## Mathematics 53

Quiz $2-07 / 30$
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This is a closed book/notes test. Calculators are not permitted

1. Find the maximum rate of change of $f(x, y)=x^{2} y+\sqrt{y}$ at the point $(2,1)$. In which direction does it occur (specify the corresponding vector)?
2. Use Lagrange multipliers to find the maximum and minimum values of $f$ subject to the given constraint.
(a)

$$
f(x, y)=x^{2} y, \quad x^{2}+y^{2}=1
$$

(b)

$$
f(x, y, z)=x y z, \quad x^{2}+y^{2}+z^{2}=3
$$

3. Find the points on the surface $x y^{2} z^{3}=2$ that are closest to the origin.
4. Consider the following function

$$
f(x, y, z)= \begin{cases}\frac{(x+y+z)^{r}}{x^{2}+y^{2}+z^{2}} & \text { if } \quad(x, y, z) \neq(0,0,0) \\ 0, & \text { if } \quad(x, y, z)=(0,0,0)\end{cases}
$$

For what values of $r$ is this function continuous on $\mathbb{R}^{3}$ ? Explain your answer.
5. (Extra Credit!) Find the maximum value of the function

$$
f(x, y)=\frac{(a x+b y+c)^{2}}{x^{2}+y^{2}+1}
$$

