On 4d/2d Correspondence

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Outline

- 4d/2d w/ 8 supercharges: what? and why?
  - ★ *Solitonic flux tubes vs. type IIA string theory*
  - ★ *(2,2) GLSM, NLSM*
  - ★ *The Dictionary*
  - ★ *Perturbation theory in GLSM*
- Less Supersymmetry
  - ★ *Heterotic deformation and Large-N solution - beyond BPS sector*
  - ★ *Omega background* (bonus)

4d/2d

'ANO' String

$U(N)$ gauge theory with fundamental matter $\quad q \to UqV \qquad U \in U(N)_G, \quad V \in SU(N)_F$

$$S = \int d^4x \ \mathrm{Tr}\left(\frac{1}{2e^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{e^2}(\mathcal{D}_\mu\phi)^2\right) + \sum_{i=1}^{N_f} |\mathcal{D}_\mu q_i|^2$$
$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{4}\mathrm{Tr}\,(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\, 1_N)^2$$

Vacuum

$$\phi = 0 \quad , \quad q^a{}_i = v\delta^a{}_i$$

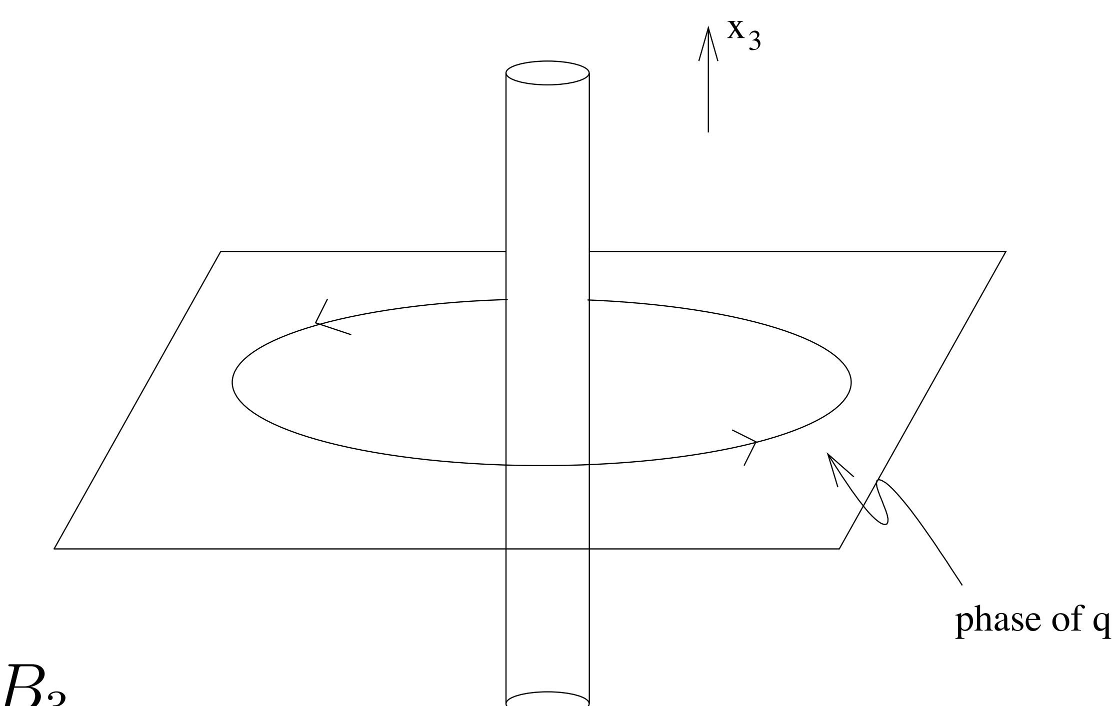
breaks symmetry

(color-flavor locking)

$$U(N)_G \times SU(N)_F \to SU(N)_{\mathrm{diag}}$$

Induces nontrivial topology $\quad \Pi_1\left(U(N) \times SU(N)/SU(N)_{\mathrm{diag}}\right) \cong \mathbf{Z}$

To find a string find need winding at infinity $\quad A_\theta \sim 1/\rho$



$$2\pi k = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} i\partial_\theta q \ q^{-1} = \mathrm{Tr}\oint_{\mathbf{S}^1_\infty} A_\theta = \mathrm{Tr}\int dx^1 dx^2 \ B_3$$

BPS equations for vortex

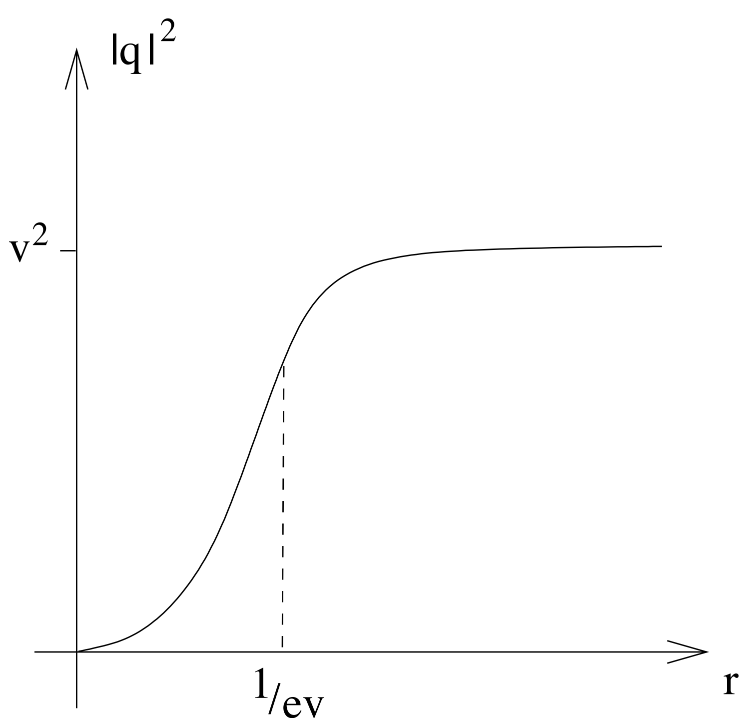
$$T_{\rm vortex} = \int dx^1 dx^2 \ {\rm Tr} \left(\frac{1}{e^2} B_3^2 + \frac{e^2}{4} (\sum_{i=1}^N q_i q_i^\dagger - v^2 \, 1_N)^2 \right) + \sum_{i=1}^N |\mathcal{D}_1 q_i|^2 + |\mathcal{D}_2 q_i|^2$$

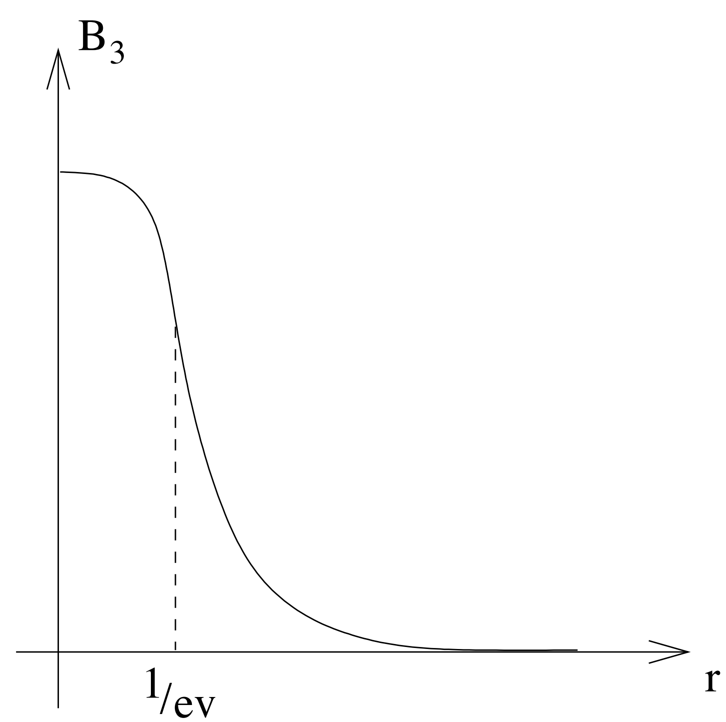
$$= \int dx^1 dx^2 \ \frac{1}{e^2} {\rm Tr} \left(B_3 \mp \frac{e^2}{2} (\sum_{i=1}^N q_i q_i^\dagger - v^2 \, 1_N) \right)^2 + \sum_{i=1}^N |\mathcal{D}_1 q_i \mp i \mathcal{D}_2 q_i|^2$$

$$\mp v^2 \int dx^1 dx^2 \ {\rm Tr} \, B_3 \quad \geq \mp v^2 \int d^2 x {\rm Tr} B_3 \ = 2\pi v^2 |k| \qquad ($$

gives

$$B_3 = \frac{e^2}{2} (\sum_i q_i q_i^\dagger - v^2 \, 1_N) \qquad (\mathcal{D}_x - i \mathcal{D}_y) q_i = 0$$





Vorticies

Simple vortex w/ N=1, k=1 (ANO) has two collective coordinates-translations in x,y directions

U(N) vortex has more moduli

$$A_z = \begin{pmatrix} A_z^\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \quad , \quad q = \begin{pmatrix} q^\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Moduli space (k=1)

$$SU(N)_{\text{diag}}/S[U(N-1)\times U(1)] \cong \mathbb{CP}^{N-1}$$

$$\mathcal{V}_{1,N} \cong \mathbf{C} \times \mathbb{CP}^{N-1}$$

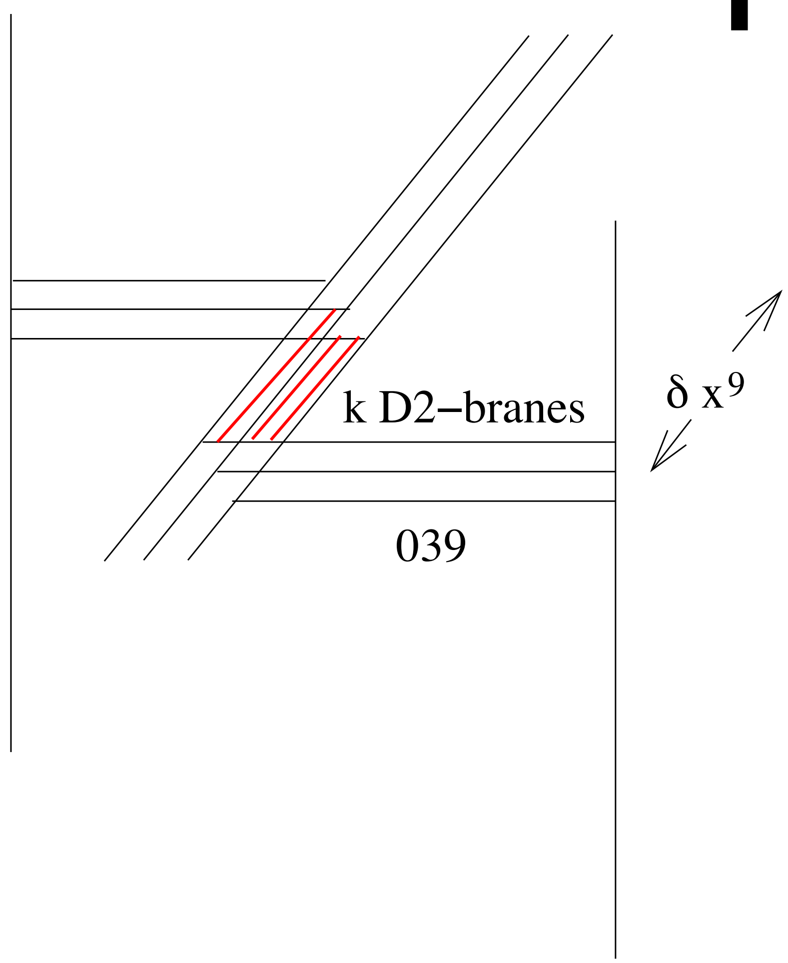
For higher k

$$\dim(\mathcal{V}_{k,N}) = 2kN$$

Again:

$T \geq 2\pi v^2 |k|$ bound saturates for BPS states

Hanany-Witten



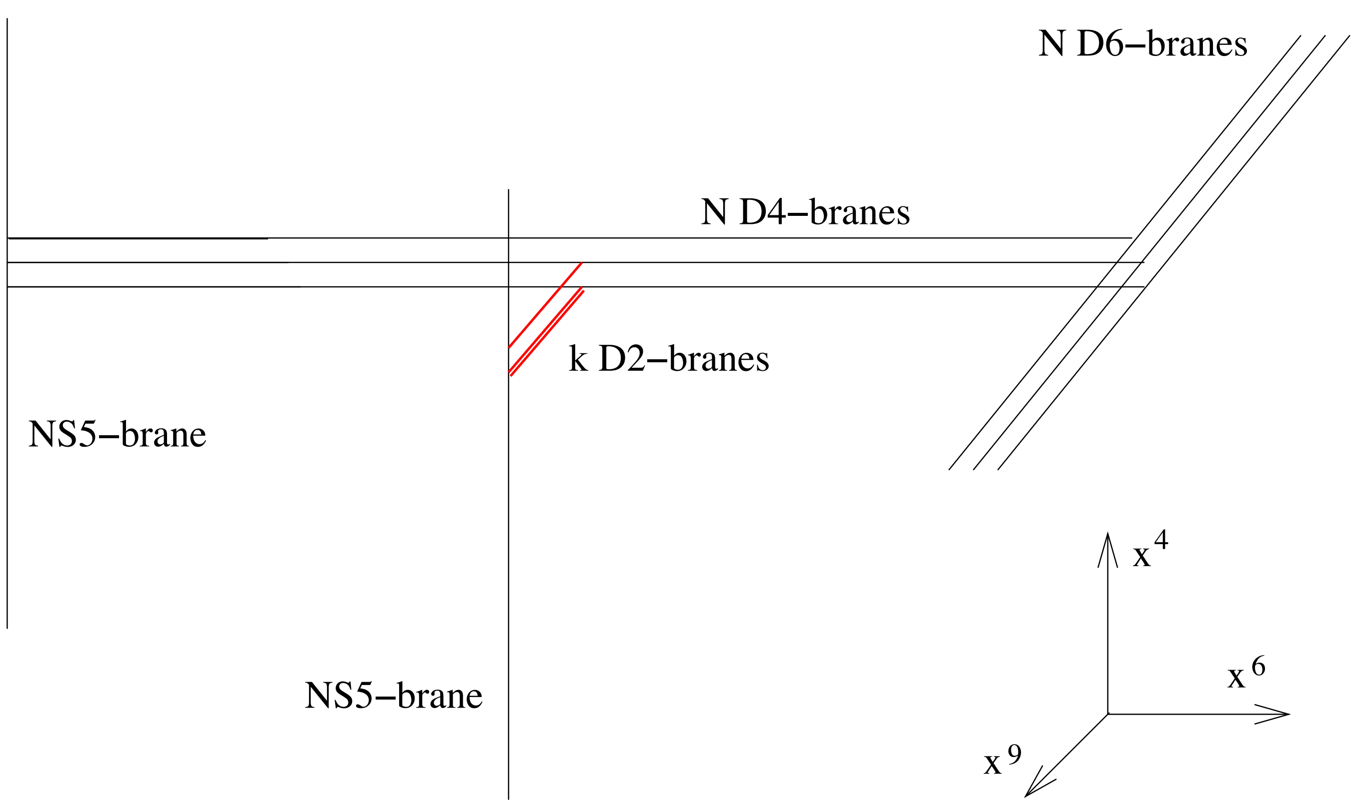
$$V = \frac{1}{g^2}\mathrm{Tr}\,|[\sigma,\sigma^\dagger]|^2 + \mathrm{Tr}\,|[\sigma,Z]|^2 + \mathrm{Tr}\,|[\sigma,Z^\dagger]|^2 + \sum_{a=1}^{N}\psi_a^\dagger\sigma^\dagger\sigma\psi_a$$

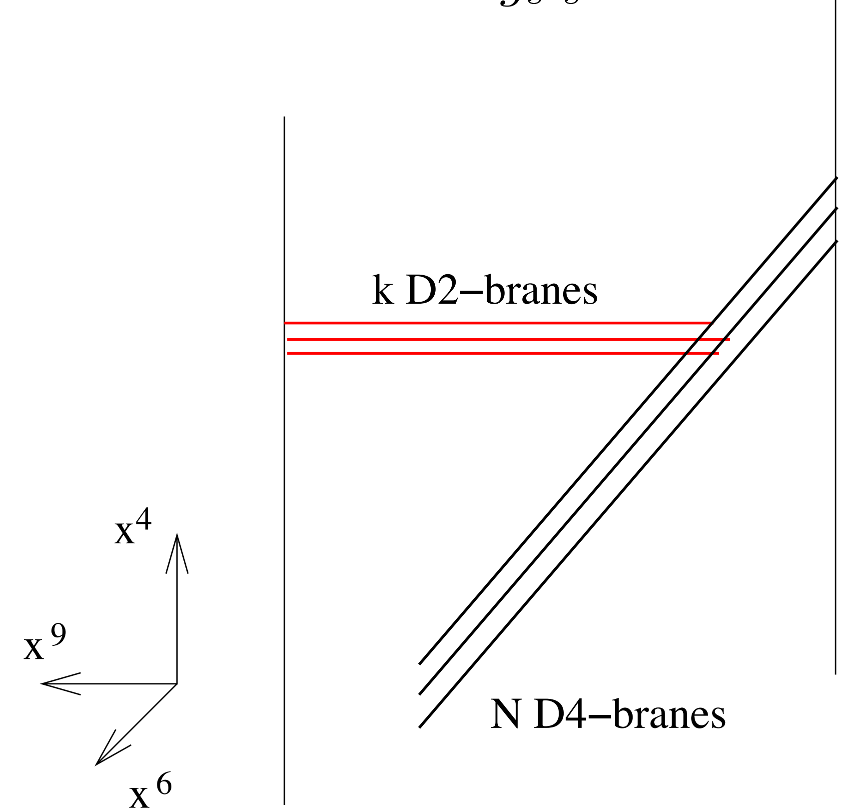
$$+\frac{g^2}{2}\,\mathrm{Tr}\,\left(\sum_a \psi_a\psi_a^\dagger + [Z,Z^\dagger] - r\,1_k\right)^2$$

$$\sigma = X^4 + iX^5 \quad , \quad Z = X^1 + iX^2$$

FI parameter

$$r = \frac{\Delta x^6}{2\pi g_s l_s} = \frac{4\pi}{e^2}$$

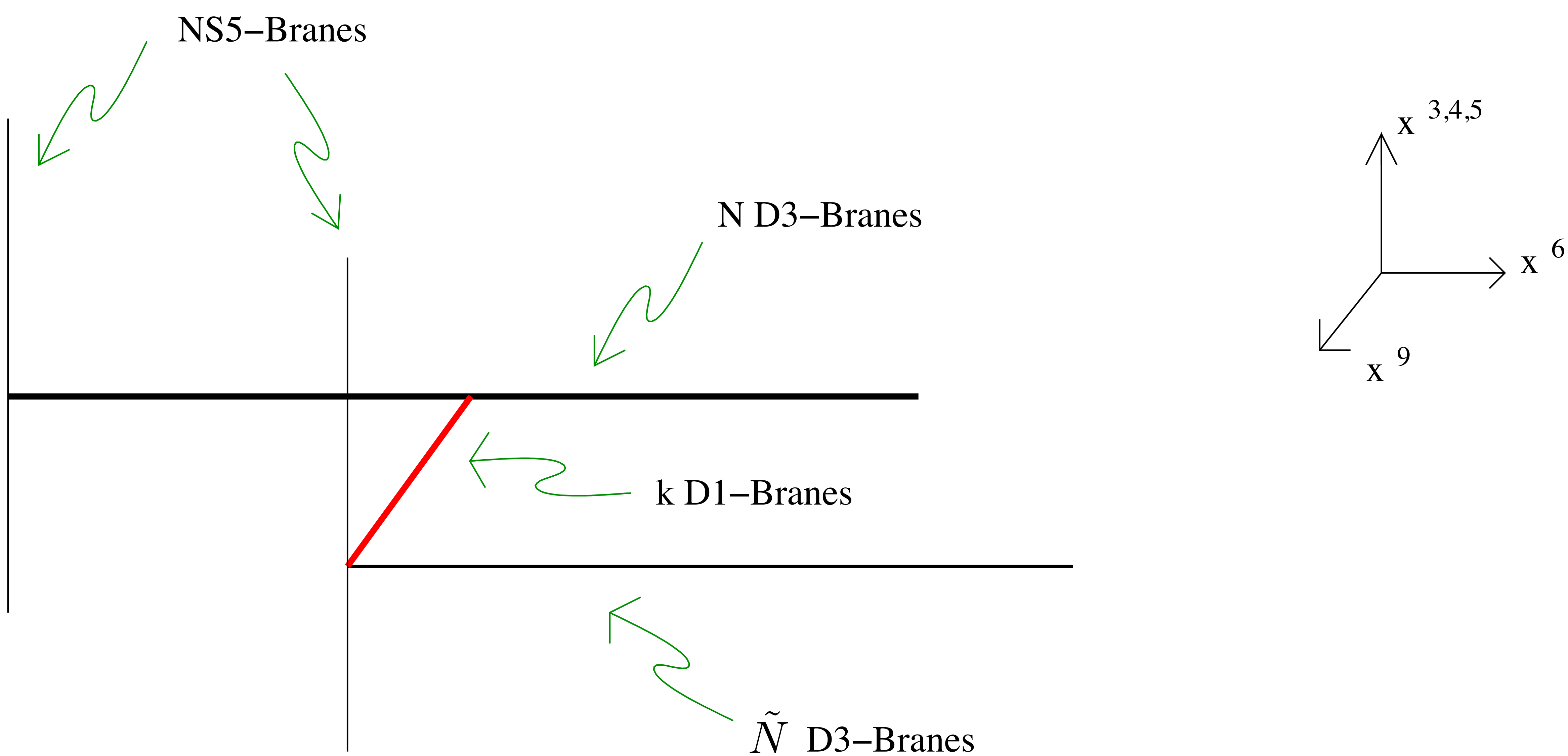




@Tong

T-dualized version

[Hanany Tong]



take k=1to get U(1) theory on D1 (semi-local vortex)

D-term

$$|Q_i|^2 - |\tilde{Q}_j|^2 = r$$

FI term -- separation of NS5s in x6

will refer to as HT model

4d / 2d duality

[Dorey Hollowood, Tong]

$\mathcal{N}=2 \quad SU(N)$ SQCD $N_f = N + \tilde{N}$ fund hypers w/ masses $m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$ $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$ on baryonic Higgs branch	(2,2) $U(1)$ GLSM e N chiral +1 $\tilde{N}$ chiral -1 w/ *twisted* masses $m_1, \ldots, m_N \quad \mu_1, \ldots, \mu_{\tilde{N}}$ $\tau = ir + \frac{\theta}{2\pi}$ vortex moduli space
BPS dyons (Seiberg-Witten)	kinks interpolating between different vacua

BPS spectra (as functions of masses, Lambda) are the same

$U(N_c)$ $\mathcal{N}=2$ $d=4$ SQCD w/ N_f quarks

$$\{Q^I_\alpha, \bar{Q}^J_{\dot\beta}\} = 2\delta^{IJ}P_{\alpha\dot\beta} + 2\delta^{IJ}Z_{\alpha\dot\beta}$$

$$\{Q^I_\alpha, Q^J_\beta\} = 2Z^{IJ}_{\alpha\beta}$$

strings

monopoles domain walls

$$\mathcal{L} = \mathrm{Im}\left[\tau\int d^4\theta\,\mathrm{Tr}\left(Q^{i\,\dagger}e^V Q_i + \tilde{Q}^{i\,\dagger}e^V\tilde{Q}_i + \Phi^\dagger e^V\Phi\right)\right]$$
$$+\mathrm{Im}\left[\tau\int d^2\theta\left(\mathrm{Tr}W^{\alpha\,2} + m^i_j\tilde{Q}_iQ^j + Q_i\Phi\tilde{Q}^i\right)\right]$$

bosonic part

FI term

$$S = \int d^4x\,\mathrm{Tr}\left\{\frac{1}{2g^2}F^2_{\mu\nu} + \frac{1}{g^2}|D_\mu\Phi|^2 + |\nabla_\mu Q|^2 + \frac{g^2}{4}(Q\bar{Q}-\xi)^2 + |\Phi Q + QM|^2\right\}$$

BPS conditions

$$B_3 - g^2(Q\bar{Q}-\xi^2) = 0$$
$$\nabla_3 Q = 0$$

String tension

$$T = \xi\int d^2x\,\mathrm{Tr}F_{12} = 2\pi\xi n$$

Nonabelian String

[Shifman Yung]

$$\varphi = U \begin{pmatrix} \phi_2(r) & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \phi_2(r) & 0 \\ 0 & 0 & \ldots & \phi_1(r) \end{pmatrix} U^{-1} ,$$

Matrix U parameterizes orientational modes

$$A_i^{\mathrm{SU}(N)} = \frac{1}{N} U \begin{pmatrix} 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 1 & 0 \\ 0 & 0 & \ldots & -(N-1) \end{pmatrix} U^{-1} \, (\partial_i \alpha) \, f_{NA}(r)$$

$$A_i^{\mathrm{U}(1)} = -\frac{1}{N} \, (\partial_i \alpha) \, f(r) , \qquad A_0^{\mathrm{U}(1)} = A_0^{\mathrm{SU}(N)} = 0 ,$$

Gauge group is broken to $\mathbb{Z}_N$

All bulk degrees of freedom massive $M^2 = e^2 v^2$

Theory is fully Higgsed

Vortex moduli space

Nf=Nc color-flavor locked phase
single SUSY vacuum

$$U(N_c) \times SU(N_f) \to SU(N)$$

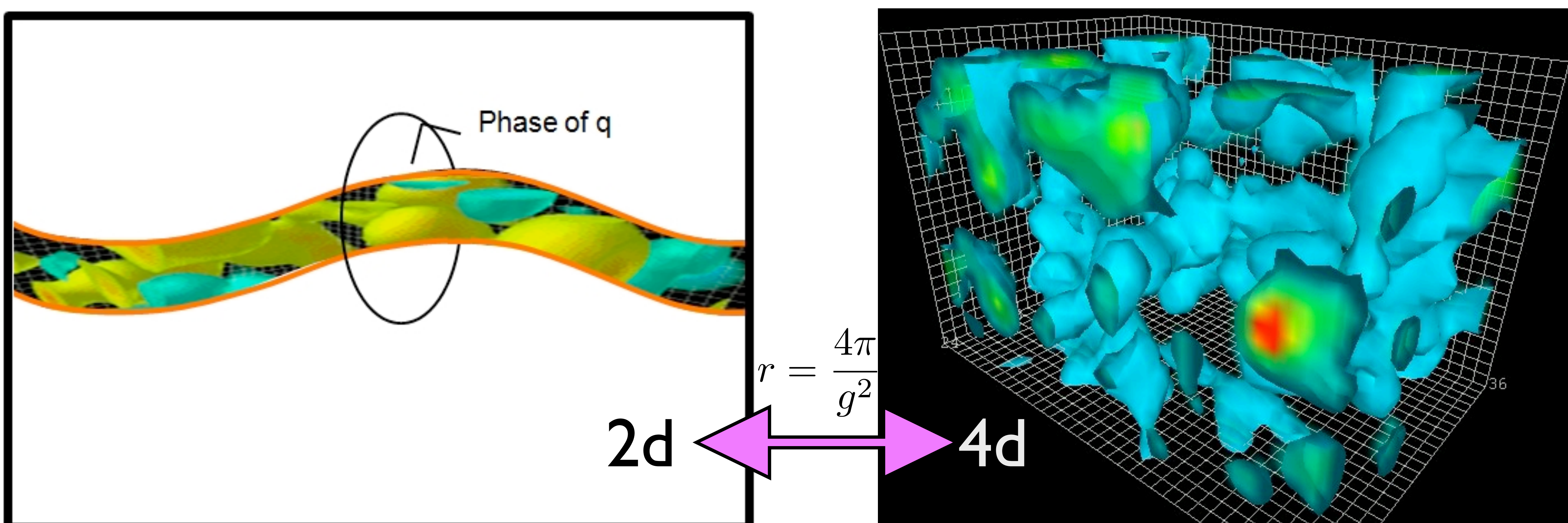
local vortex

$$\frac{SU(N)}{SU(N-1) \times U(1)} = \mathbb{CP}^{N-1}$$

Nf>Nc semilocal

$$\pi_2(\mathcal{M}_{vac}) = \pi_2\left(\frac{SU(N+\tilde{N})}{SU(N) \times SU(\tilde{N}) \times U(1)}\right) = \mathbb{Z}$$

Duality between two strongly coupled theories

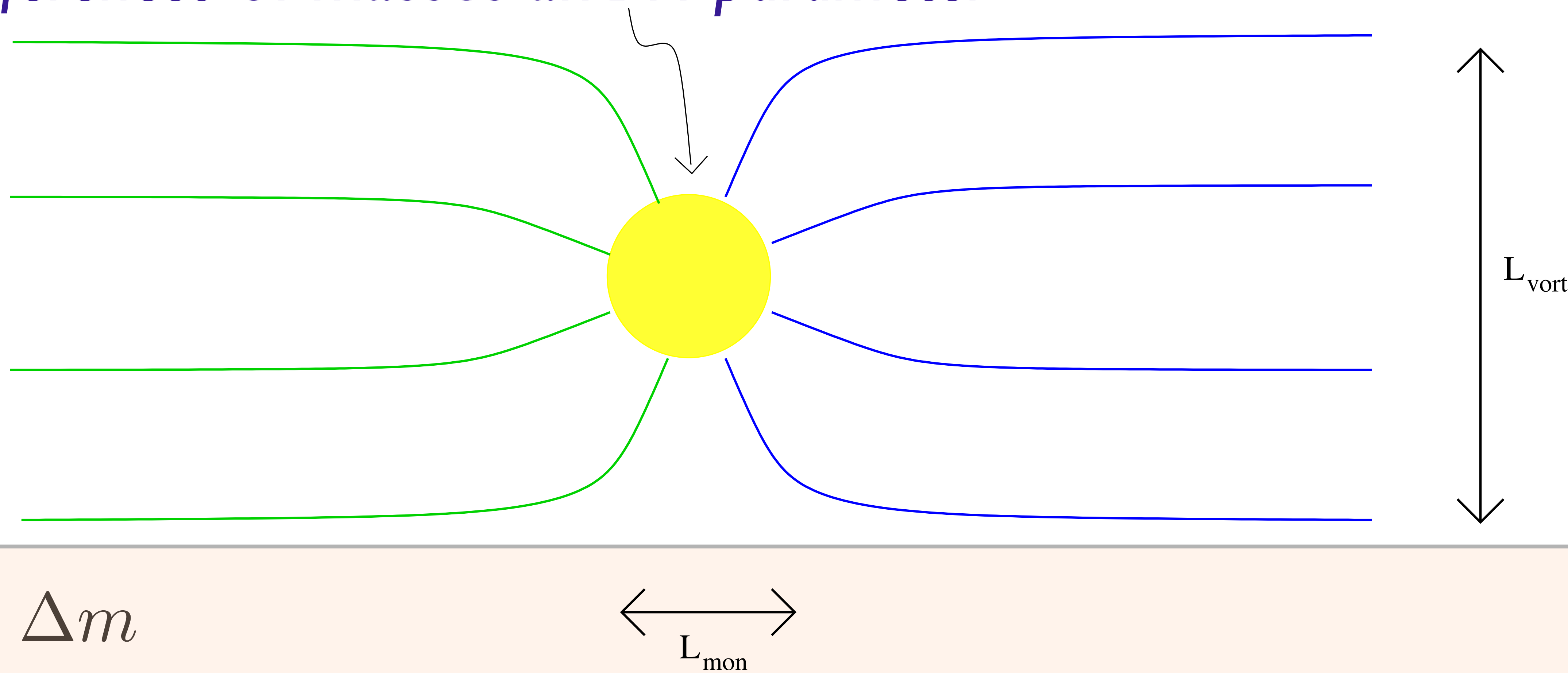


Monopoles in Higgs Phase

[Shifman, Yung]
[Tong]

Add masses. New vacuum $\phi = \mathrm{diag}(m_i) \quad , \quad q^a{}_i = v\delta^a{}_i \quad , \quad \tilde{q}^a{}_i = 0$

Pattern of symmetry breaking depends on the relationship between the *differences* of masses and *FI parameter*



$ev \gg \Delta m$

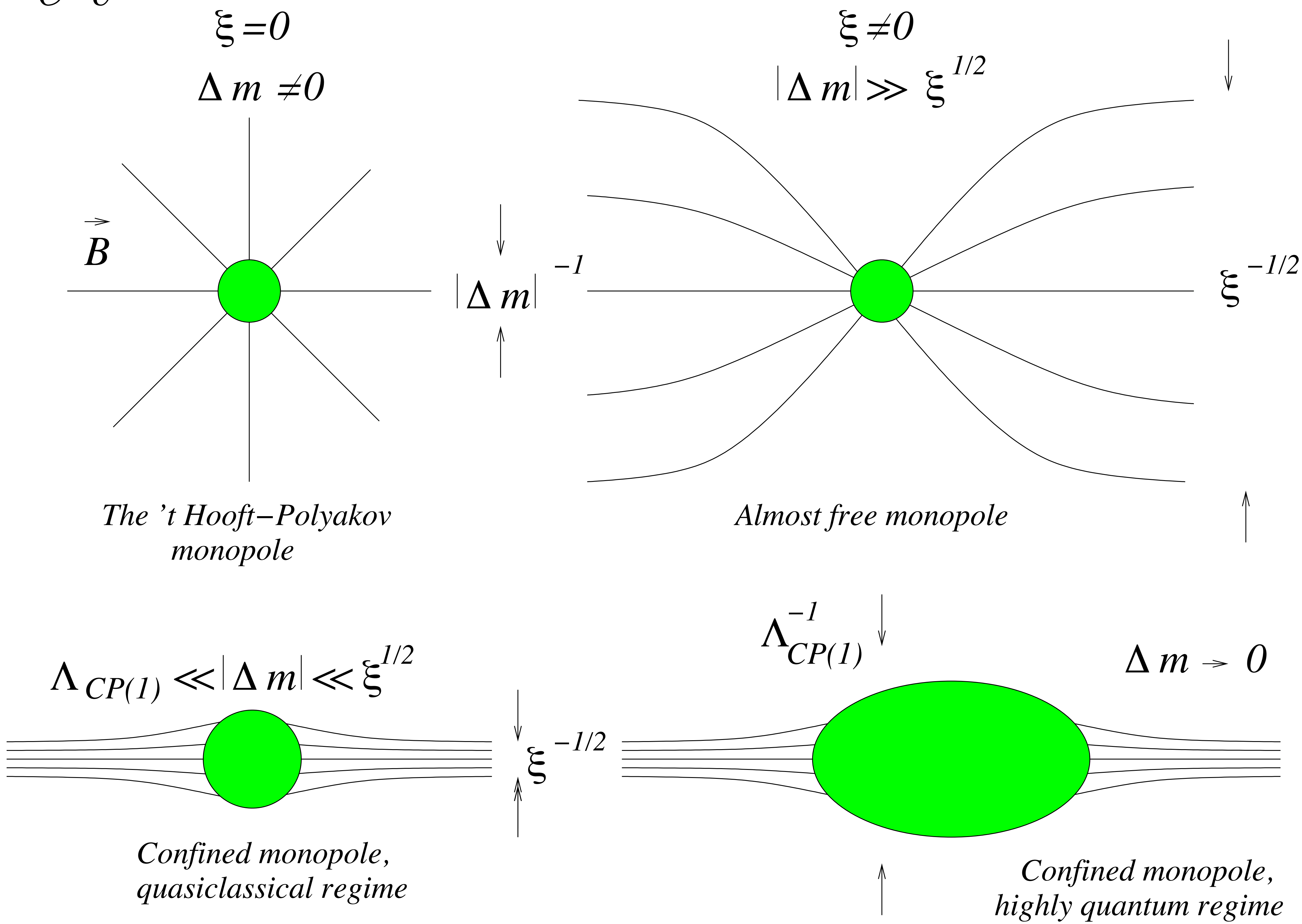
$$U(N)_G \times SU(N)_F \xrightarrow{v} SU(N)_{\mathrm{diag}} \xrightarrow{m} U(1)^{N-1}_{\mathrm{diag}}$$

$ev \ll \Delta m$

$$U(N)_G \times SU(N)_F \xrightarrow{m} U(1)^N_G \times U(1)^{N-1}_F \xrightarrow{v} U(1)^{N-1}_{\mathrm{diag}}$$

Confined monopoles

$$\xi = e^2 v^2$$



$\frac{(\Delta m)^2}{\xi}$ becomes 2d FI term r

BPS dyons

$$Z = \sum_{a=1}^{N_c} \phi_a (j_a + \tau h_a) + \sum_{i=1}^{N_f} m_i s_i$$

Central charge

$$Z = \sum_{i=1}^{N_c} m_i (S_i + \tau h_i)$$

At baryonic root of Higgs branch

$$F(t,u) = \left(t - \prod_{i=1}^{N_c} (u - m_i) \right) (u - \Lambda^{N_c})$$

SW curve degenerates
has Nc branching pts

$$Z = \sum_{i=1}^{N_c} (m_i S_i + m_{Di} h_i)$$

all quantum corrections in mD

$$m_{Dl} - m_{Dk} = \frac{1}{2\pi} N_c (e_l - e_k) + \frac{1}{2\pi} \sum_{i=1}^{N_c} m_i \log \left(\frac{e_l - m_i}{e_k - m_i} \right)$$

(2,2) 2d GLSM

[Witten]

Consider U(1) gauge theory

$$\mathcal{L}_{\text{vortex}} = \frac{1}{2g^2}\left(F_{01}^2 + |\partial\sigma|^2\right) + \sum_{i=1}^{N_c}\left(|\mathcal{D}\psi_i|^2 + |\sigma - m_i|^2|\psi_i|^2\right) + \frac{g^2}{2}\left(\sum_{i=1}^{N_c}|\psi_i|^2 - r\right)^2$$

$$\text{Vacuum } i: \quad \sigma = m_i \quad , \quad |\psi_j|^2 = r\delta_{ij}$$

for vortex embedded into i's U(1) subgroup

FI term runs

$$r(\mu) = r_0 - \frac{N_c}{2\pi}\log\left(\frac{M_{UV}}{\mu}\right) \Rightarrow \Lambda = \mu\exp\left(-\frac{2\pi r(\mu)}{N_c}\right)$$

Effective twisted superpotential

$$\mathcal{W}(\Sigma) = \frac{i}{2}\tau\Sigma - \frac{1}{4\pi}\sum_{i=1}^{N_c}(\Sigma - m_i)\log\left(\frac{2}{\mu}(\Sigma - m_i)\right)$$

Vacua $\exp\dfrac{\partial\widetilde{\mathcal{W}}}{\partial\sigma} = 1$

Central charge $\quad Z = -i\sum_{i=1}^{N_c}(m_i S_i + m_{D\,i}T_i)$

$$m_{D\,i} = -2i\mathcal{W}(e_i) = \frac{1}{2\pi i}N_c e_i + \frac{1}{2\pi i}\sum_{j=1}^{N_c}m_j\log\left(\frac{e_i - m_j}{\Lambda}\right)$$

Hanany-Tong model as U(1) GLSM

$$\mathcal{L} = \int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger \, \mathrm{e}^{\mathcal{V}} \Phi_i + \sum_{i=1}^{\tilde{N}} \widetilde{\Phi}_i^\dagger \, \mathrm{e}^{-\mathcal{V}} \widetilde{\Phi}_i - r\mathcal{V} + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

$$V = \theta^+ \bar{\theta}^+ (A_0 + A_3) + \theta^- \bar{\theta}^- (A_0 - A_3) - \theta^- \bar{\theta}^+ \sigma - \theta^- \bar{\theta}^+ \bar{\sigma} + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta} \bar{\lambda} + \bar{\theta} \theta \bar{\theta} \theta D$$

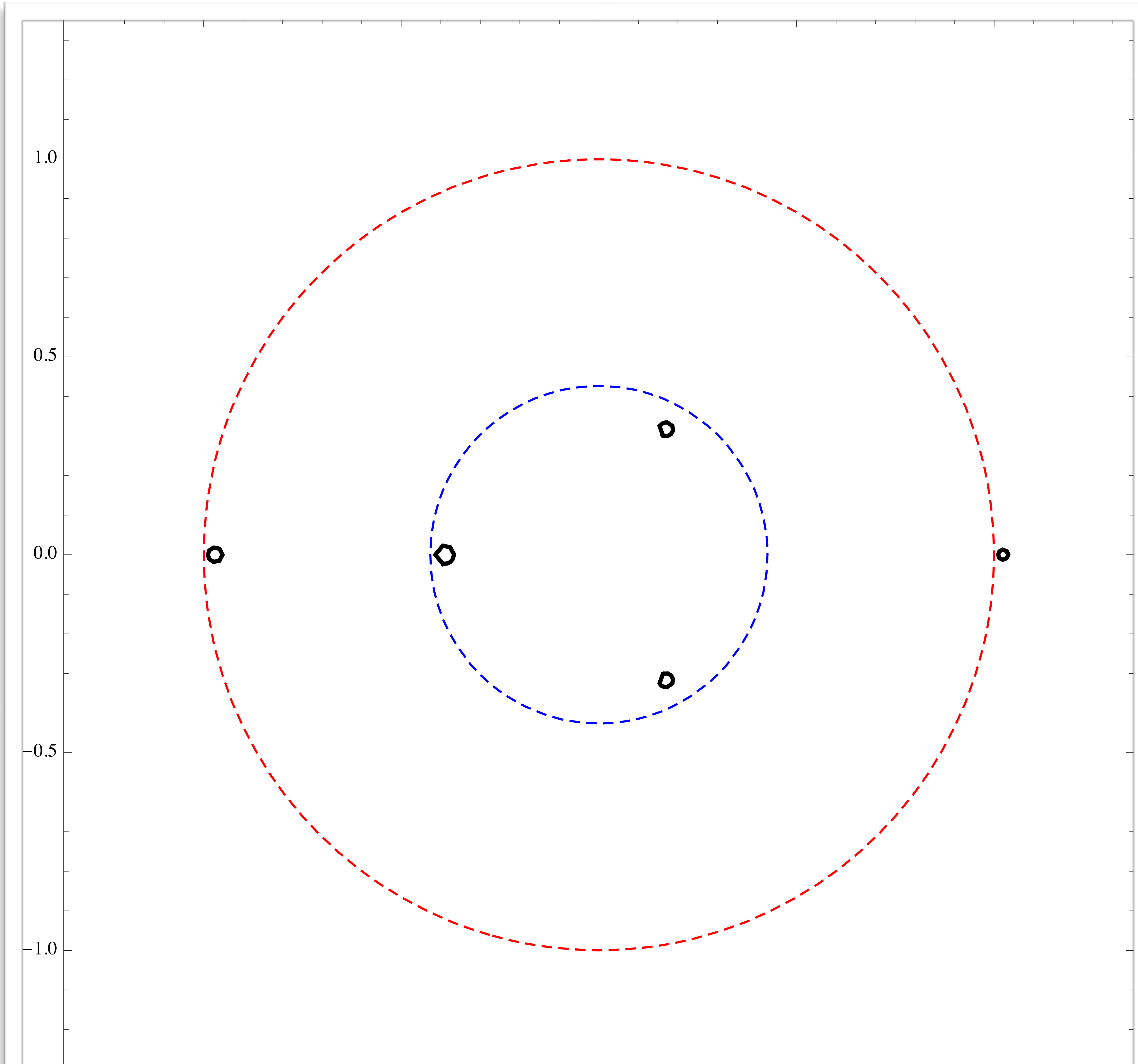
One loop twisted effective superpotential is exact in (2,2)

$$\begin{aligned} \widetilde{W}_{\text{eff}} &= -\frac{1}{2\pi} \sum_{i=1}^{N} (\sqrt{2}\sigma + m_i) \left(\log \frac{\sqrt{2}\sigma + m_i}{\Lambda} - 1 \right) + \\ &+ \frac{1}{2\pi} \sum_{j=1}^{\widetilde{N}} (\sqrt{2}\sigma + \widetilde{m}_j) \left(\log \frac{\sqrt{2}\sigma + \widetilde{m}_j}{\Lambda} - 1 \right) . \end{aligned}$$

gives vacua of the theory and its BPS spectrum !!

[PK Monin Vinci]

N=5 Nf=8



Worldsheet sigma model from the 4d theory

Derivation from 4d theory [Shifman Vinci Yung]

Brane construction is not sensitive to IR physics

Blind to deformations within the same universality class

Need to know explicit metric on the vacuum manifold in order to go beyond BPS sector

Let's see if GLSMs from brane picture are the same as sigma models what live on vortex

From GLSM

$$\mathcal{L} = \int d^4\theta \left(\left(|X_1|^2 + |X_2|^2\right) e^V - rV + \frac{1}{e^2}|\Sigma|^2 \right)$$

Take limit $e \to \infty$ solve for V

Kahler potential $K = r \log(1 + |X|^2)$ $X = X_2/X_1$

For HT model

$$\mathcal{L}_{\mathrm{HT}} = \int d^4\theta \, \left(|\mathcal{N}_i|^2 \mathrm{e}^V + |\mathcal{Z}_j|^2 \mathrm{e}^{-V} - rV\right)$$

Limit $e \to \infty$ defines vacuum manifold

$$\begin{array}{c} \mathcal{O}(-1)^{\tilde{N}} \\ \downarrow \\ \mathbb{CP}^{N-1} \end{array}$$

Kahler potential

$$K_{\mathrm{HT}} = \sqrt{r^2 + 4r|\zeta|^2} - r \log\left(r + \sqrt{r^2 + 4r|\zeta|^2}\right) + r\log(1 + |\Phi_i|^2)$$

$$|\zeta|^2 \equiv |\mathfrak{z}_j|^2 (1 + |\Phi_i|^2) \qquad \mathfrak{z}_j \;=\; r^{-1/2} \mathcal{N}_N \mathcal{Z}_j \,, \quad j = 1, \ldots, \widetilde{N}$$

Let's see what is the metric on the vortex sigma model

1/2 BPS vortices

[Shifman Vinci Yung]

$$
\begin{aligned}
S &= \int d^4x \,\mathrm{Tr}\left\{ \frac{1}{g^2}\left(F_{12} + \frac{g^2}{2}(Q\bar{Q} - \xi)\right)^2 + \right. \\
&+ |\nabla_1 Q + i\nabla_2 Q|^2 + |\Phi Q + QM|^2 + \xi\, F_{12} + \\
&+ \left. \frac{1}{g^2}(F_{ik})^2 + (\nabla_k Q)^*(\nabla_k Q) + \frac{1}{g^2}(F_{kl})^2 \right\}, \\
& i = 1,2, \quad k,l = 0,3\,.
\end{aligned}
$$

String tension

$$T = \xi \int d^2x \,\mathrm{Tr}\, F_{12} = 2\pi\xi$$

Ansatz

$$
Q_0 = \left(\begin{array}{cccc|c}
\phi_1(r) & 0 & 0 & 0 & 0 \\
0 & \ddots & \vdots & \vdots & \vdots \\
\vdots & \dots & \phi_1(r) & 0 & 0 \\
0 & \dots & 0 & \phi_2(r)e^{i\theta} & \phi_3(r)
\end{array}\right)
\qquad
A_{0,i} = \epsilon_{ij}\frac{x_j}{r^2} f(r) \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}
$$

Bogomol'ny equations

reduce to Abelian Higgs

$$r\phi_1'(r) = 0\,,$$

$$r\phi_2'(r) - f(r)\,\phi_2(r) = 0\,,$$

$$r\phi_3'(r) - (f(r)-1)\phi_3(r) = 0\,,$$

$$\frac{1}{r}f'(r) + \frac{g^2}{2}(\phi_2^2(r) + |\phi_3(r)|^2 - \xi) = 0\,.$$

modulus

$$\phi_1(r) = \sqrt{\xi}, \quad \phi_3 = \frac{\rho}{r}\phi_2$$

can solve the rest of equations analytically provided that

$$\frac{1}{g\sqrt{\xi}|\rho|} << 1$$

e.g. gauge field

$$A_k \;=\; -i\Big(\partial_k n\, n^* - n\,\partial_k n^* - 2n\,n^*(n^*\partial_k n)\Big)\omega(r)$$

$$-i\,n\,n^*\Big(\rho^*\partial_k\rho - \rho\partial_k\rho^* + 2|\rho|^2(n^*\partial_k n)\Big)\gamma(r)$$

after some work [Shifman Vinci Yung] we get...

Effective action

$$\mathcal{L}_{\rm eff} = \pi\xi\left(\ln\frac{L^2}{|\rho|^2}\right)\left|\partial_k(\rho\, n)\right|^2 - \pi\xi|\partial_k\rho + \rho\,(n^*\partial_k n)|^2$$

$$+ \frac{2\pi}{g^2}\left[\partial_k n^*\partial_k n + (\partial_k n^* n)^2\right].$$

for large L can insert Log under derivative

already includes
subleading corrections

$$z = \rho\left[2\pi\xi\,\ln\frac{L}{|\rho|}\right]^{1/2}$$

$$L \sim |\Delta m|^{-1}$$

Arrive to a new model (ZN) with Kahler potential

$$K_{zn} = r|\zeta|^2 + r\log(1+|\Phi_i|^2)$$

$$|\zeta|^2 \equiv |\mathfrak{z}_j|^2(1+|\Phi_i|^2)$$

$$\Phi_i = \frac{\mathcal{N}_i}{\mathcal{N}_N}, \quad i = 1,\ldots,N-1,$$

$$\mathfrak{z}_j = r^{-1/2}\mathcal{N}_N\mathcal{Z}_j, \quad j = 1,\ldots,\widetilde{N}.$$

for one extra flavor reduces
to blow up of $\mathbb{C}^N$

ZN model vs HT model

$$K_{\mathrm{HT}} = \sqrt{r^2 + 4r|\zeta|^2} - r\log\left(r + \sqrt{r^2+4r|\zeta|^2}\right) + r\log(1+|\Phi_i|^2)$$

$$\xrightarrow{\zeta \to 0}$$

$$K_{zn} = r|\zeta|^2 + r\log(1+|\Phi_i|^2)$$

$$K_{\mathrm{HT}} = K_{zn} + \mathcal{O}(|\zeta|^2)$$

IR physics of ZN and HT models is the same

BPS spectra are the same, but otherwise **different**

Perturbation theory

Perturbation theory

Gel-Mann-Low function

$$\beta_{i\bar{\jmath}} = a^{(1)} R^{(1)}_{i\bar{\jmath}} + \frac{1}{2r} a^{(2)} R^{(2)}_{i\bar{\jmath}} + \dots$$

$$R^{(1)}_{i\bar{\jmath}} = R_{i\bar{\jmath}}\,,$$

$$R^{(2)}_{i\bar{\jmath}} = R_{i\bar{k}l\bar{m}} R^{\bar{k}}{}_{\bar{\jmath}}{}^{l\bar{m}}$$

Kaehler metric

$$g_{i\bar{\jmath}} = \partial_i \bar{\partial}_{\bar{\jmath}} K(z_i, \bar{z}_i)$$

Ricci tensor

$$R_{i\bar{\jmath}} = -\partial_i \bar{\partial}_{\bar{\jmath}} \log\det(g_{i\bar{\jmath}})$$

$$-\log\det(g^{(\mathrm{HT})}_{i\bar{\jmath}}) = \log(1 + |\Phi_i|^2) - \log\left(1 + \frac{r}{\sqrt{r^2 + 4r|\zeta|^2}}\right)$$

FI term renormalization (GLSM)

$$r_{\rm ren}(\mu) = r_0 - \frac{N - \widetilde{N}}{2\pi} \log \frac{M}{\mu}$$

$$r_{\rm ren} = 0 \Longrightarrow r_0 = \frac{N - \widetilde{N}}{2\pi} \log \frac{M}{\Lambda}$$

$$c_1(M_{\rm HT})\Big|_{\mathbb{CP}^{N-1}} = (N - \widetilde{N})\,[\omega_{\mathbb{CP}^{N-1}}]$$

Kaehler class is renormalized only at one loop, hence the result above should be the full answer for the coupling renormalization

If so what does the extra term in the last formula on the previous slide mean?

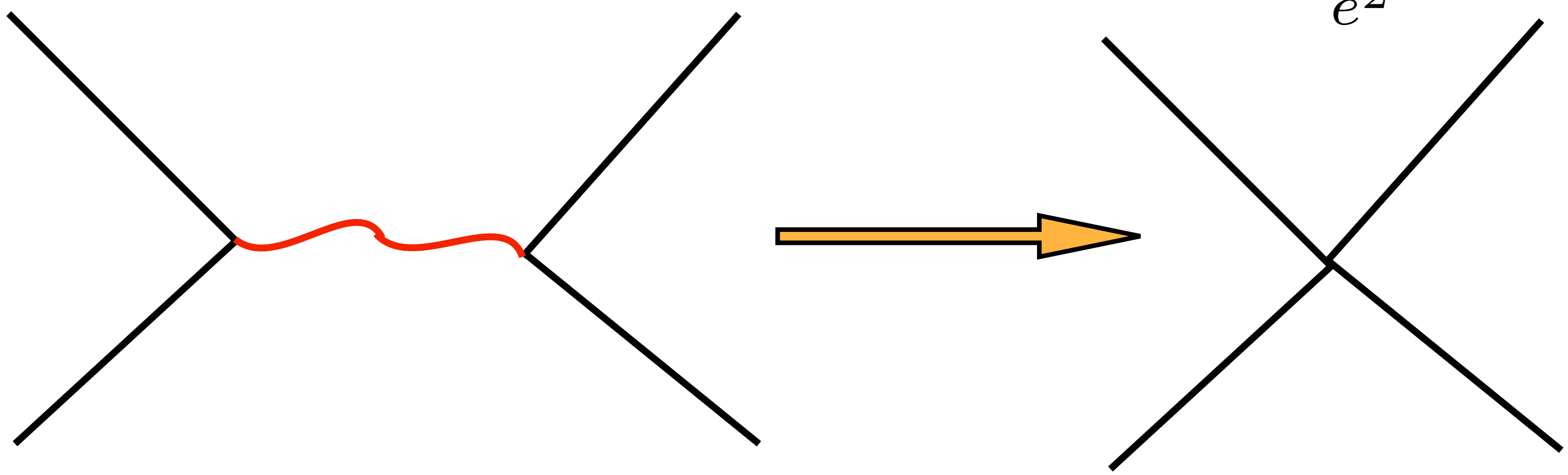
To understand why we need to compare renormalization schemes used in both calculations

GLSM vs NLSM

$$\int d^2x \int d^4\theta \left(|\Phi|^2 e^V - rV + \frac{1}{e^2}|\Sigma|^2 \right)$$

V-massive vector field w/ propagator

$$\frac{1}{\frac{p^2}{e^2} - M^2} \xrightarrow{p \ll e} \frac{1}{-M^2}$$



Integrating out V

$$-\log\det(g_{i\bar{\jmath}}) = (N - \widetilde{N})\log(1 + |\Phi_i|^2) - (N-1)|\zeta|^2 + \mathcal{O}(|\zeta|^4)\,.$$

Dimensional regularization (GLSM perturbation theory) mixes up UV and IR divergencies. Need to single out the UV piece out, IR contribution is not seen in the GLSM limit

Less SUSY I
Heterotic deformation

(0,2) Theory

[Gorsky Shifman Yung]

In 4d introduce masses $\int d^2\theta\, \mu^2 (\Phi^a)^2$

breaks $\mathcal{N}=2$ to $\mathcal{N}=1$ On the flux tube $(2,2) \longmapsto (0,2)$

obtain heterotic sigma model [Edalati Tong][Shifman Yung] [Distler Kachru]

$$\mathcal{L} = \int d^4\theta \left(\Phi_i^\dagger e^V \Phi^i - rV - \mathcal{B}V \right) \qquad \mathbb{CP}^{N-1} \times \mathbb{C}$$

B-right handed superfield

can be treated as model w/ field dependent FI term

$$K = (r + \mathcal{B}) \log(1 + |\phi^i|^2)$$

Geometry becomes non-Kahler
due to generation of H field (field dependent theta term)

Can do CP(N-1) as well

[PK Monin]

$$\mathcal{L}_{\mathbb{CP}^N} = \int d^2\theta \Big[\tfrac{1}{2}\varepsilon_{\beta\alpha}(\mathcal{D}_\alpha + i\mathcal{A}_\alpha)\mathcal{N}_i^\dagger(\mathcal{D}_\beta - i\mathcal{A}_\beta)\mathcal{N}_i + i\mathcal{S}(\mathcal{N}_i^\dagger\mathcal{N}_i - r_0)$$

$$+ \tfrac{1}{4}\varepsilon_{\beta\alpha}\mathcal{D}_\alpha\mathcal{B}^\dagger\,\mathcal{D}_\beta\mathcal{B} + \left(i\,\omega\,\mathcal{B}(\mathcal{S} - \tfrac{i}{2}\overline{\mathcal{D}}\gamma^5\mathcal{A}) + \text{H.c.}\right)\Big],$$

Isovector $\quad \mathcal{N}^i = n^i + \bar\theta\xi^i + \tfrac{1}{2}\bar\theta\theta F^i,$

Spinor $\quad \mathcal{A}_\alpha = -i(\gamma^\mu\theta)_\alpha A_\mu + \sqrt{2}(\gamma^5\theta)_\alpha\sigma_2 + \sqrt{2}\bar\theta\theta\, v_\alpha,$

Constraint $\quad \mathcal{S} = \sqrt{2}\sigma_1 + \sqrt{2}\bar\theta u + \tfrac{1}{2}\bar\theta\theta D$

complex fields $\quad \sigma = \sigma_1 + i\sigma_2\,,\qquad \lambda_\alpha = u_\alpha + iv_\alpha$

if negatively charged fields are included

$$\begin{aligned}
\mathcal{L}^{\mathrm{w}}_{\mathbb{CP}^N} &= |\nabla_\mu n_i|^2 + |\nabla_\mu\rho_i|^2 + i\bar\xi^i_L\nabla_R\xi^i_L + i\bar\xi^i_R\nabla_L\xi^i_R + i\bar\eta^i_L\nabla_R\eta^i_L + i\bar\eta^i_R\nabla_L\eta^i_R \\
&- 2|\sigma|^2|n_i|^2 - 2|\sigma|^2|\rho_i|^2 - D\left(|n_i|^2 - |\rho_i|^2 - r_0\right) - 4|\omega|^2|\sigma|^2 \\
&+ \left[i\sqrt{2}\bar n_i\left(\lambda_L\xi^i_R - \lambda_R\xi^i_L\right) - i\sqrt{2}\sigma\bar\xi^i_R\xi^i_L + \text{H.c.}\right] \\
&+ \left[-i\sqrt{2}\bar\rho_i\left(\bar\lambda_L\eta^i_R - \bar\lambda_R\eta^i_L\right) + i\sqrt{2}\bar\sigma\bar\eta^i_R\eta^i_L + \text{H.c.}\right] \\
&+ \tfrac{i}{2}\bar\zeta_R\partial_L\zeta_R - \left[i\sqrt{2}\omega\lambda_L\zeta_R + \text{H.c.}\right],
\end{aligned}$$

(0,2) GLSM

[PK Monin Vinci]

$$\int d^4\theta \left[\sum_{i=1}^{N_c} \Phi_i^\dagger e^V \Phi_i + \sum_{i=1}^{N_c - N_f} \tilde{\Phi}_i^\dagger e^{-V} \tilde{\Phi}_i - (r + \mathcal{B}) V + \frac{1}{2e^2} \Sigma^\dagger \Sigma \right]$$

$$\Phi^i = n^i + \bar{\theta}\xi^i + \theta\bar{\xi}^i + \bar{\theta}\theta F^i\,, \quad i = 1, \ldots, N_c$$
$$\widetilde{\Phi}^j = \rho^j + \bar{\theta}\eta^j + \theta\bar{\eta}^j + \bar{\theta}\theta\tilde{F}^j\,, \quad j = 1, \ldots, \tilde{N}$$

$$\Sigma = \sigma + i\theta^+\bar{\lambda}_+ - i\bar{\theta}^-\lambda_- + \theta^+\bar{\theta}^-(D - iF_{01})$$

$$\mathcal{B} = \omega(\bar{\theta}\zeta_R + \bar{\theta}\theta\bar{\mathcal{F}}\mathcal{F})$$

deformation adds

$$\mathcal{L}^{het} = \mathcal{L} + \bar{\zeta}_R \partial_L \zeta_R - |\omega|^2 |\sigma|^2 - [i\omega\lambda_L\zeta_R + \text{H.c.}]$$

Not enough SUSY non-pert. corrections out of control

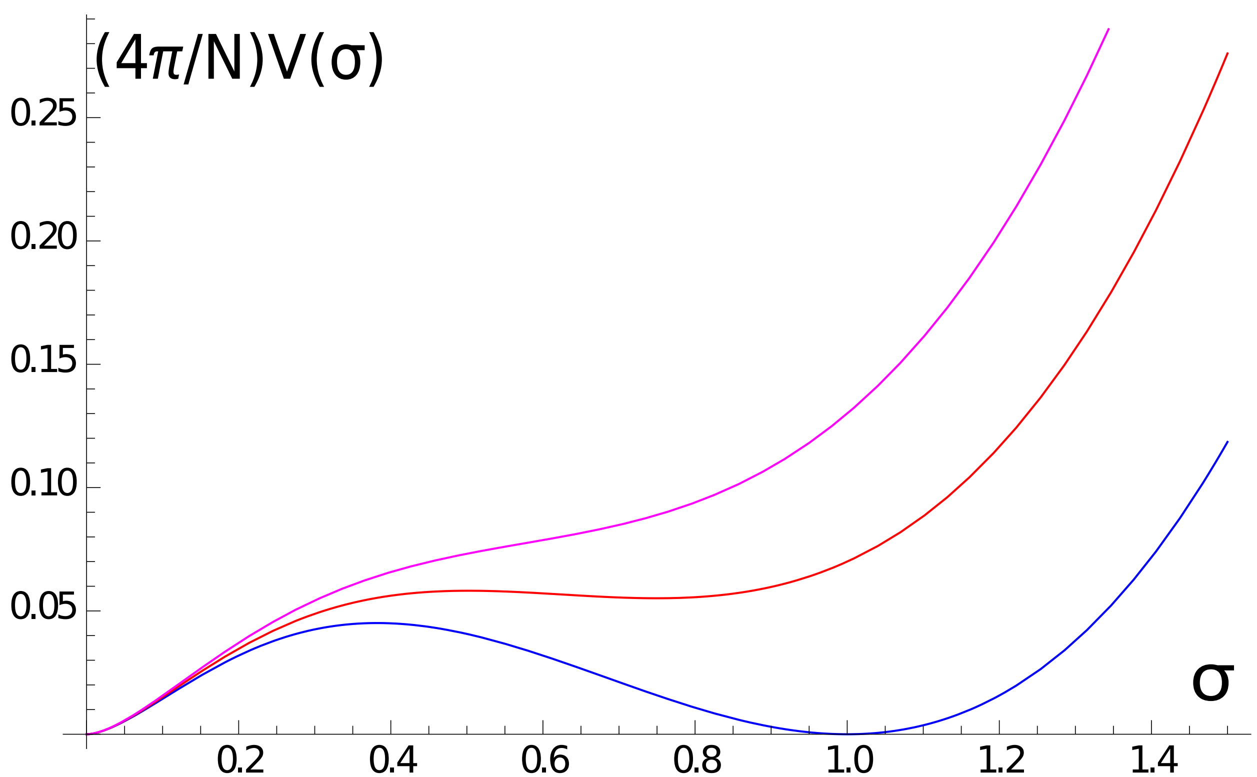
Have to dwell on large-N approach

Large-N solution of (0,2)

$$V_{1-loop} = \frac{1}{4\pi}\sum_{i=1}^{N-1}\left(-\left(D+|\sigma-m_i|^2\right)\log\frac{|\sigma-m_i|^2+D}{\Lambda^2}+|\sigma-m_i|^2\log\frac{|\sigma-m_i|^2}{\Lambda^2}\right)$$
$$-\frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1}\left(-\left(D-|\sigma-\mu_j|^2\right)\log\frac{|\sigma-\mu_j|^2-D}{\Lambda^2}-|\sigma-\mu_j|^2\log\frac{|\sigma-\mu_j|^2}{\Lambda^2}\right)$$
$$+\frac{N-\tilde{N}}{4\pi}D\,.$$

$$V_{eff} = V_{1-loop}+\left(|\sigma-m_0|^2+D\right)|n_0|^2+\left(|\sigma-\mu_0|^2-D\right)|\rho_0|^2+\frac{uN}{4\pi}|\sigma|^2$$

for zero masses



Symmetric masses

$$m_k = m\,e^{2\pi i\frac{k}{N}}\,,\quad k=0,\ldots,N-1\,,$$
$$\mu_l = \mu\,e^{2\pi i\frac{l}{\tilde{N}}}\,,\quad l=0,\ldots,\tilde{N}-1\,.$$

Vacuum equations

$$
\begin{aligned}
&\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0\,, \\
&\frac{1}{4\pi}\sum_{i=1}^{N-1} \log\frac{|\sigma - m_i|^2 + D}{\Lambda^2} - \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} \log\frac{|\sigma - \mu_j|^2 - D}{\Lambda^2} = |n_0|^2 - |\rho_0|^2\,, \\
&\frac{1}{4\pi}\sum_{i=1}^{N-1} (\sigma - m_i)\log\frac{|\sigma - m_i|^2 + D}{|\sigma - m_i|^2} + \frac{1}{4\pi}\sum_{j=1}^{\tilde{N}-1} (\sigma - \mu_j)\log\frac{|\sigma - \mu_j|^2 - D}{|\sigma - \mu_j|^2} = \\
&= (\sigma - m_0)\,|n_0|^2 + (\sigma - \mu_0)\,|\rho_0|^2 + \frac{uN}{4\pi}\,\sigma\,.
\end{aligned}
$$

Solution of (2,2) model

Phase transitions -- artifact of large-N

$$\left(|\sigma - m_0|^2 + D\right) n_0 = 0\,, \quad \left(|\sigma - \mu_0|^2 - D\right) \rho_0 = 0$$

Higgs in n (Hn)

$$\rho_0 = 0 \quad D = -|\sigma - m|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{m}{\Lambda}\,, & \mu < m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m\,. \end{cases}$$

Higgs in rho (Hrho)

$$n_0 = 0 \quad D = |\sigma - \mu|^2$$

$$r = \begin{cases} \frac{N-\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu > m \\ \frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{\tilde{N}}{2\pi} \log \frac{\mu}{\Lambda}\,, & \mu < m \end{cases}$$

Coulomb (C)

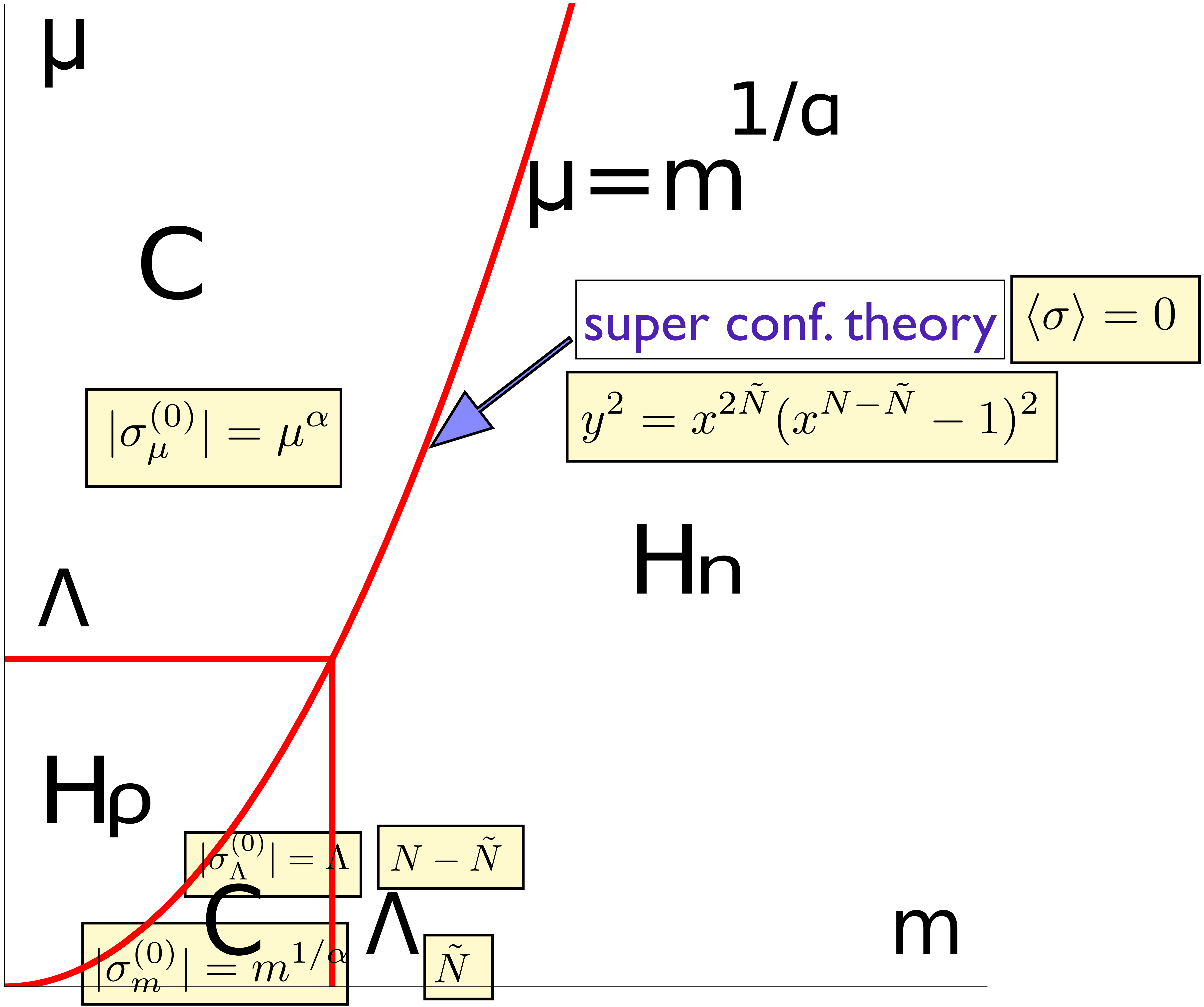
$$n_0 = \rho_0 = 0$$

renormalized FI term vanishes in C phase

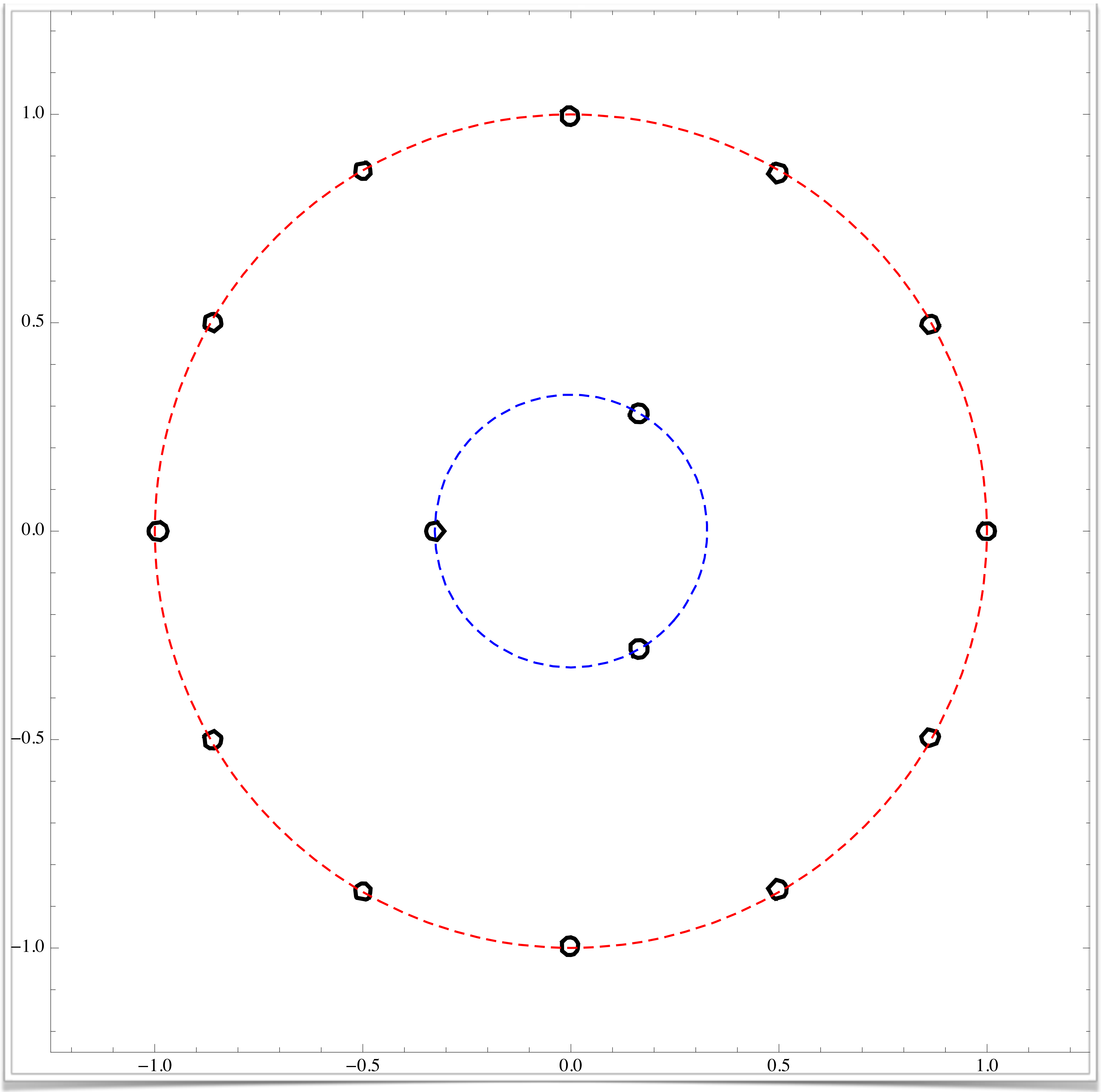
in (2,2) from exact superpotential

$$\frac{\prod_i (\sigma - m_i)}{\prod_i (\sigma - \mu_j)} = \Lambda^{N-\tilde{N}} \qquad \sigma = 0$$

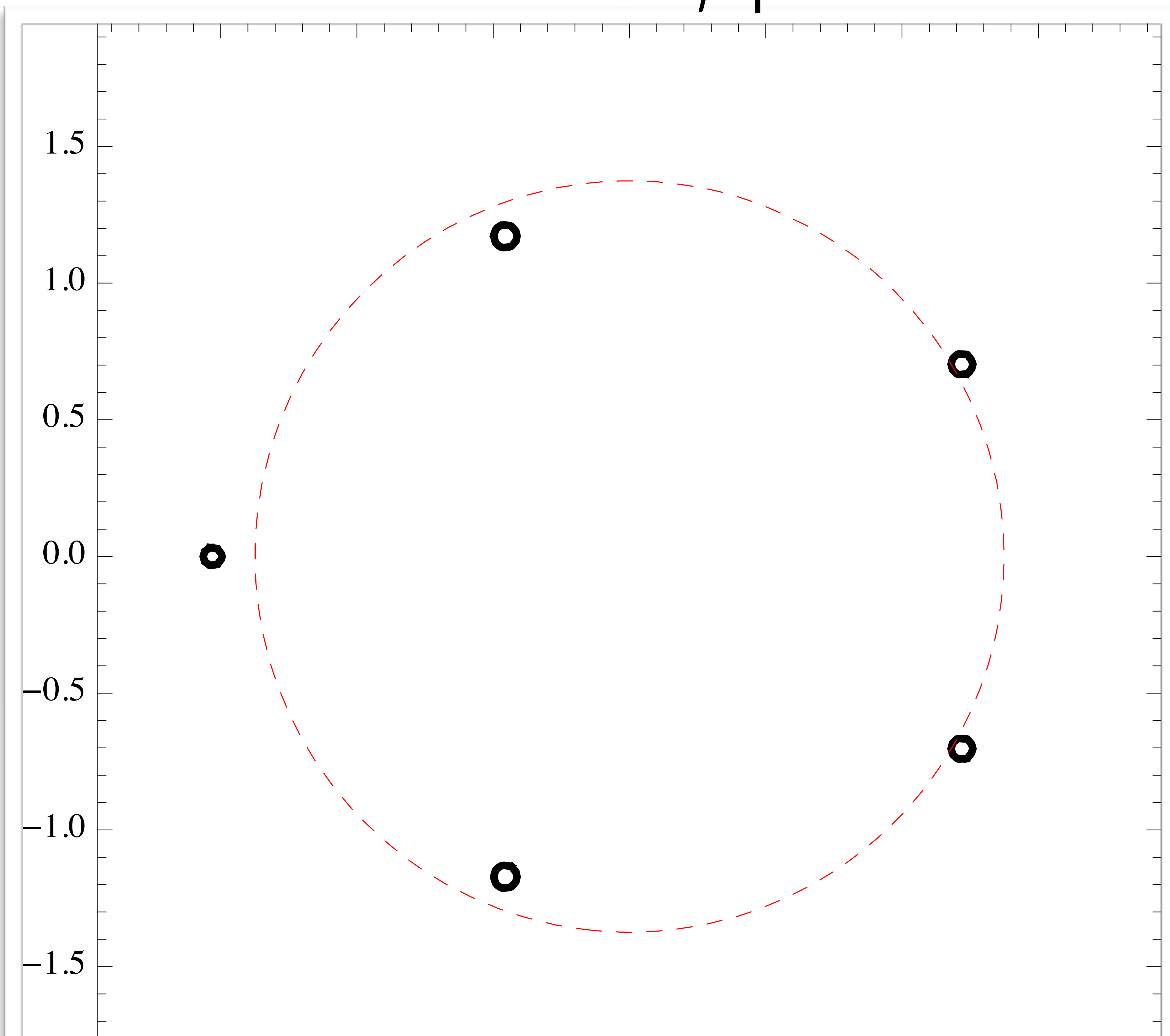
is one of the solutions...

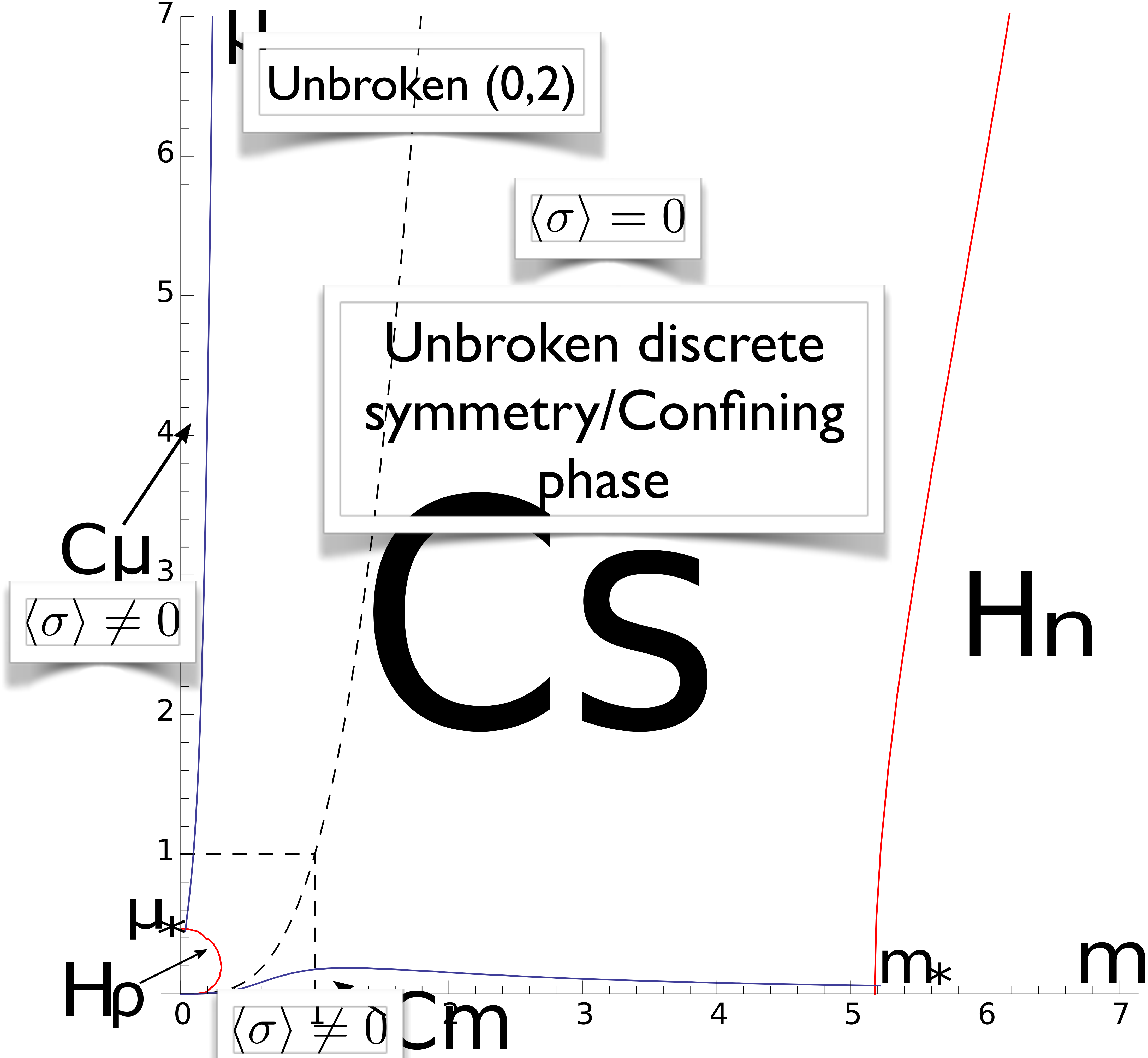

μ
1/α
μ=m
C
super conf. theory
$\langle\sigma\rangle = 0$
$y^2 = x^{2\tilde{N}}(x^{N-\tilde{N}} - 1)^2$
$|\sigma_\mu^{(0)}| = \mu^\alpha$
Hn
Λ
Hp
$|\sigma_\Lambda^{(0)}| = \Lambda$
$N - \tilde{N}$
C
Λ
$\tilde{N}$
$|\sigma_m^{(0)}| = m^{1/\alpha}$
m

N=15 Nf=18



Nf=5 Cμ phase





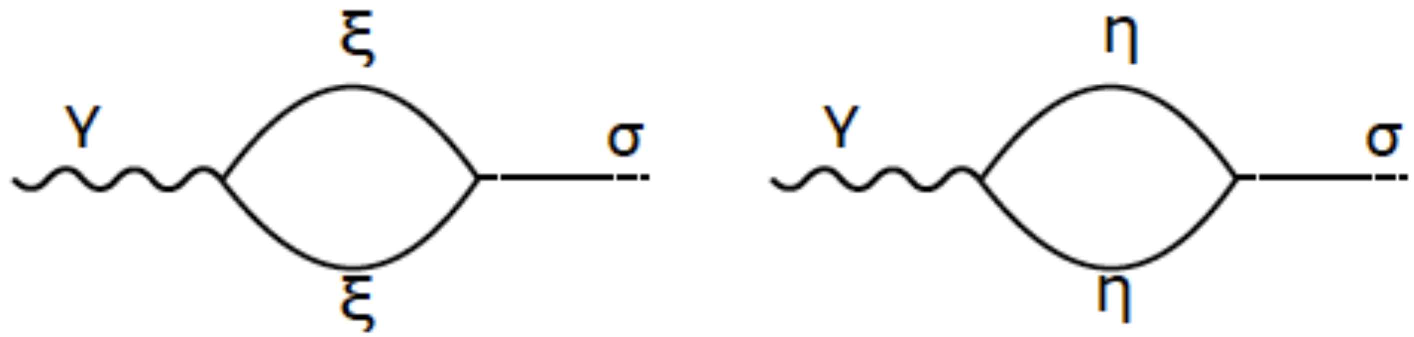
μ
Unbroken (0,2)
$\langle\sigma\rangle = 0$
Unbroken discrete symmetry/Confining phase
Cμ
$\langle\sigma\rangle \neq 0$
Cs
Hn
μ_*
Hp
m_*
m
$\langle\sigma\rangle \neq 0$
Cm
0
1
2
3
4
5
6
7

Spectrum

[Bolokhov Shifman Yung]
[PK Monin Vinci]

$$\mathcal{L} = -\frac{1}{4e_\gamma^2}F_{\mu\nu}^2 + \frac{1}{e_{\sigma\,1}^2}(\partial_\mu \mathfrak{Re}\,\sigma)^2 + \frac{1}{e_{\sigma\,2}^2}(\partial_\mu \mathfrak{Im}\,\sigma)^2 + i\mathfrak{Im}(\bar{b}\,\delta\sigma)\epsilon_{\mu\nu}F^{\mu\nu} - V_{\text{eff}}(\sigma) + \text{Fermions}$$

Anomaly



$$b = \frac{N}{4\pi}\left(\frac{1}{N}\sum_{i=1}^{N-1}\frac{1}{\bar{\sigma}_0 - \bar{m}_i} - \alpha\frac{1}{\tilde{N}}\sum_{i=1}^{\tilde{N}-1}\frac{1}{\bar{\sigma}_0 - \bar{\mu}_i}\right)$$

$$m_\gamma = e_{\sigma\,2}e_\gamma|b|$$

Photon becomes massless in Cs phase!! Confinement!

Note that Lambda vacua disappear at large deformations

Need to sit in zero-vacua

e.g. in Cm phase

$$m_\gamma = \sqrt{6}\,\Lambda\left(\frac{\Lambda}{m}\right)^{1/\alpha}\left(\left(\frac{m}{\Lambda}\right)^{2/\alpha} - \left(\frac{\mu}{\Lambda}\right)^2 e^{u/\alpha}\right)\mathrm{e}^{-\frac{u}{2\alpha}}$$

Massless goldstino in fermionic sector

Conclusions and open questions

- Study BPS (and beyond) spectrum of SQCD can effectively be done using 2d NLSM (and GLSM)
- Rich variety of phases in (0,2) model at strong coupling
- Other heterotic deformations $\bar{D}\Phi_+ \sim \bar{D}\Phi_-$
- Are there flux tubes in theories without FI term? (e.g. SU(N)) Omega deformed 4d theory may have such solutions...
- Connections to integrable systems in 2d...
- Relationship w/ another 4d/2d duality [Vafa et al]