Mathematics 53

Quiz 5 – 08/06

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This is a closed book/notes test. Calculators are not permitted

1. Evaluate the following integral

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$$

2. Give five other iterated integrals that are equal to

$$\int_0^2 \int_0^{y^3} \int_0^{y^2} f(x, y, z)\, dz\, dx\, dy$$

3. Evaluate the integral over $D \subset \mathbb{R}^2$ which is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$.

$$\iint_D xy^2 \, dA$$

4. Calculate the integral

$$\iint_R ye^{xy}\,dA\,, \qquad \text{where} \qquad R = \{(x,y) \in \mathbb{R}^2 | 0 \le x \le 2,\, 0 \le y \le 3\}\,.$$

5. Show that when Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

when written in cylindrical coordinates $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ becomes

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2} = 0\,.$$

6. (Extra Credit!) The plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1\,, \qquad a > 0,\, b > 0,\, c > 0$$

cuts the solid ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\,,$$

into two pieces. Find their volumes.