$\begin{array}{l} \text{Mathematics 53} \\ \text{Quiz } 2-07/30 \\ \text{Peter Koroteev} \end{array}$

This is a closed book/notes test. Calculators are not permitted

1. Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point (2, 1). In which direction does it occur (specify the corresponding vector)?

- 2. Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint.
 - (a)

$$f(x,y) = x^2 y$$
, $x^2 + y^2 = 1$,

(b)

$$f(x, y, z) = xyz$$
, $x^2 + y^2 + z^2 = 3$.

3. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

4. Consider the following function

$$f(x, y, z) = \begin{cases} \frac{(x + y + z)^r}{x^2 + y^2 + z^2} & \text{if } (x, y, z) \neq (0, 0, 0) \\ 0, & \text{if } (x, y, z) = (0, 0, 0) . \end{cases}$$

For what values of r is this function continuous on \mathbb{R}^3 ? Explain your answer.

5. (Extra Credit!) Find the maximum value of the function

$$f(x,y) = \frac{(ax + by + c)^2}{x^2 + y^2 + 1}$$