Non-Abelian Vortices and 4d/2d Correspondence

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work in progress with A.Gorsky, H. Chen

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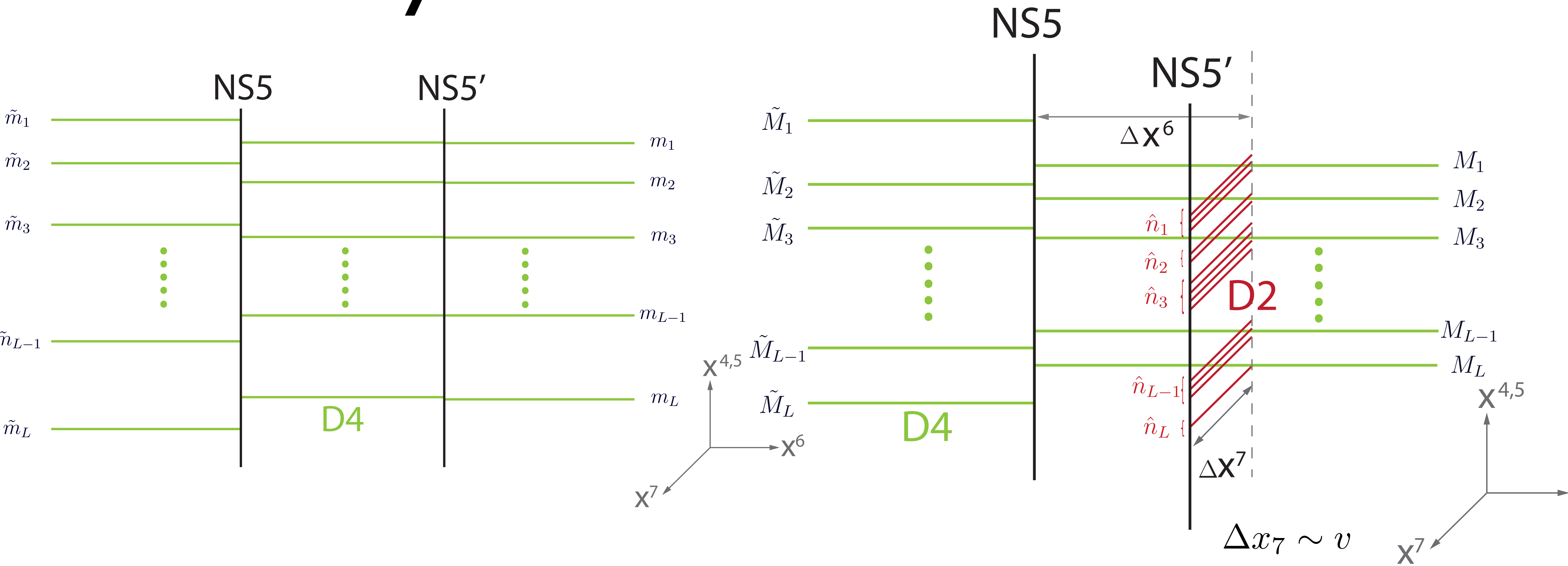
Outline

- 4d/2d correspondence in a nutshell
  - ⋆ *From brane construction*
  - ⋆ *The Dictionary of 4d/2d*
  - ⋆ *Derivation from vortices*
- 4d/2d in Omega background
  - ⋆ *N=2 theory in Nekrasov-Shatashvili limit*
  - ⋆ *Monopoles vortices and strings in Omega background*

Hanany-Witten construction



SQCD $N_f = 2N_c$

2d FI parameter $r = \dfrac{\Delta x^6}{2\pi g_s l_s} = \dfrac{4\pi}{e^2}$

Higgs branch root

$$\sigma = X^4 + iX^5 \quad , \quad Z = X^1 + iX^2$$

Color-flavor locked phase of SQCD

$$V = \frac{1}{g^2}\mathrm{Tr}\,|[\sigma,\sigma^\dagger]|^2 + \mathrm{Tr}\,|[\sigma,Z]|^2 + \mathrm{Tr}\,|[\sigma,Z^\dagger]|^2 + \sum_{a=1}^{N} \psi_a^\dagger \sigma^\dagger \sigma \psi_a$$

$$+\frac{g^2}{2}\mathrm{Tr}\left(\sum_a \psi_a \psi_a^\dagger + [Z,Z^\dagger] - r\,1_k\right)^2$$

In the simplest case $\mathbb{CP}^{N-1}$ model

@Dorey

4d / 2d duality

[Dorey Hollowood, Tong]

$\mathcal{N}=2 \quad SU(N)$ SQCD	(2,2) $U(1)$ GLSM e
$N_f = N + \tilde{N}$ fund hypers w/ masses	N chiral +1 $\tilde{N}$ chiral -1 w/ *twisted* masses
$m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$	$m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$
$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$	$\tau = ir + \frac{\theta}{2\pi}$
on baryonic Higgs branch	vortex moduli space
BPS dyons (Seiberg-Witten)	kinks interpolating between different vacua

BPS spectra (as functions of masses, Lambda) are the same

4d / 2d duality

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w/ masses	w/ *twisted* masses
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Nonabelian vortices help to understand it from pure field theory constructions

$U(N_c)$ $\mathcal{N}=2$ $d=4$ SQCD w/ N_f quarks

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\beta}\} = 2\delta^{IJ}P_{\alpha\dot\beta} + 2\delta^{IJ}Z_{\alpha\dot\beta}$$

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$$\mathcal{L} = \mathrm{Im}\left[\tau\int d^4\theta\,\mathrm{Tr}\left(Q^{i\,\dagger}e^V Q_i + \tilde{Q}^{i\,\dagger}e^V\tilde{Q}_i + \Phi^\dagger e^V\Phi\right)\right]$$
$$+\mathrm{Im}\left[\tau\int d^2\theta\left(\mathrm{Tr}W^{\alpha\,2} + m^i_j\tilde{Q}_iQ^j + Q_i\Phi\tilde{Q}^i\right)\right]$$

bosonic part

FI term

$$S = \int d^4x\,\mathrm{Tr}\left\{\frac{1}{2g^2}F^2_{\mu\nu} + \frac{1}{g^2}|D_\mu\Phi|^2 + |\nabla_\mu Q|^2 + \frac{g^2}{4}(Q\bar{Q}-\xi)^2 + |\Phi Q + QM|^2\right\}$$

BPS conditions

$$B_3 - g^2(Q\bar{Q}-\xi^2) = 0$$
$$\nabla_3 Q = 0$$

String tension

$$T = \xi\int d^2x\,\mathrm{Tr}F_{12} = 2\pi\xi n$$

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monopoles domain walls

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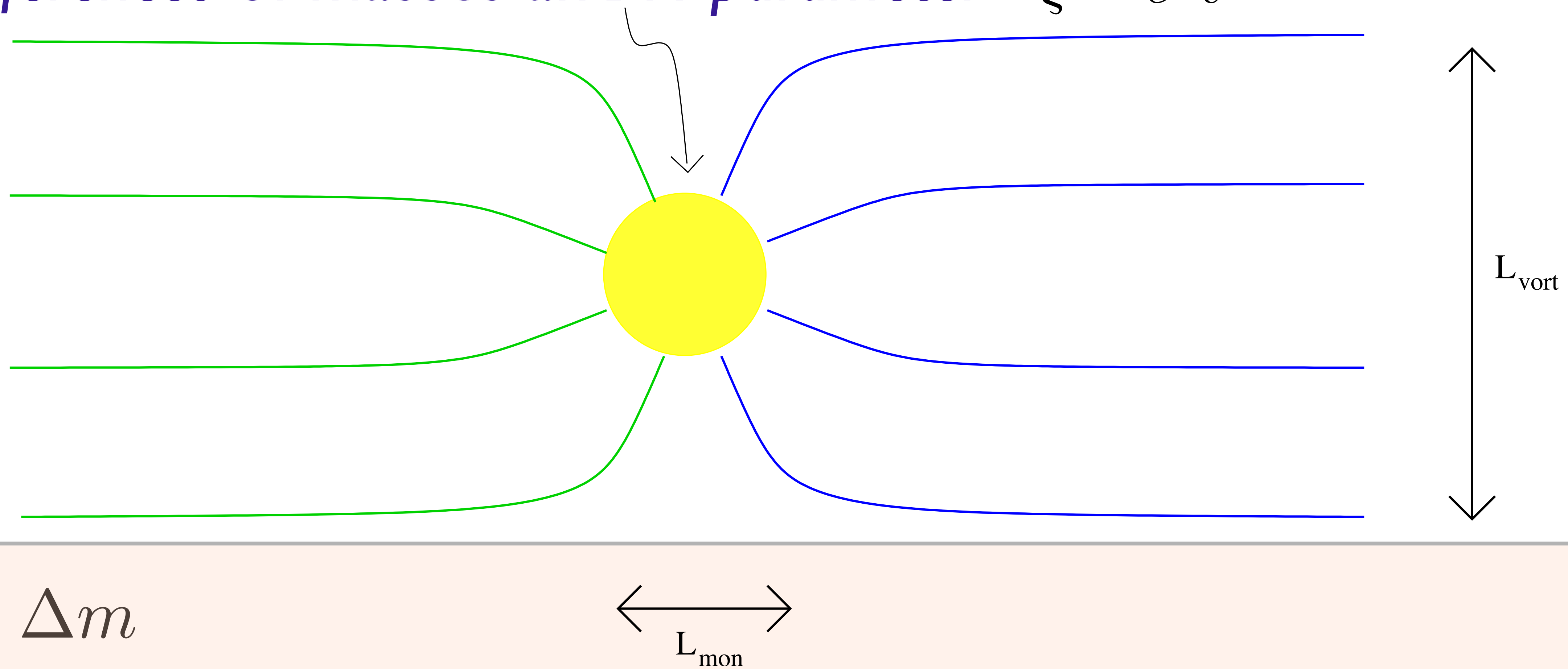
Monopoles on Higgs Phase

[Shifman, Yung]
[Tong]

Higgs branch condition $\phi = \mathrm{diag}(m_i) \quad , \quad q^a{}_i = v\delta^a{}_i \quad , \quad \tilde{q}^a{}_i = 0$

Pattern of symmetry breaking depends on the relationship between the *differences* of masses and *FI parameter* $\xi = e^2 v^2$



$ev \gg \Delta m$

$$U(N)_G \times SU(N)_F \xrightarrow{v} SU(N)_{\mathrm{diag}} \xrightarrow{m} U(1)^{N-1}_{\mathrm{diag}}$$

$ev \ll \Delta m$

$$U(N)_G \times SU(N)_F \xrightarrow{m} U(1)^N_G \times U(1)^{N-1}_F \xrightarrow{v} U(1)^{N-1}_{\mathrm{diag}}$$

(2,2) 2d GLSM

[Witten]

Consider U(1) gauge theory

$$\mathcal{L}_{\text{vortex}} = \frac{1}{2g^2}\left(F_{01}^2 + |\partial\sigma|^2\right) + \sum_{i=1}^{N_c}\left(|\mathcal{D}\psi_i|^2 + |\sigma - m_i|^2|\psi_i|^2\right) + \frac{g^2}{2}\left(\sum_{i=1}^{N_c}|\psi_i|^2 - r\right)^2$$

$$\text{Vacuum } i : \quad \sigma = m_i \quad , \quad |\psi_j|^2 = r\delta_{ij}$$

for vortex embedded into i's U(1) subgroup

FI term runs

$$r(\mu) = r_0 - \frac{N_c}{2\pi}\log\left(\frac{M_{UV}}{\mu}\right) \Rightarrow \Lambda = \mu\exp\left(-\frac{2\pi r(\mu)}{N_c}\right)$$

Effective twisted superpotential

$$\mathcal{W}(\Sigma) = \frac{i}{2}\tau\Sigma - \frac{1}{4\pi}\sum_{i=1}^{N_c}(\Sigma - m_i)\log\left(\frac{2}{\mu}(\Sigma - m_i)\right)$$

Vacua $\exp\frac{\partial\widetilde{\mathcal{W}}}{\partial\sigma} = 1$

Central charge

$$Z = -i\sum_{i=1}^{N_c}(m_i S_i + m_{D\,i}T_i)$$

$$m_{D\,i} = -2i\mathcal{W}(e_i) = \frac{1}{2\pi i}N_c e_i + \frac{1}{2\pi i}\sum_{j=1}^{N_c} m_j\log\left(\frac{e_i - m_j}{\Lambda}\right)$$

'ANO' String

$U(N)$ gauge theory with fundamental matter $q \to UqV \quad U \in U(N)_G, \quad V \in SU(N)_F$

$N_f = N_c$

$$S = \int d^4x \ \mathrm{Tr}\left(\frac{1}{2e^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2\right) + \sum_{i=1}^{N_f}|\mathcal{D}_\mu q_i|^2$$

$$-\sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{4}\,\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\, 1_N)^2$$

Vacuum

$\phi = 0 \quad , \quad q^a{}_i = v\delta^a{}_i$

breaks symmetry

(color-flavor locking)

$U(N)_G \times SU(N)_F \to SU(N)_{\mathrm{diag}}$

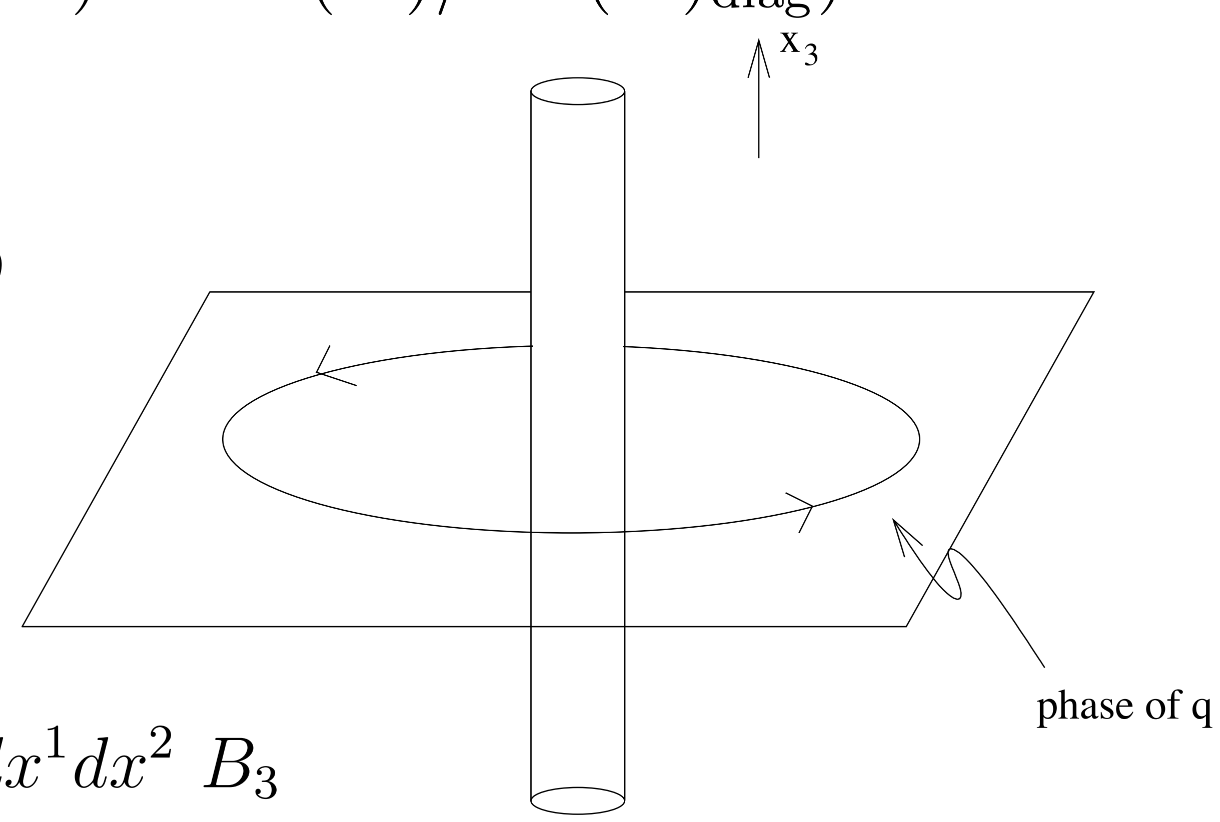
Induces nontrivial topology on moduli space

$$\Pi_1\left(U(N) \times SU(N)/SU(N)_{\mathrm{diag}}\right) \cong \mathbf{Z}$$

To find a string need winding at infinity

$$q_N \sim q\,\mathrm{e}^{ik\theta}$$

$$A_\theta \sim \frac{k}{\rho}$$



$$2\pi k = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} i\partial_\theta q\ q^{-1} = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} A_\theta = \mathrm{Tr}\int dx^1 dx^2\ B_3$$

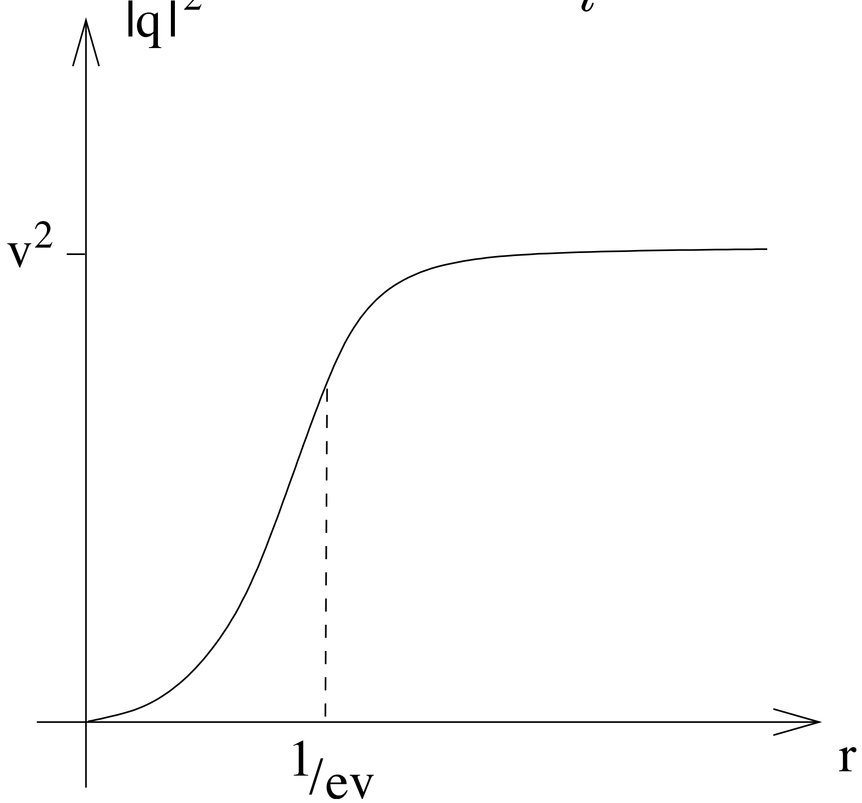
BPS equations for vortex

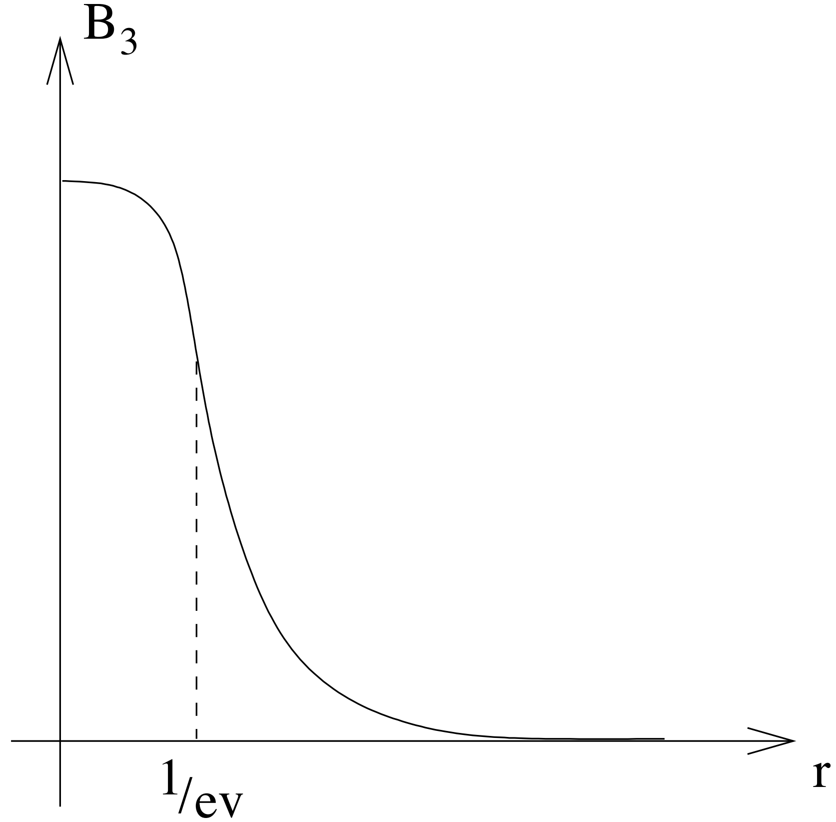
$$T_{\text{vortex}} = \int dx^1 dx^2 \ \text{Tr} \left(\frac{1}{e^2} B_3^2 + \frac{e^2}{4} (\sum_{i=1}^{N} q_i q_i^\dagger - v^2 \, 1_N)^2 \right) + \sum_{i=1}^{N} |\mathcal{D}_1 q_i|^2 + |\mathcal{D}_2 q_i|^2$$

$$= \int dx^1 dx^2 \ \frac{1}{e^2} \text{Tr} \left(B_3 \mp \frac{e^2}{2} (\sum_{i=1}^{N} q_i q_i^\dagger - v^2 \, 1_N) \right)^2 + \sum_{i=1}^{N} |\mathcal{D}_1 q_i \mp i \mathcal{D}_2 q_i|^2$$

$$\mp v^2 \int dx^1 dx^2 \ \text{Tr} \, B_3 \quad \geq \mp v^2 \int d^2 x \text{Tr} B_3 \quad = 2\pi v^2 |k| \qquad ($$

gives $$B_3 = \frac{e^2}{2} (\sum_i q_i q_i^\dagger - v^2 \, 1_N) \qquad (\mathcal{D}_x - i\mathcal{D}_y) q_i = 0$$





Vortices

Simple vortex w/ N=1, k=1 (ANO) has two collective coordinates-translations in x,y directions

U(N) vortex has more moduli

$$A_z = \begin{pmatrix} A_z^\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \quad , \quad q = \begin{pmatrix} q^\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Moduli space (k=1)

$$SU(N)_{\rm diag}/S[U(N-1)\times U(1)] \cong \mathbb{CP}^{N-1}$$

For higher k

$$\dim(\mathcal{V}_{k,N}) = 2kN \qquad \mathcal{V}_{1,N} \cong \mathbf{C}\times\mathbb{CP}^{N-1}$$

Non-Abelian String

[Auzzi, Bolognesi, Evslin, Konishi, Yung]

[Shifman Yung]

$$\varphi = U \begin{pmatrix} \phi_2(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi_2(r) & 0 \\ 0 & 0 & \dots & \phi_1(r) \end{pmatrix} U^{-1} ,$$

Take Abelian string solution
Make global rotation

$$A_i^{\mathrm{SU}(N)} = \frac{1}{N} U \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} U^{-1} \, (\partial_i \alpha) \, f_{NA}(r)$$

Matrix U parameterizes orientational modes

$$A_i^{\mathrm{U}(1)} = -\frac{1}{N} \, (\partial_i \alpha) \, f(r) \, , \qquad A_0^{\mathrm{U}(1)} = A_0^{\mathrm{SU}(N)} = 0 \, ,$$

Gauge group is broken to $\mathbb{Z}_N$

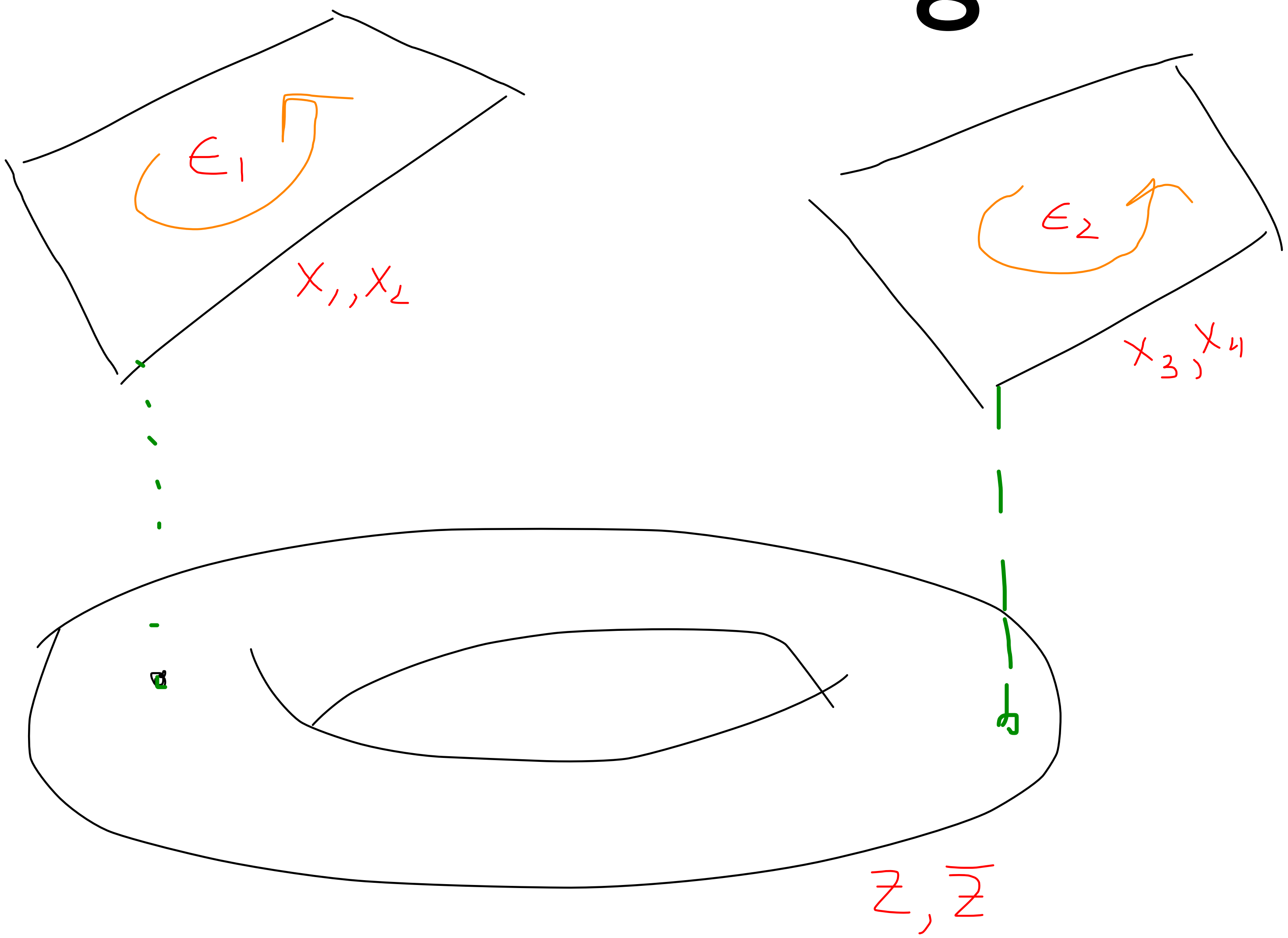
$$\frac{SU(N)}{SU(N-1) \times U(1)} = \mathbb{CP}^{N-1}$$

All bulk degrees of freedom massive $M^2 = e^2 v^2$

Theory is fully Higgsed

Omega background

[Nekrasov et al]



Rotational symmetry broken to maximal torus

$$SO(4) \to SO(2) \times SO(2)$$

6d Metric

$$G_{AB}dx^A dx^B = Adzd\bar{z} + (dx^m + \Omega^m dz + \bar{\Omega}^m d\bar{z})^2$$

We will be interested in Nekrasov-Shatashvili limit

$$\Omega^m = (-i\epsilon x^2, i\epsilon x^1, 0, 0)$$

$$\epsilon_2 \to 0$$

4d/2d in Omega background

[Dorey Hollowood Lee]

N=2 SQCD in Omega background in NS limit with Nf=2Nc

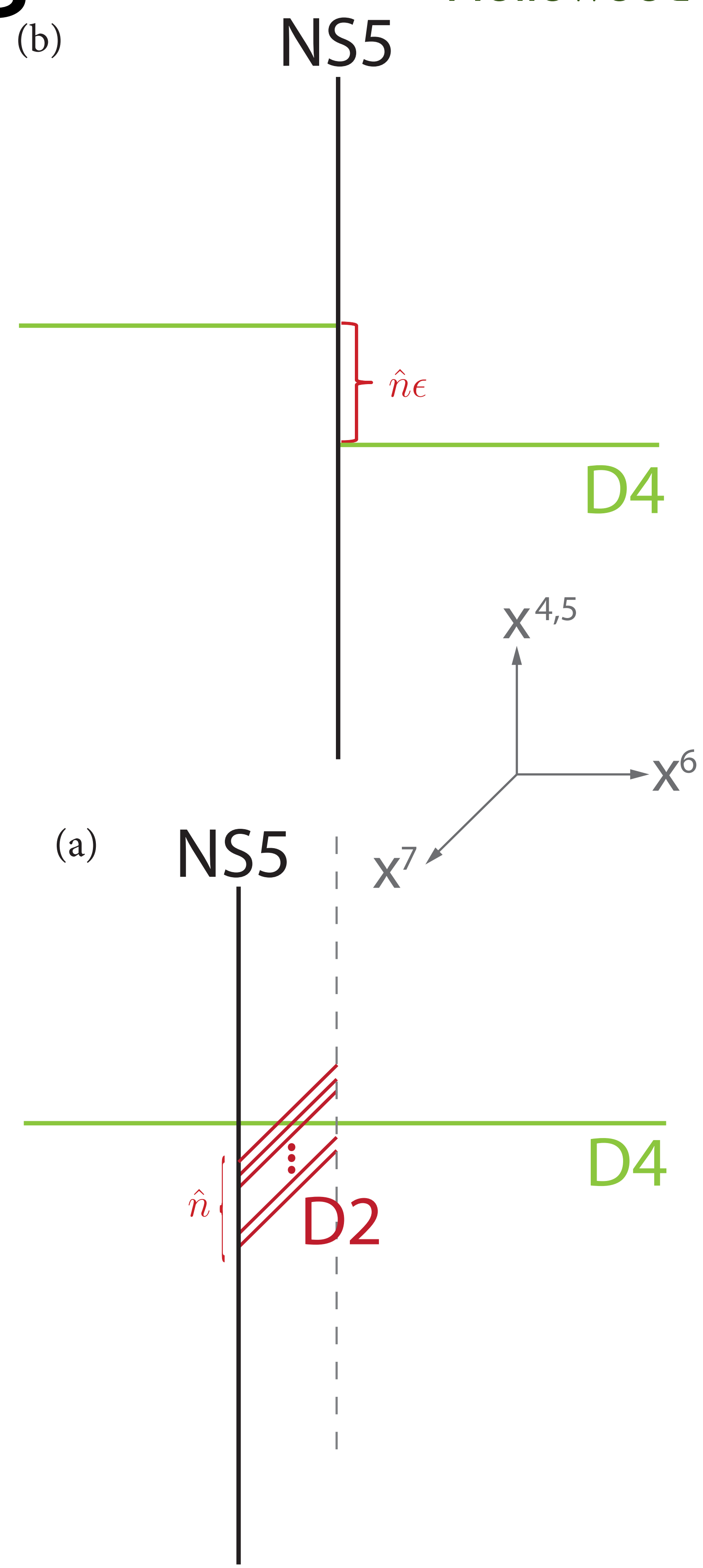
$$\vec{a} = \vec{m}_F - \vec{n}\epsilon \qquad \vec{n} = (n_1, \ldots, n_L) \in \mathbb{Z}^L$$

$$\mathcal{W}^{(I)} \overset{\text{on-shell}}{\equiv} \mathcal{W}^{(II)}$$

(2,2) GLSM w/ gauge group U(K) massive adjoint and twisted masses

$$\vec{M}_F = \vec{m}_F - \frac{3}{2}\vec{\epsilon}\,, \qquad \vec{M}_{AF} = \vec{m}_{AF} + \frac{1}{2}\vec{\epsilon}\,.$$

$$M_{adj} = \epsilon \qquad K = \sum_{i=1}^{N} n_i - N$$



4d/2d in Omega background

[Dorey Hollowood Lee]

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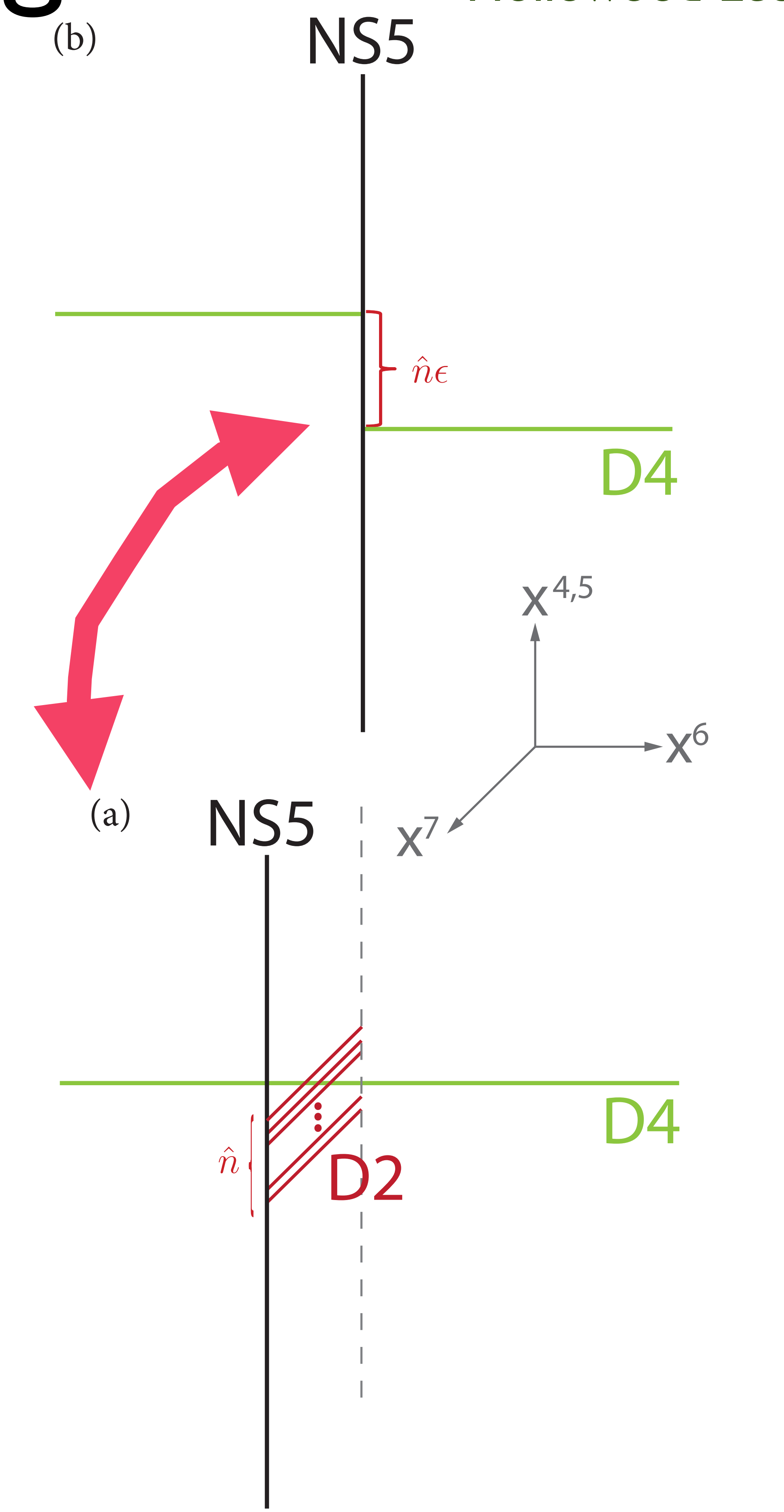

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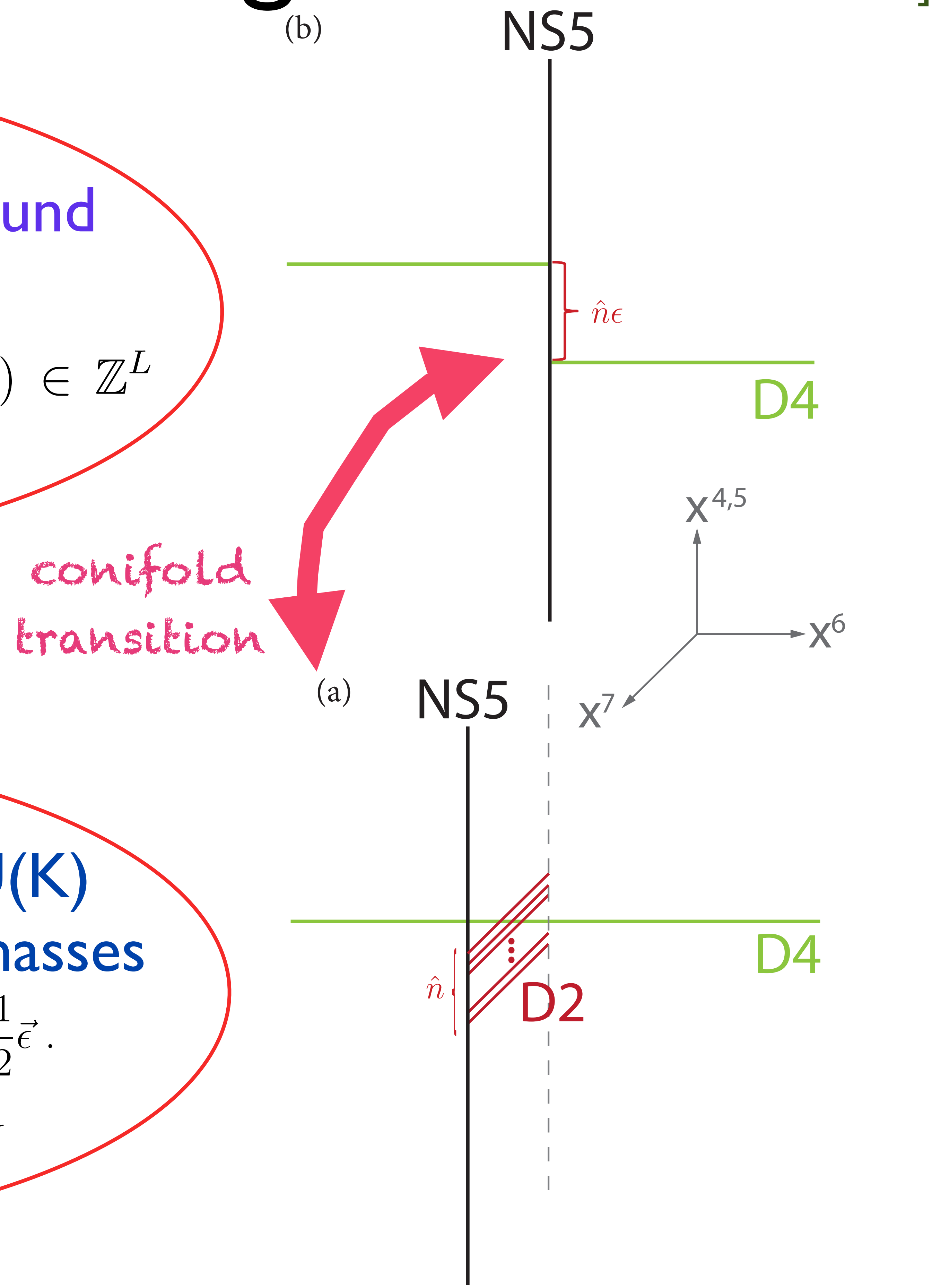

Vortices in Omega background [PK Gorsky Chen] in progress

Symmetry breaking pattern

$$SU(2)_c \times SU(2)_R \times SU(2)_{\mathcal{R}} \to U(1)_c \times SU(2)_{R+\mathcal{R}}$$

SUSY transform pure SYM

$$\delta\Lambda^I_\alpha = \zeta^I_\beta((\sigma^{mn})^\beta_\alpha F_{mn} + i[\phi,\bar\phi]\delta^\beta_\alpha + \nabla_m(\bar\Omega^m\phi - \Omega^m\bar\phi)\delta^\beta_\alpha) + \bar\zeta^I_{\dot\beta}(\sigma^m)^{\dot\beta}_\alpha(\nabla_m\phi - F_{mn}\Omega^n)$$

String central charge current

$$\zeta_3 = \tfrac{1}{2}\partial_m\left((\phi^a\bar\Omega^m - \bar\phi^a\Omega^m)B^a_3\right)\sigma^3_{\alpha\dot\alpha}\delta^{IJ} = \tfrac{i}{2}B^a_3\partial_\varphi(\phi^a\bar\epsilon - \bar\phi^a\epsilon)\sigma^3_{\alpha\dot\alpha}\delta^{IJ}$$

yields for a string of tension ~ epsilon

$$\mathcal{L} = \tfrac{1}{2}|B^a_z + \phi\tau^a\bar\phi - i\nabla_m(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a)|^2 + \tfrac{1}{2}|\mathcal{D}_1\phi^a + i\mathcal{D}_2\phi^a - (\Omega_2 - i\Omega_1)B^a_z|^2 + \partial_m(B^a_z(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a)) \geq \partial_m(B^a_z(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a))\,.$$

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String central charge current

$$\zeta_3 = \tfrac{1}{2}\partial_m\left((\phi^a\bar\Omega^m - \bar\phi^a\Omega^m)B^a_3\right)\sigma^3_{\alpha\dot\alpha}\delta^{IJ} = \tfrac{i}{2}B^a_3\partial_\varphi(\phi^a\bar\epsilon - \bar\phi^a\epsilon)\sigma^3_{\alpha\dot\alpha}\delta^{IJ}$$

yields for a string of tension ~ epsilon

$$\mathcal{L} = \tfrac{1}{2}|B^a_z + \phi\tau^a\bar\phi - i\nabla_m(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a)|^2 + \tfrac{1}{2}|\mathcal{D}_1\phi^a + i\mathcal{D}_2\phi^a - (\Omega_2 - i\Omega_1)B^a_z|^2 + \partial_m(B^a_z(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a)) \geq \partial_m(B^a_z(\Omega^m\bar\phi^a - \bar\Omega^m\phi^a))\,.$$

Searching for the field theoretical explanation of the new duality

Conclusions and open questions

- Nonabelian vortices to study BPS spectrum of SQCD
- Generalization of the 4d/2d duality to theories in Omega background
- Connections to integrable systems in 2d...
- Relationship w/ another 4d/2d duality [Vafa et al]
- Holography for Non-Abelian vortices