\,, \quad \alpha_1 = \tfrac{1}{2}Q + \widetilde{\mu}_1$$

*Conformal block matches with instanton partition function*

$$\mathcal{Z}_{\text{inst}}(a, \mu_0, \widetilde{\mu}_0, \mu_1, \widetilde{\mu}_1) = (1-q)^{2\mu_0(Q-\mu_1)}\mathcal{F}_{\alpha_0\ \alpha\ \alpha_1}^{\ \mu_0\ \mu_1}(q)$$

$b = \epsilon_1 = 1/\epsilon_2$

**In NS limit** $b \to \infty$

# But the proof already exists! [Mironov, Morozov]

at large c conformal block becomes a hypergeometric function

[Zamolodchikov]

$$B_{\Delta;\Delta_1\Delta_2\Delta_3\Delta_4}(x) \xrightarrow{c\to\infty} {}_2F_1\Big(\Delta+\Delta_1-\Delta_2, \Delta+\Delta_3-\Delta_4; 2\Delta; x\Big) =$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!} \prod_{k=0}^{n-1} \frac{(\Delta+\Delta_1-\Delta_2+k)(\Delta+\Delta_3-\Delta_4+k)}{2\Delta+k}$$

Only chiral Nekrasov functions contribute

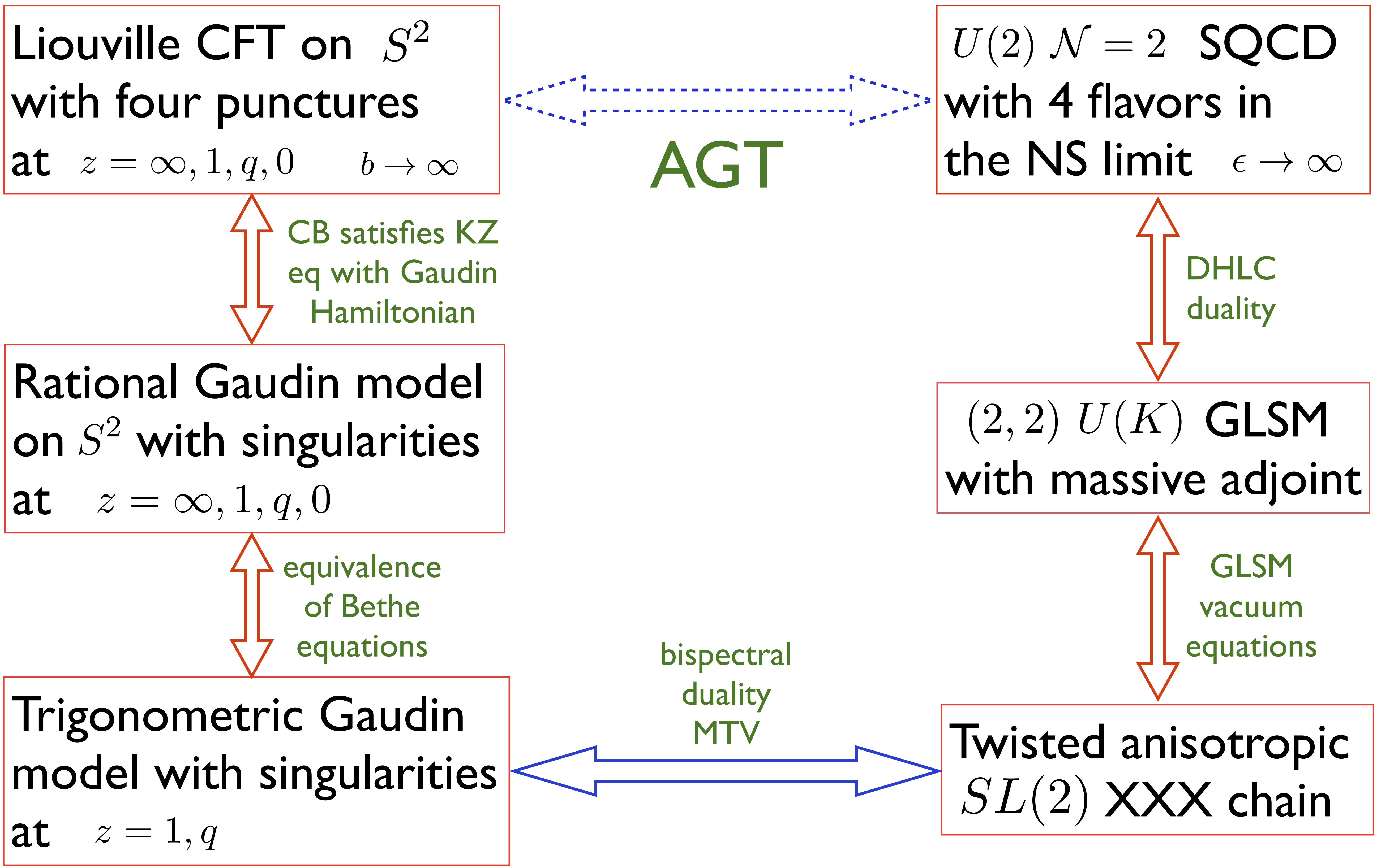
$$(Y, Y') = ([1^n], \emptyset) \text{ or } (\emptyset, [1^n])$$

One can identify each term of the expansion in the instanton number with the Taylor series in x for 2F1

Similar to Fateev-Litvinov conformal blocks

*Both proofs are rather formal and deal with each term in the series. Need more physical understanding...*

# Roadmap to proof



# 4d/2d in Omega background

[Dorey Hollowood Lee]

N=2 SQCD in Omega background
in NS limit with Nf=2Nc

$$\vec{a} = \vec{m}_F - \vec{n}\epsilon \qquad \vec{n} = (n_1, \ldots, n_L) \in \mathbb{Z}^L$$

$$\mathcal{W}^{(I)} \overset{\text{on-shell}}{\equiv} \mathcal{W}^{(II)}$$

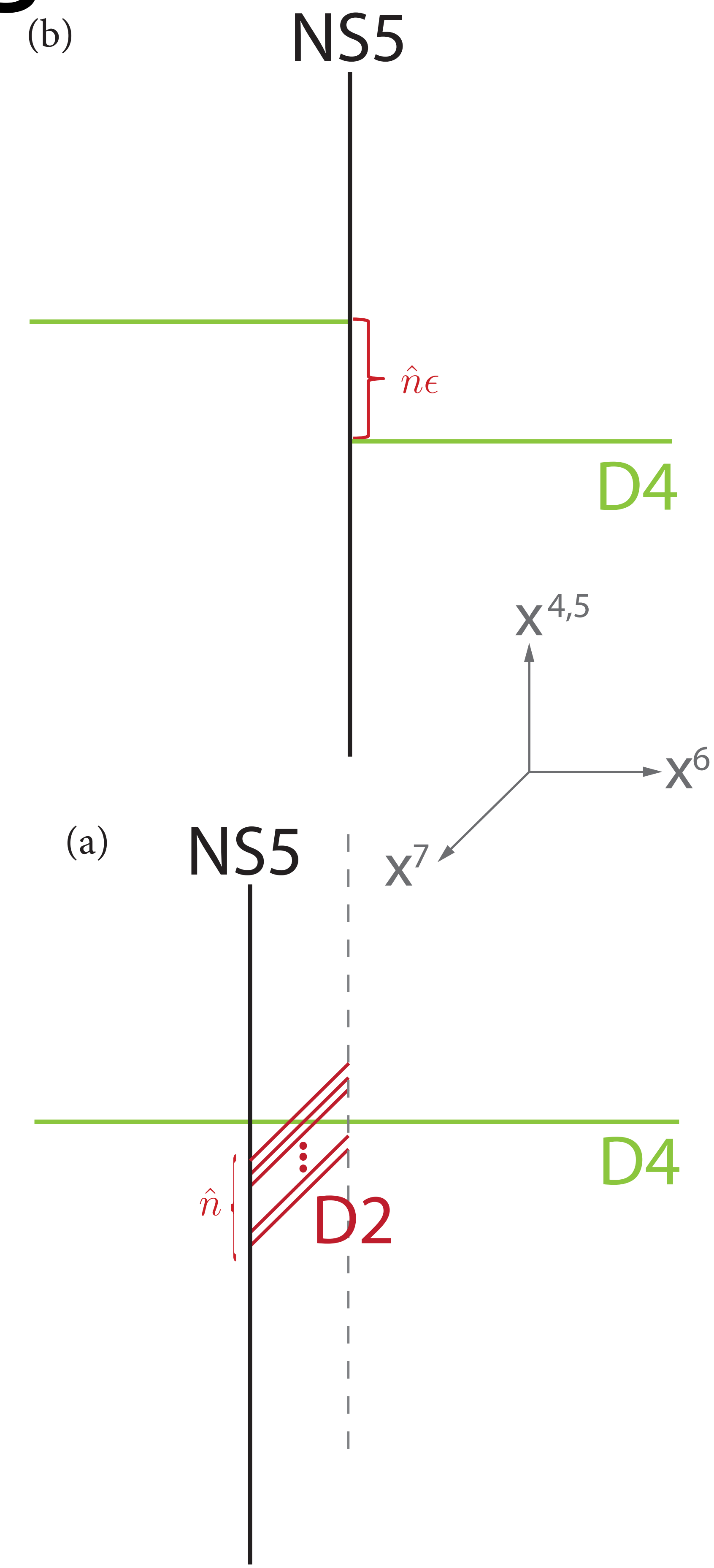
[CDHL]

exact proof

(2,2) GLSM w/ gauge group U(K)
massive adjoint and twisted masses

$$\vec{M}_F = \vec{m}_F - \frac{3}{2}\vec{\epsilon}\,, \qquad \vec{M}_{AF} = \vec{m}_{AF} + \frac{1}{2}\vec{\epsilon}\,.$$

$$M_{adj} = \epsilon \qquad K = \sum_{i=1}^{N} n_i - N$$



# Nekrasov-Shatashvili quantization

From 4d prepotential to 2d twisted superpotential

$$\widetilde{\mathcal{W}}(a,\epsilon) = \lim_{\epsilon_2 \to 0} \frac{\mathcal{F}(a,\epsilon,\epsilon_2)}{\epsilon_2} = \frac{\partial \mathcal{F}(a,\epsilon,\epsilon_2)}{\partial \epsilon_2}\Big|_{\epsilon_2=0}$$

at small epsilon

$$\widetilde{\mathcal{W}}(a,\epsilon) = \frac{\mathcal{F}(a)}{\epsilon} + \dots$$

Twisted superpotential is multivalued on Coulomb branch

$$\mathcal{W}^{(I)}(\vec{a},\epsilon) \;=\; \frac{1}{\epsilon}\mathcal{F}(\vec{a},\epsilon) \;-\; 2\pi i\,\vec{k}\cdot\vec{a}$$

Supersymmetric vacua

$$\exp\left(\frac{\partial \widetilde{W}(a)}{\partial a_i}\right) = 1$$

Quantization of a-m cycle

$$\frac{1}{2\pi}\oint_{\alpha_l} \lambda_{\mathrm{SW}} \;=\; \hbar\,\hat{n}_l$$

# Vortex interpretation

## SQCD in NS Omega background

$$\mathcal{L} = \frac{1}{4g^2}F^2_{mn} + |\nabla_m\phi - F_{mn}\bar{\Omega}^n|^2 + \frac{g^2}{2}|\phi\tau^a\bar{\phi} - i\nabla_m(\Omega^m\bar{\phi}^a - \bar{\Omega}^m\phi^a) + \bar{q}\tau^a q - \widetilde{q}\tau^a\bar{\widetilde{q}}|^2$$
$$+ \frac{1}{2}|\nabla_m q|^2 + \frac{1}{2}|\nabla_m\widetilde{q}|^2 + \frac{1}{2}|(\phi - m_i - i\Omega^m\nabla_m)q_i|^2 + \frac{1}{2}|(\phi - \widetilde{m}_i - i\Omega^m\nabla_m)\widetilde{q}_i|^2$$
$$+ 2g^2|\widetilde{q}\tau^a q|^2 + \frac{g^2}{2}|\widetilde{q}_i q_i - N\xi_{FI}|^2 + \frac{g^2}{8}(|q|^2 - |\widetilde{q}|^2)^2 \,. \qquad ($$

## Vacua

$$\phi^a = m^a - n^a\epsilon$$

## (2,2) SUSY is the same as for BPS vortices

## Generalized FI terms

$$\Xi^{a\,f}_{g} = i\nabla_{\alpha\dot{\alpha}}(\bar{\Omega}^{\alpha\dot{\alpha}}\phi^a - \Omega^{\alpha\dot{\alpha}}\bar{\phi}^a)\delta^f_g + \xi^f_{FI\,g}\delta^a_{N^2}$$

## BPS equations

$$B^a_3 + g^2(\bar{q}_i\tau^a q^i - \Xi^a) = 0$$
$$(\nabla_1 + i\nabla_2)q^i = 0$$
$$(\nabla_1 + i\nabla_2)\phi^a - (\Omega_2 - i\Omega_1)B^a_3 = 0$$

# XXX vs Gaudin

[Nekrasov Shatashvili]

## Gaudin model - Hitchin system on S2 with punctures

[Nekrasov]

Effective twisted superpotential

$$\widetilde{W}^{2d}_{\text{eff}}(\lambda) = \epsilon \sum_{a=1}^{K} \sum_{i=1}^{N} f\left(\frac{\lambda_a - M_i}{\epsilon}\right) - \epsilon \sum_{a=1}^{K} \sum_{i=1}^{N} f\left(\frac{\lambda_a - \widetilde{M}_i}{\epsilon}\right) + \epsilon \sum_{a,b=1}^{K} f\left(\frac{\lambda_a - \lambda_b - \epsilon}{\epsilon}\right) + 2\pi i \hat{\tau} \sum_{a=1}^{K} \lambda_a \,,$$

Ground state equations

Heisenberg SL(2) magnet

twisted and anisotropic

$$\prod_{a=1}^{N} \frac{\lambda_i - \nu_a + \frac{\epsilon}{2} S_a}{\lambda_i - \nu_a - \frac{\epsilon}{2} S_a} = q \prod_{\substack{j=1 \\ j \neq i}}^{K} \frac{\lambda_i - \lambda_j - \epsilon}{\lambda_i - \lambda_j + \epsilon}$$

Large anisotropy limit

*rational* Gaudin model

$$\lambda_i \mapsto x\lambda_i, \quad \nu_a \mapsto x\nu_a, \quad \hat{\tau} \mapsto \frac{\hat{\tau}}{x}$$

$$\frac{\log q}{\epsilon} - \sum_{a=1}^{N} \frac{S_a}{\lambda_i - \nu_a} = \sum_{\substack{j=1 \\ j \neq i}}^{K} \frac{2}{\lambda_i - \lambda_j}$$

Bethe equations obtained by diagonalizing (4 sites)

$$S(u) = \sum_{a=1}^{4} \frac{\mathcal{H}_a}{u - z_a} + \sum_{a=1}^{4} \frac{\Delta(\nu_a)}{(u - z_a)^2}$$

Gaudin Hamiltonians

$$\mathcal{H}_a = \sum_{b \neq a} \sum_{\alpha,\beta=1}^{\dim(\mathfrak{g})} \frac{\mathfrak{J}^{(b)}_{\alpha} \mathfrak{J}^{\alpha\,(b)}}{z_a - z_b}$$

# Bispectral duality

[Mukhin Tarasov Varchenko]

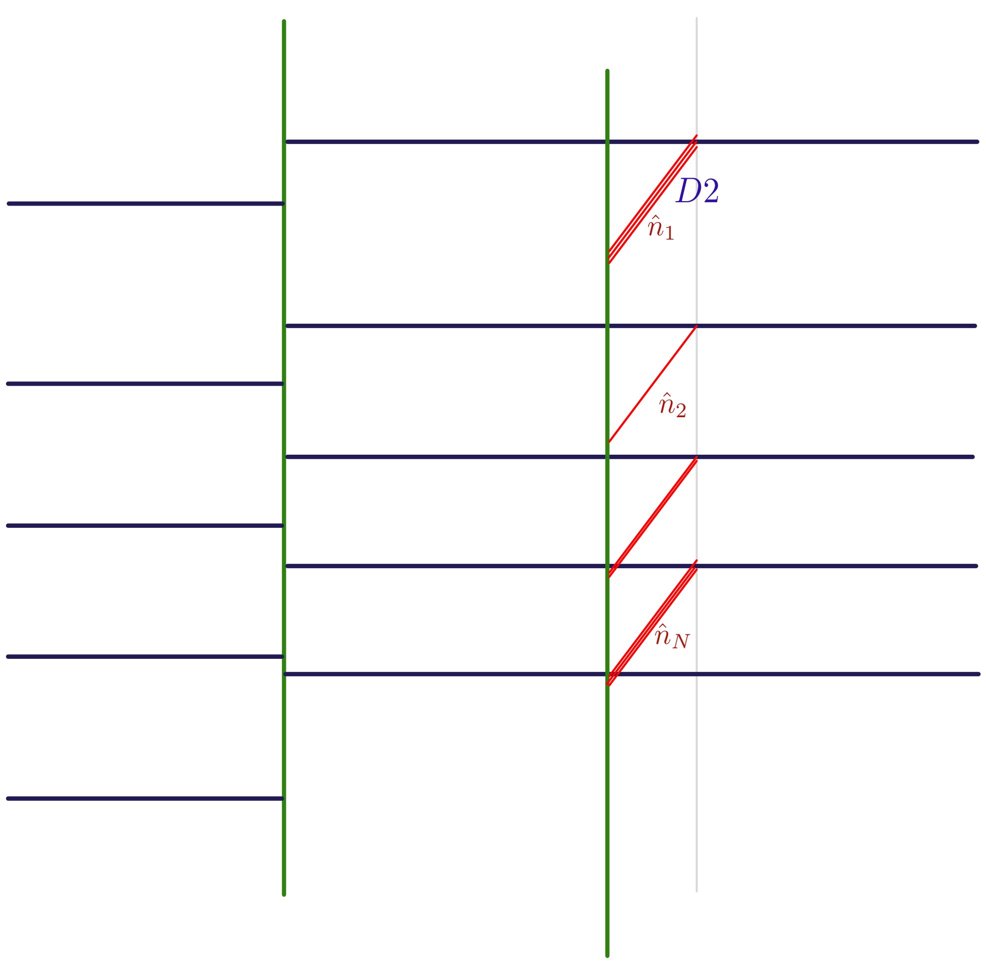
## Trigonometric Gaudin vs XXX magnet

$$\frac{\mathcal{M}_1 - \mathcal{M}_2 - \epsilon}{t_i} + \sum_{b=1}^{2} \frac{\nu_b \epsilon}{t_i - z_b} - \sum_{\substack{j=1 \\ j\neq i}}^{\kappa_2} \frac{2\epsilon}{t_i - t_j} = 0, \quad i = 1, \ldots, \kappa_2\,,$$

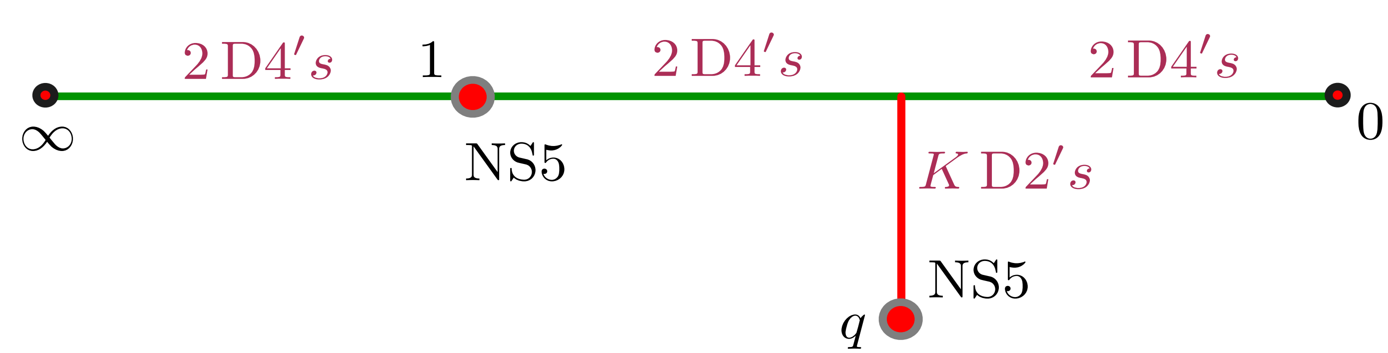
Equations have isomorphic spaces of solutions

$$\prod_{a=1}^{2} \frac{\lambda_i + \mathcal{M}_a}{\lambda_i + \mathcal{M}_a + \kappa_a \epsilon} = \frac{z_2}{z_1} \prod_{\substack{j=1 \\ j\neq i}}^{\nu_2} \frac{\lambda_i - \lambda_j - \epsilon}{\lambda_i - \lambda_j + \epsilon}\,, \quad i = 1, \ldots, \nu_2$$

$$\kappa_1 + \kappa_2 = \nu_1 + \nu_2$$




Nice brane interpretation
Rotation by 90 degrees



# From Liouville to Gaudin

Gaudin Hamiltonian in KZ equation

$$b^2 \frac{d\Psi(z_i)}{dz_i} = \mathcal{H}_{Gaud}\Psi(z_i)\,, \quad i = 1, \ldots, L$$

[Babujian Flume]

Liouville CB satisfies 2nd order ODE which in the NS limit becomes KZ equation with Gaudin Hamiltonian

[Teschner]

with rescaled conformal dimensions

$$\delta_i = -\frac{\Delta_i}{b^2}$$

$$\delta_1 = \left(\frac{\widetilde{\mu}_0}{b} - \frac{1}{2}\right)\left(\frac{\widetilde{\mu}_0}{b} + \frac{1}{2}\right)$$

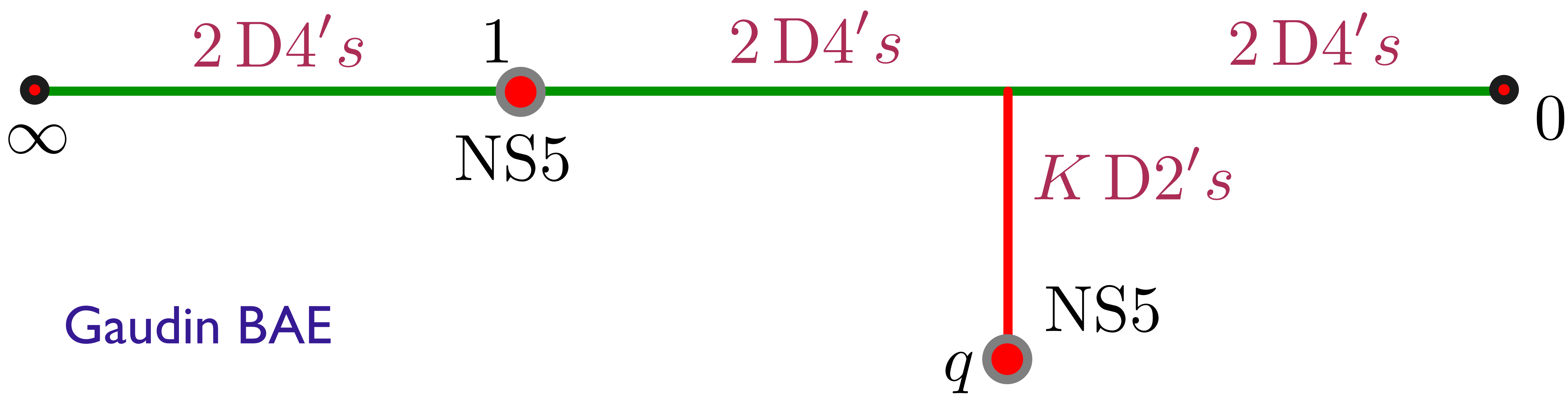
$$\delta_2 = \left(\frac{\mu_0}{b} - 1\right)\frac{\mu_0}{b}\,,$$

$$\delta_3 = \left(\frac{\mu_1}{b} - 1\right)\frac{\mu_1}{b}\,,$$

$$\delta_4 = \left(\frac{\widetilde{\mu}_1}{b} - \frac{1}{2}\right)\left(\frac{\widetilde{\mu}_1}{b} + \frac{1}{2}\right)$$

*take home message: CB in Liouville - wave function in Gaudin*

# The Duality




**Gaudin BAE**

$$\sum_{b=1}^{4} \frac{\nu_b \epsilon}{t_i - z_b} - \sum_{\substack{j=1\\ j\neq i}}^{\kappa_2} \frac{2\epsilon}{t_i - t_j} = 0$$

$$\epsilon\nu_1 = 0, \quad \epsilon\nu_2 = K, \quad \epsilon\nu_3 = m_3 - m_4 - \epsilon = 2\widetilde{\mu}_1 - \epsilon$$

**Higgs branch root**

$$a_a = m_{2+a} - n_a\epsilon\,, \quad a = 1, 2$$

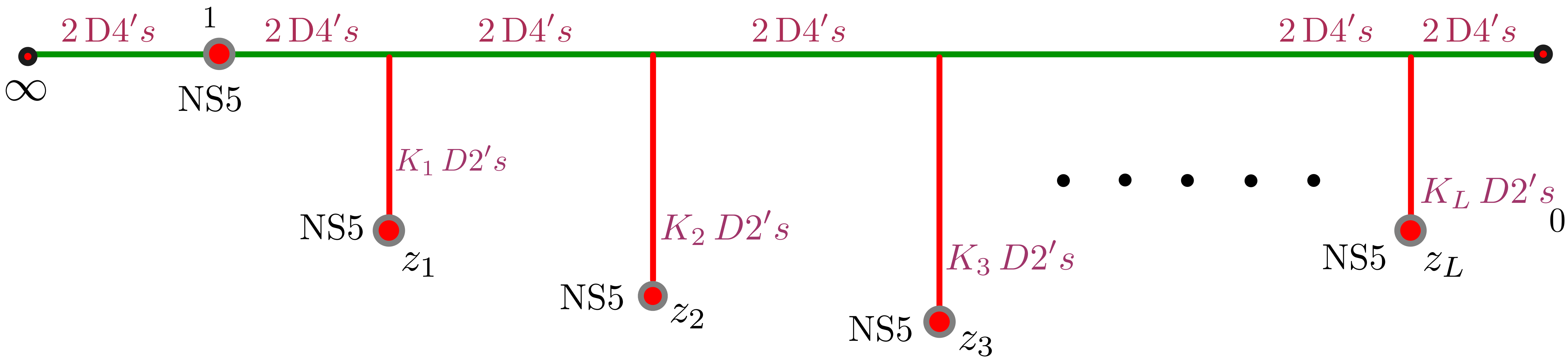
**U(1) condition**

$$\frac{\mu_1}{\epsilon} = \frac{n_1 + n_2}{2}$$

# AGT in NS limit

| | |
|---|---|
| Liouville conformal block at $b \to \infty$ on $S^2$ with four punctures | $U(2)\,, N_f = 4$ SQCD instanton partition function in the NS limit |
| Rational Gaudin model from KZ equation on conformal blocks | $SL(2)$ spin chain from the ground state equation for the 2d GLSM dual to 4d theory |
| Puncture's positions $z_2/z_1$ | Instanton number $q$ |
| $\mathfrak{sl}_2$ spin at $z_2 = q$ | $U(1)$ condition, number of D2 branes emerging at NS5 brane at $z_2 = q$ |
| Conformal dimensions of chiral operators at points $z_2 = q$, $z_3 = 0$ | Quadratic $\mathfrak{sl}(2)$ Casimir eigenvalues on spin $\frac{1}{2}\hat{n}_1 + \frac{1}{2}\hat{n}_2$ and $-\frac{1}{2}\hat{n}_1 + \frac{1}{2}\hat{n}_2 - \frac{1}{2}$ representations |
| Gaudin Hilbert space sectors with different number $\kappa_a$ of Bethe roots | Higgs branch lattice $\{n_a\}$ |

# Quiver Generalizations




## conformal dimensions

$$\alpha_0(Q-\alpha_0)\,, \quad \mu_0(Q-\mu_0)\,, \ldots, \quad m_L(Q-m_L)\,, \quad \alpha_{L+1}(Q-\alpha_{L+1})$$

## Higgs branch

$$a_a^{(p)} = m_a^{(p)} + n_a^{(p)}\epsilon + \sum_{k=1}^{L} \mu_k^{(p)}$$

## Casimir eigenvalues and spins of representations at each NS5 from the number of D2 branes

$$\frac{K_1}{2}\left(\frac{K_1}{2}+1\right), \quad \ldots, \quad \frac{K_L}{2}\left(\frac{K_L}{2}+1\right) \qquad K_i = \hat{n}_1^{(i)} + \hat{n}_2^{(i)}$$

# Strings Domain Walls and Monopoles

# SUSY algebra

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\beta}\} = 2\delta^{IJ} P_{\alpha\dot\beta} + 2\delta^{IJ} Z_{\alpha\dot\beta}$$ ← *strings*

$$\{Q^I_\alpha, Q^J_\beta\} = 2Z^{IJ}_{\alpha\beta}$$ ← *monopoles* *domain walls*

In N=2 SYM we only find dyons as BPS solitons in the low energy effective theory

Let us see what happens in Omega background

Four supercharges remain

$$\bar{Q}_{\dot\alpha J} = \tfrac{1}{2}\epsilon_{\dot\alpha J}\bar{Q} + \tfrac{1}{2}(\bar\sigma_{mn})_{\dot\alpha J}\bar{Q}^{mn}$$

$$Q_{12},\ Q_{21},\ \bar{Q}_{\dot 1 2},\ \bar{Q}_{\dot 2 1}\,.$$

Symmetry breaking pattern

$$SU(2)_c \times SU(2)_R \times SU(2)_{\mathcal{R}} \to U(1)_c \times SU(2)_{R+\mathcal{R}}$$

SUSY transform

$$\delta\phi = \zeta^I_\alpha(\lambda^\alpha_I - \Omega^m(\sigma_m)^{\alpha\dot\alpha}\bar\lambda_{I\dot\alpha}) + \bar\zeta^I_{\dot\alpha}\Omega^m(\bar\sigma_m)^{\alpha\dot\alpha}\lambda_{I\alpha}\,,$$

$$\delta\lambda_{I\alpha} = \zeta_{I\beta}((\sigma^{mn})^\beta_\alpha F_{mn} + i[\phi,\bar\phi]\delta^\beta_\alpha + \nabla_m(\bar\Omega^m\phi - \Omega^m\bar\phi)\delta^\beta_\alpha) + \bar\zeta_{I\dot\beta}(\sigma^m)^{\dot\beta}_\alpha(\nabla_m\phi - F_{mn}\Omega^n)\,.$$

# Vortices in Omega background

[PK Gorsky Chen] in progress

## Can view Omega deformation as formal replacement

$$\phi \mapsto \phi - i\Omega^m \nabla_m + \tfrac{i}{2}\Omega^{mn} S_{mn}$$

## Lagrangian

$$\mathcal{L} = \tfrac{1}{4g^2}(F^a_{mn})^2 + |\nabla_m \phi^a - F^a_{mn}\bar{\Omega}^n|^2 + \tfrac{1}{2}|\phi\tau^a\bar{\phi} - i\nabla_m(\Omega^m\bar{\phi}^a - \bar{\Omega}^m\phi^a) + i\bar{\Omega}^m\Omega^n F^a_{mn}|^2$$

## String central charge current

$$\zeta_3 = \tfrac{1}{2}\partial_m\left((\phi^a\bar{\Omega}^m - \bar{\phi}^a\Omega^m)B^a_3\right)\sigma^3_{\alpha\dot{\alpha}}\delta^{IJ} = \tfrac{i}{2}B^a_3\partial_\varphi(\phi^a\bar{\epsilon} - \bar{\phi}^a\epsilon)\sigma^3_{\alpha\dot{\alpha}}\delta^{IJ}$$

## Bogomolny completion

$$\mathcal{L} = \tfrac{1}{2}|B^a_z + \phi\tau^a\bar{\phi} - i\nabla_m(\Omega^m\bar{\phi}^a - \bar{\Omega}^m\phi^a)|^2 + \tfrac{1}{2}|\mathcal{D}_1\phi^a + i\mathcal{D}_2\phi^a - (\Omega_2 - i\Omega_1)B^a_z|^2$$
$$+ \partial_m(B^a_z(\Omega^m\bar{\phi}^a - \bar{\Omega}^m\phi^a)) \geq \partial_m(B^a_z(\Omega^m\bar{\phi}^a - \bar{\Omega}^m\phi^a))\,.$$

central charge
is nonzero
if conical singularity
is present

$$Z_{\text{string}} = \int d^3x\,\zeta_3 = \int dz \int d\rho\,\rho \int_0^{2\pi} d\varphi\,\partial_\varphi(\Re e(\epsilon\bar{\phi}^a)B^a_3)$$

*similar to cosmic string*

*we call it epsilon-string*

# Monopoles and domain walls

Let’s try to find a monopole

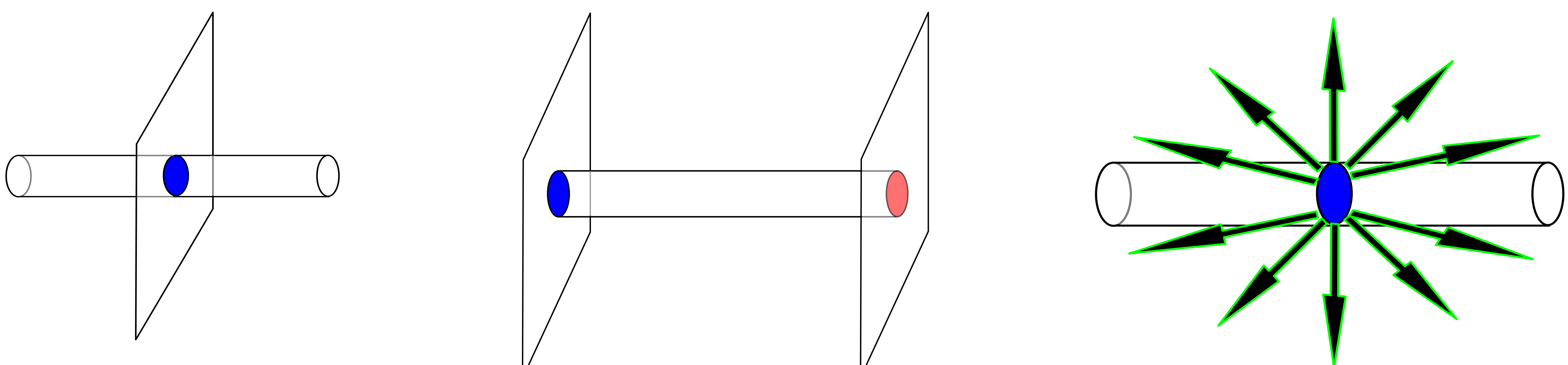
[Ito, Kamashita, Sasaki]

$$B_3^a - \nabla_3 \phi^a - \epsilon\, x^m B_m^a = 0$$
$$B_m^a - \nabla_m \phi^a + \epsilon\, x_m B_3^a = 0$$

On the solution adjoint scalar interpolates between different values at large and small z, magnetic field pattern is spherically symmetric

Naturally suggests that this monopole is located on a domain wall separating two vacua

Tension $$T = 2\epsilon \left( (v+\epsilon)^2 - (v-\epsilon)^2 \right) = 8v\epsilon^2$$



# Conclusions and open questions

- Study of SQCD BPS (and beyond) spectrum can effectively be done using 2d GLSM
- 4d/2d duality helps to understand AGT in NS limit by reducing it to bispectral duality
- Study dynamics of new solitons (eps strings, d.w.)
- Generalize to other AGT pairs