# 6D SCFTs, 4D SCFTs, Conformal Matter and Spin Chains

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# Based On

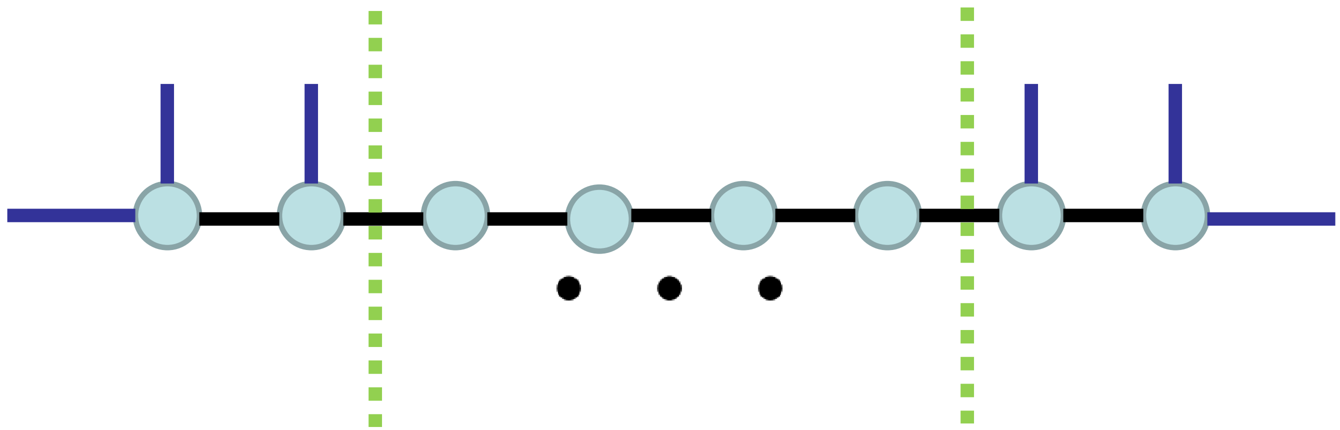
hep-th/2007.07262 w/ Baume and Lawrie

hep-th/2007.08545

hep-th/????.????? w/ Baume and Lawrie

# Main Idea:

6D SCFTs Look like 1D Systems:



1D States $\Rightarrow$ 6D Operators

# Outline

- Why Study 6D SCFTs?
- 1D Structures / Spin Chains
- Conclusions

# Why Study 6D SCFTs?

# What's a 6D CFT?

A physical theory in $5 + 1$ Dimensions

(basically no distances, only angles)

Spacetime Symmetries: $SO(6, 2)$

Lorentz (i.e. $SO(5, 1)$) + a few more:

Includes Translations and Scaling + ...

# Why Are They Interesting?

Basic Reason:

Not Obvious $\exists$ interacting 6D CFTs...

# Do they Exist?

Not from pert$^n$ of Gaussian fixed point

# Do they Exist?

Try: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \text{perturbations}$

Unstable: $V(\phi) = \phi^3$

Irrelevant: $V(\phi) = \phi^4 \;\; ([\phi] = 2...)$

# String Theory Says:

Yes, decouple most string modes

$\Rightarrow$ theory with local $T_{\mu\nu}$

so, expect a CFT in IR

# SUSY Helps: 6D SCFTs

A 6D CFT + supersymmetry:

$$Q^2 = 0$$

$$QQ' + Q'Q = \text{6D Translation}$$

Two Possibilities:

- 8 Independent $Q$'s: The "(1,0) Theories"
- 16 Independent $Q$'s: The "(2,0) Theories"

# So... Interesting?

## Intrinsic Reasons:

Nahm: SCFTs only for D $\leq 6$

No examples until mid 1990's!
(required input from string theory)

All interacting examples "non-Lagrangian"
¿¿¿Operators and Correlators???

# Conceptually Important

Major Issue: ¿¿¿*Define* QFT???

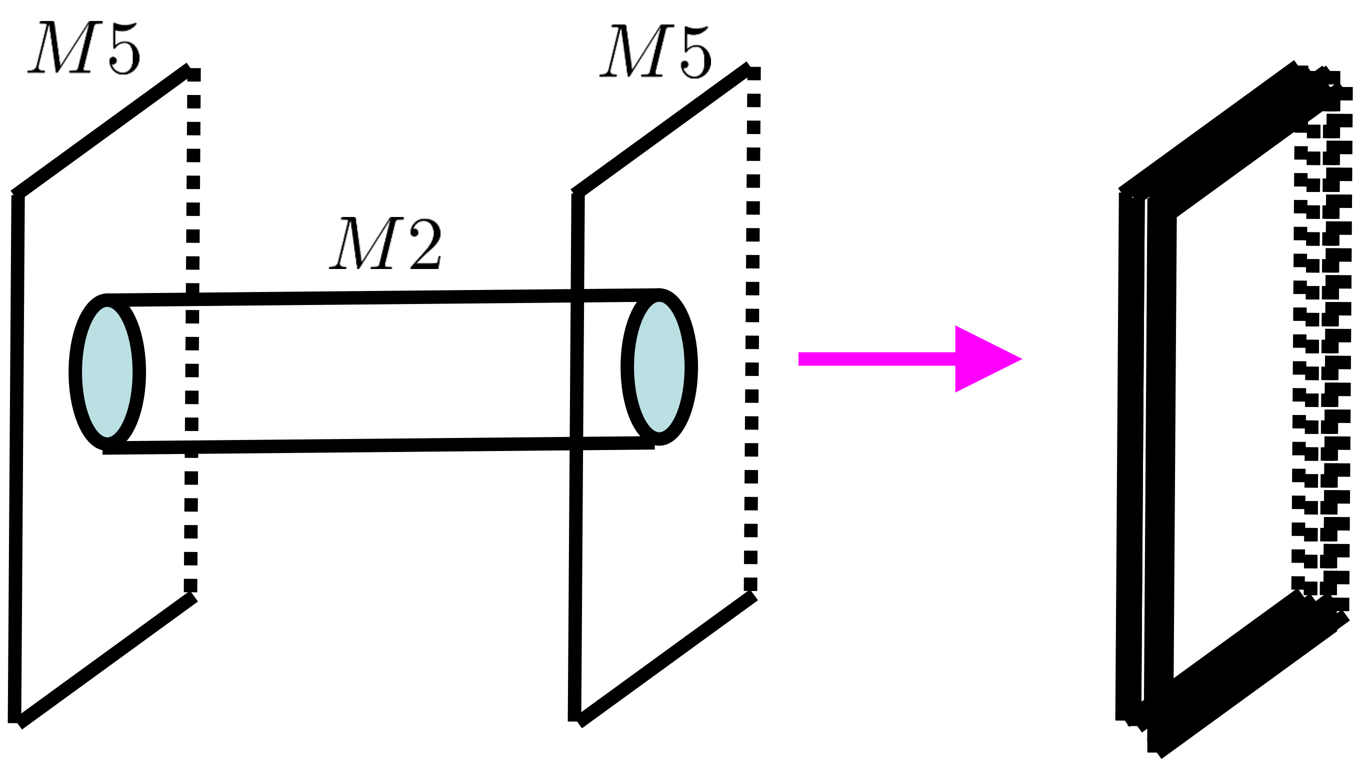
More Examples $\Rightarrow$ More Clues

# First Examples
# The $(2,0)$ Theories

# Use M5-Branes

Witten '95, Strominger '95

Consider $N$ M5-Branes on $\mathbb{R}^{5,1} \times \mathbb{R}^5_\perp$



# Classifying $(2,0)$ Theories

Witten '95, Strominger '95

Type IIB on $\mathbb{C}^2/\Gamma$

$(2,0)$ SUSY $\Rightarrow \Gamma$ a discrete subgroup of $SU(2)$

Such subgroups are classified: $A_K$, $D_K$, $E_6$, $E_7$, $E_8$

(symmetries of platonic solids)

# Geometric Resolution

Witten '95, Strominger '95

Type IIB on $\mathbb{C}^2/\Gamma_{ADE}$

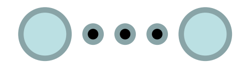
Resolution Involves:

Bouquet of $\mathbb{CP}^1$'s

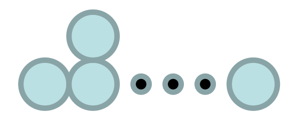
$\mathbb{CP}^1_i \cap \mathbb{CP}^1_j = -\text{Dynkin}_{ij}$

Note: $\mathbb{CP}^1_i \cap \mathbb{CP}^1_i = -2$

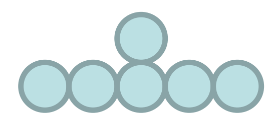
$A_N$



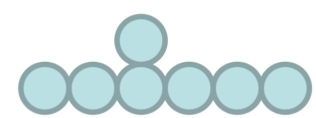
$D_N$



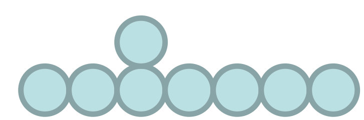
$E_6$



$E_7$



$E_8$



# More Examples
# The $(1,0)$ Theories

# Multiplets

Suppose we have a 6D SUSY theory...

Multiplets we might consider:

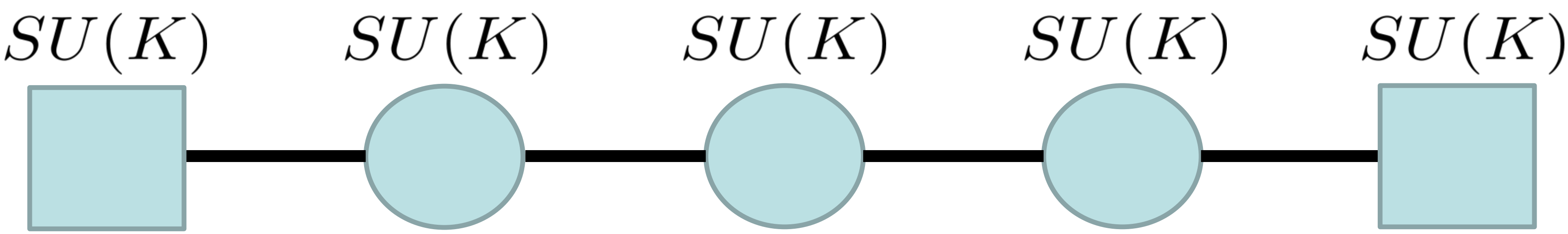
Vector Multiplet(s) $(A_\mu, + fermions)$

Hypermultiplet(s) $(X_{\mathcal{R}}, Y_{\mathcal{R}^c} + fermions)$

Tensor Multiplet(s) $(t,\ B^-_{\mu\nu}, + fermions)$

# 6D Quiver SCFTs

1) Construct consistent 6D Quiver w/ cutoff $\Lambda_{UV}$

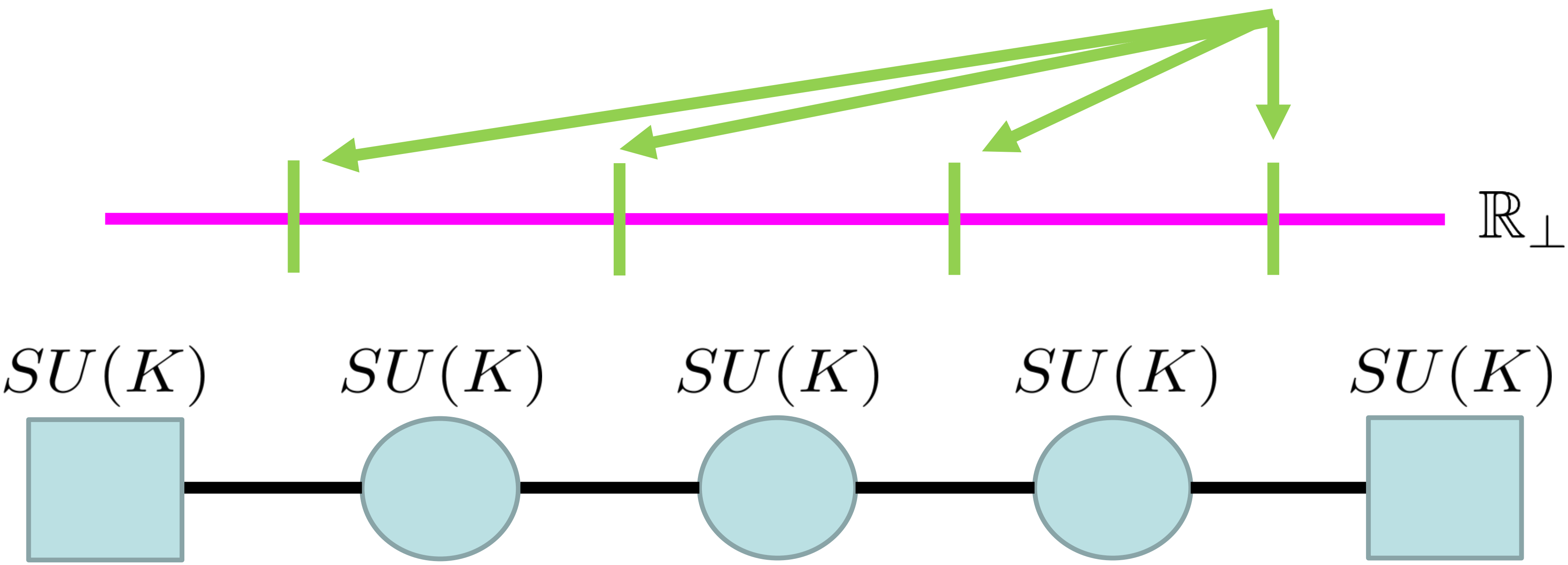



2) Go to strong coupling: $\frac{1}{g^2} \to 0$

# M-Theory Realization

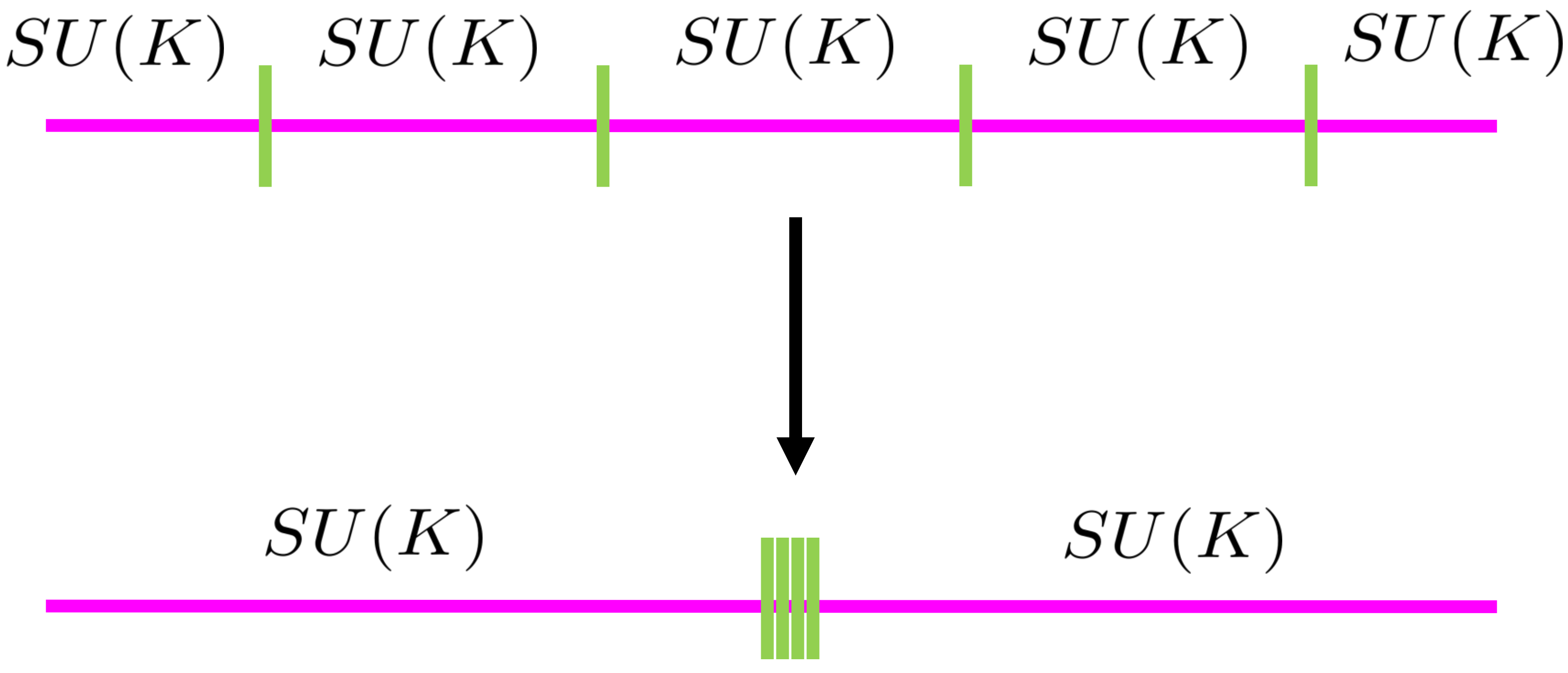
M-theory on $\mathbb{R}^{5,1} \times \mathbb{R}_{\perp} \times \mathbb{C}^2/\mathbb{Z}_K \Rightarrow$ 7D $SU(K)$ SYM

Introduce $N$ domain walls $=$ $N$ M5-brane probes



# SCFT Limit

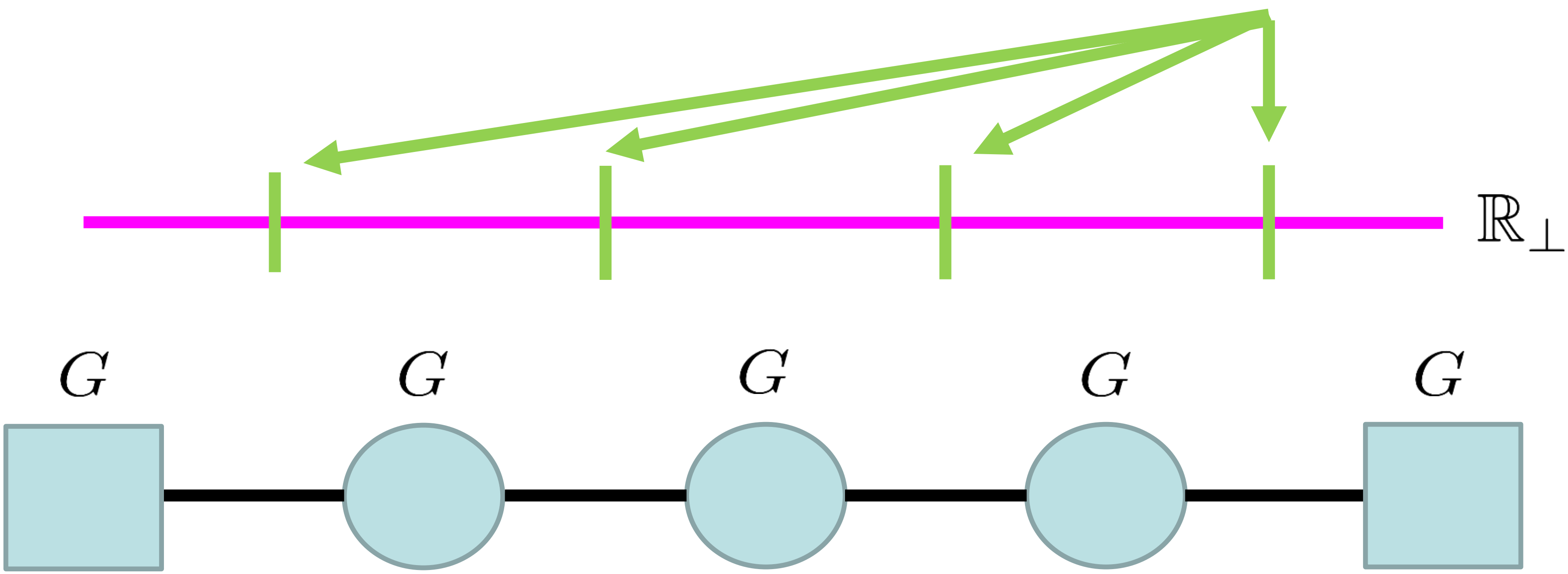
$$\text{Interval Length} = \frac{1}{g^2} \to 0$$



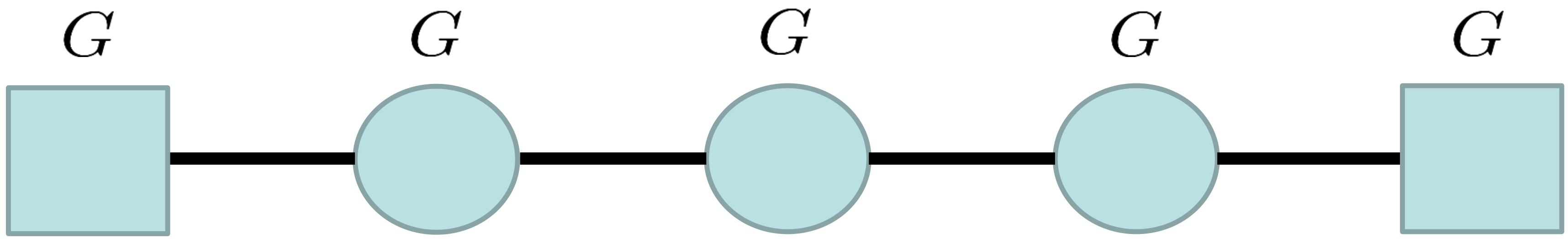
# Generalizations

M-theory on $\mathbb{R}^{5,1} \times \mathbb{R}_\perp \times \mathbb{C}^2/\Gamma_{ADE} \Rightarrow$ 7D $G_{ADE}$ SYM

Introduce $N$ domain walls $=$ $N$ M5-brane probes



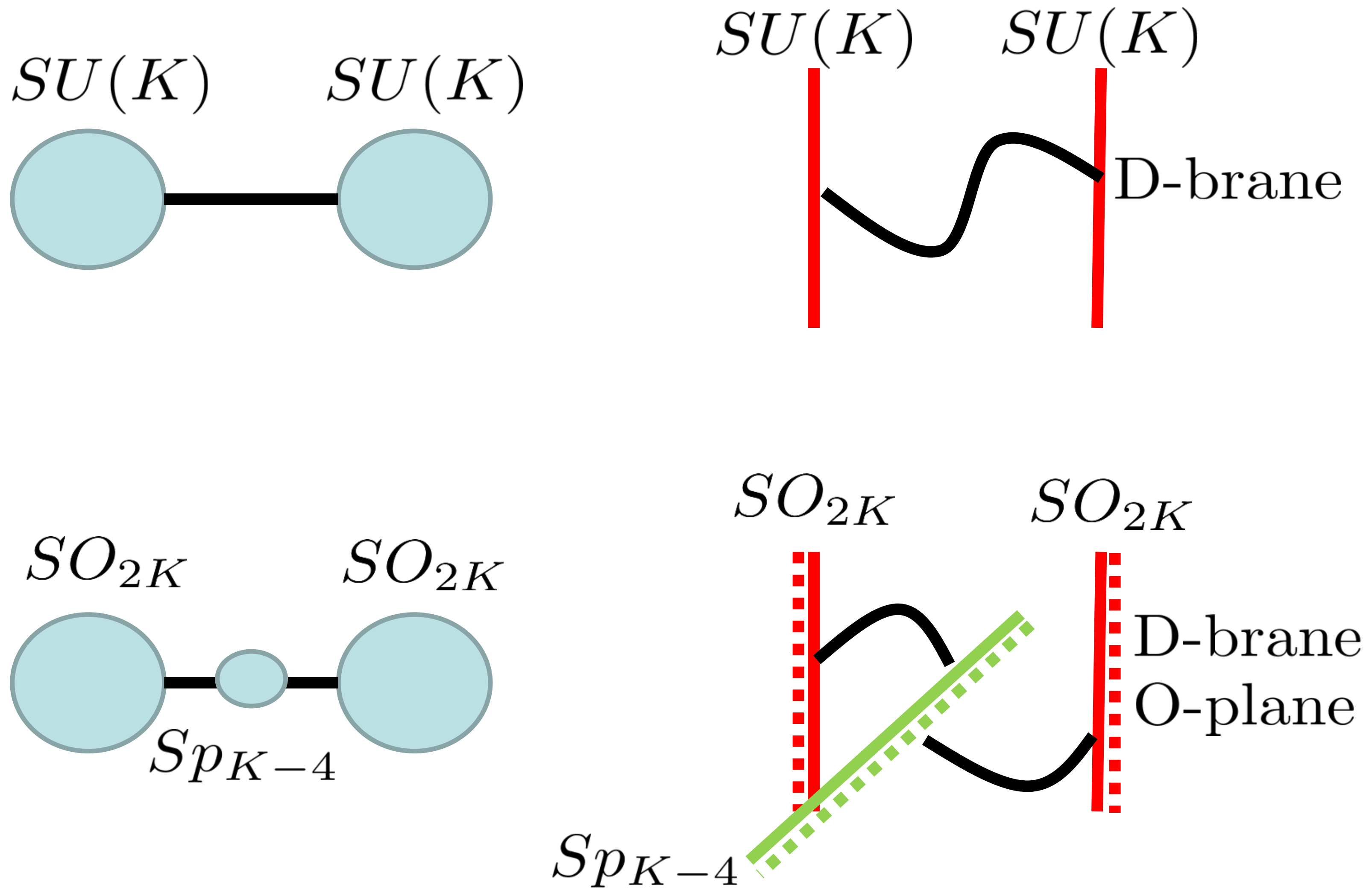
# Is this a Quiver?



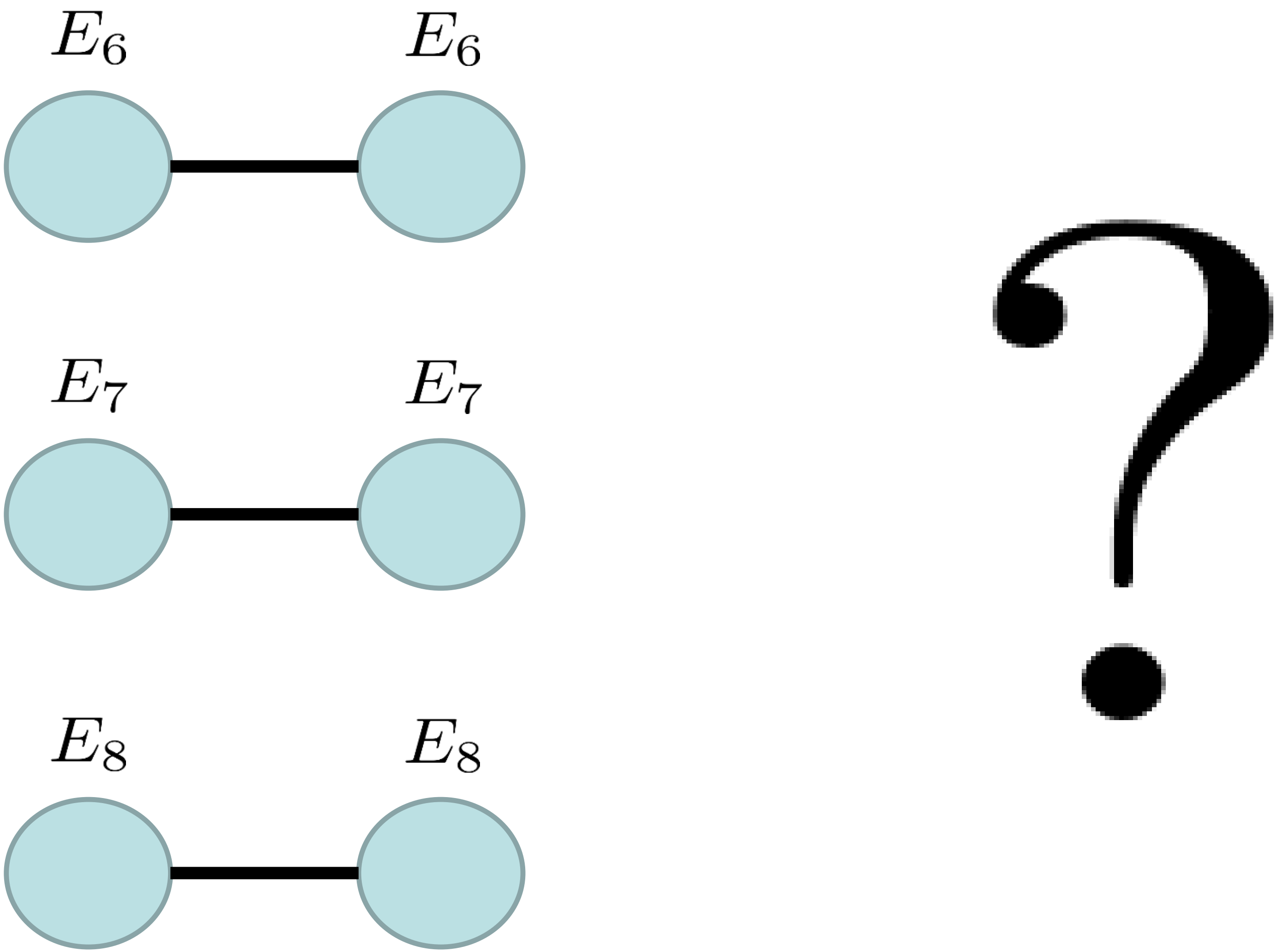

Issue: Link Fields aren't hypermultiplets!

Links known as "conformal matter"

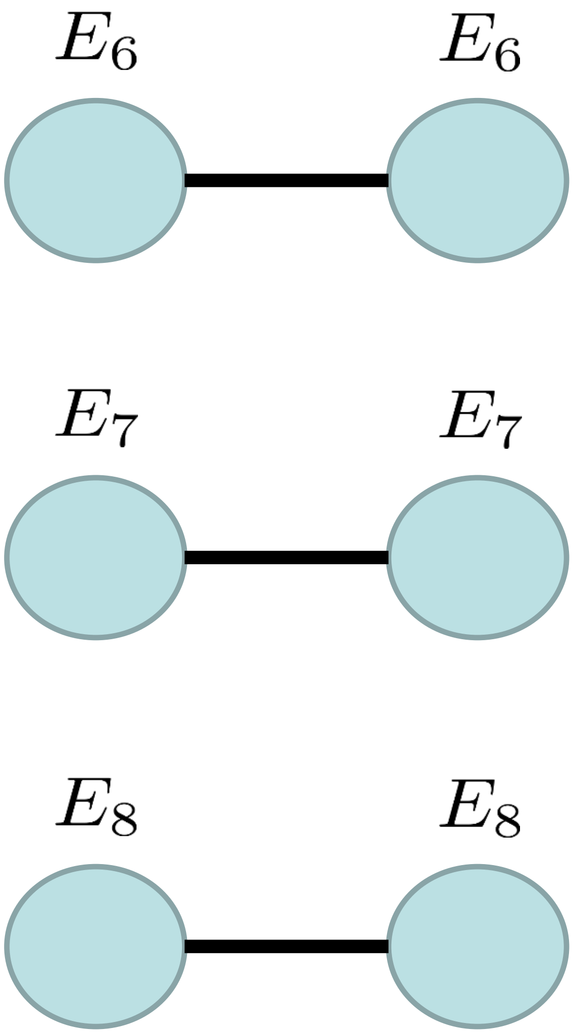
# $SU(K)$ vs $SO_{2K}$ $(K \geq 4)$



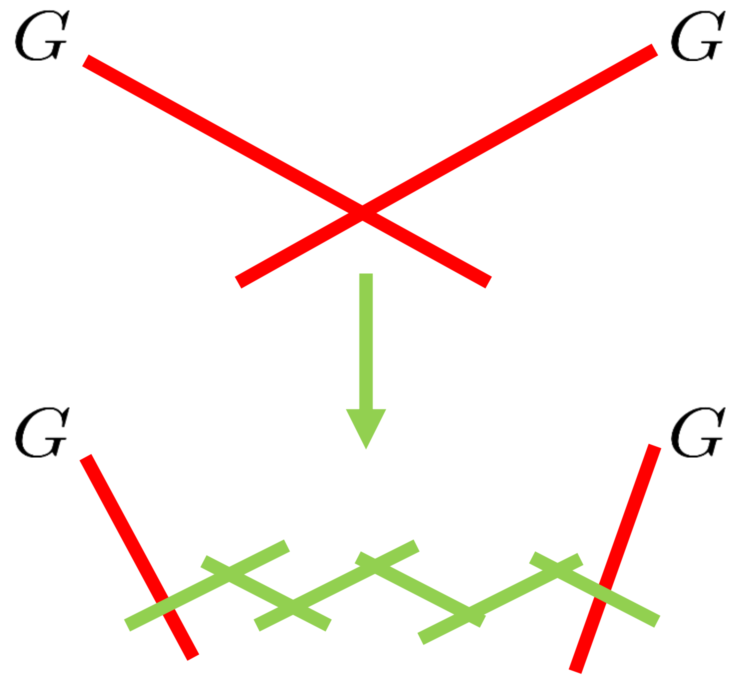
# E-Type Quivers?



# E-Type Quivers?



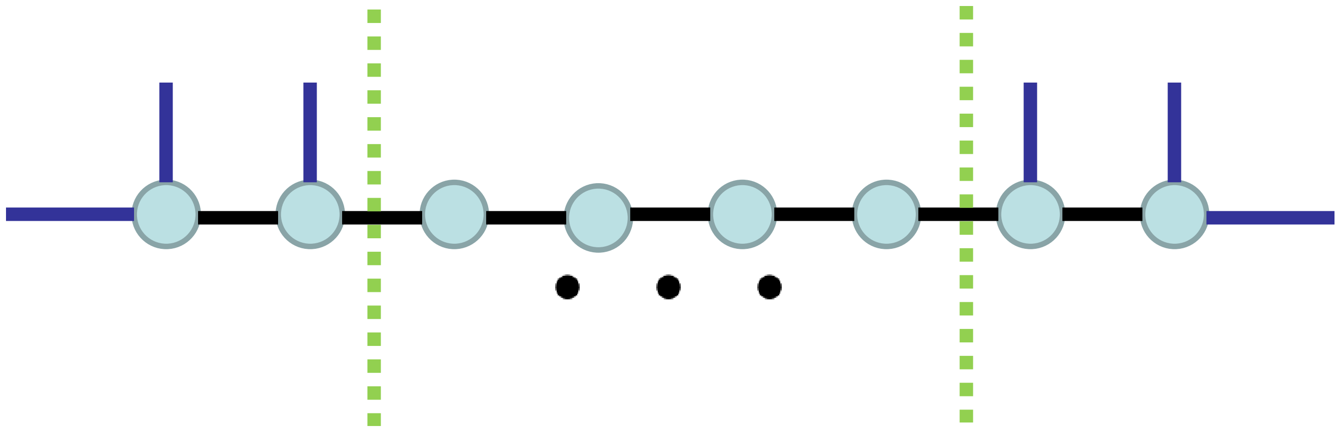

Use F-theory!



# Classification

JJH Morrison Vafa ’13; JJH Morrison Rudelius Vafa ’15;

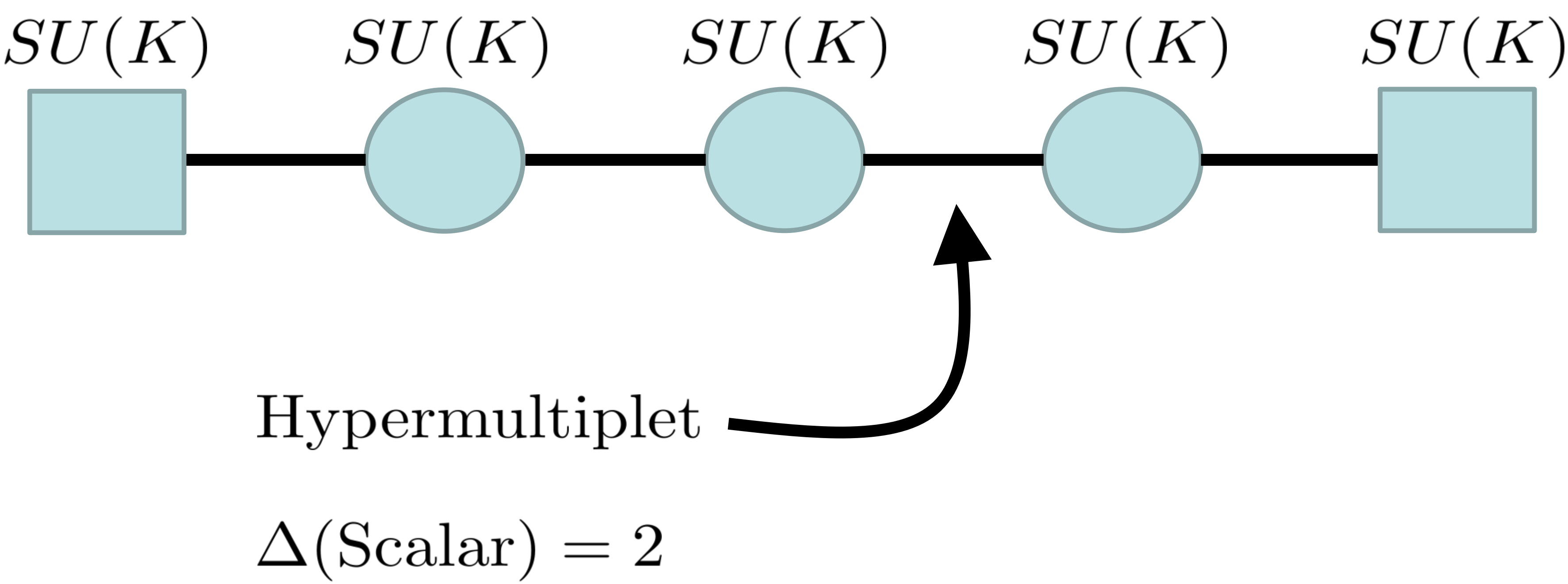
JJH Rudelius Tomasiello ’19



Looks like a 1D system...

# Operators and Conformal Matter

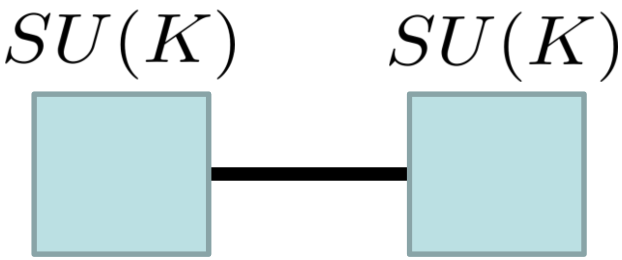
# Warmup... Consider:



Geometrically?

# Via Calabi-Yau Geometry

Argyres Plesser Seiberg Witten '95; JJH '14




$$xy = (uv - X \cdot Y)^{K+1}$$

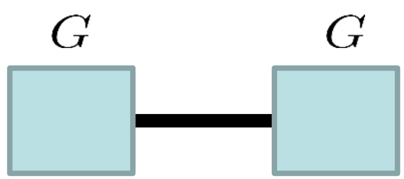
$$\Delta\left(\frac{dx \wedge du \wedge dv}{y}\right) = 4$$

$$\Rightarrow \Delta\left(X \cdot Y\right) = 4$$

$$\Rightarrow \Delta\left(\text{Hyper}\right) = 2$$

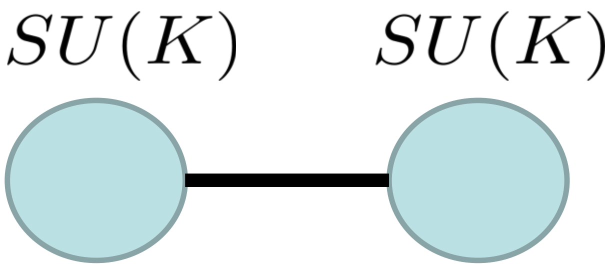
$SU(2)_{\mathcal{R}}$ irrep: spin 1/2

# Conformal Matter

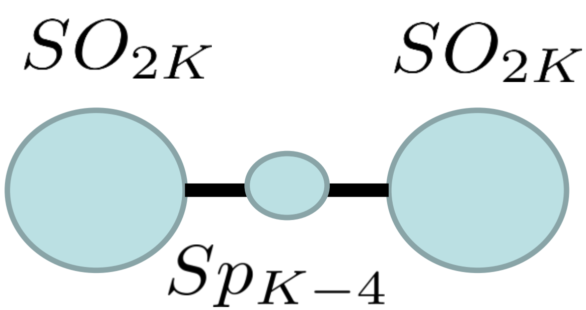



| | $\Delta$ | $SU(2)_{\mathcal{R}}$ |
|---|---|---|
| $SU(K)$ | 2 | 1/2 |
| $SO(2K)$ | 4 | 1 |
| $E_6$ | 6 | 3/2 |
| $E_7$ | 8 | 2 |
| $E_8$ | 12 | 3 |

# Compositeness...

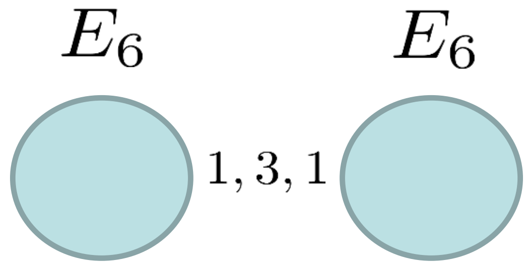



$$1/2$$

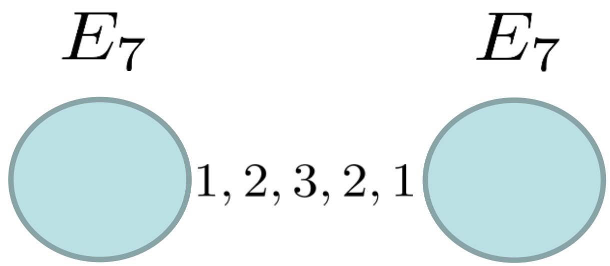



$$1/2 \otimes 1/2 = 1 \oplus 0$$

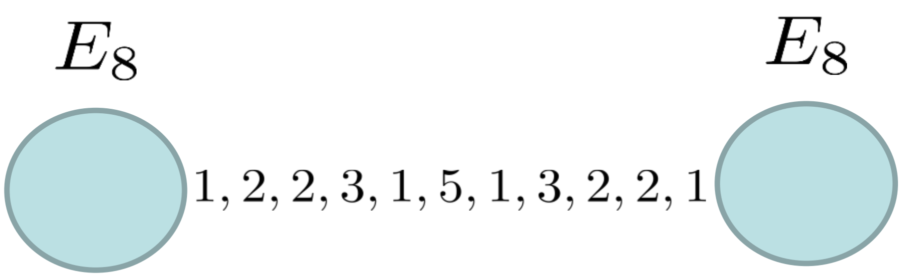
# More Compositeness...




$$(1/2)^{\otimes 3} = 3/2 \oplus \ldots$$

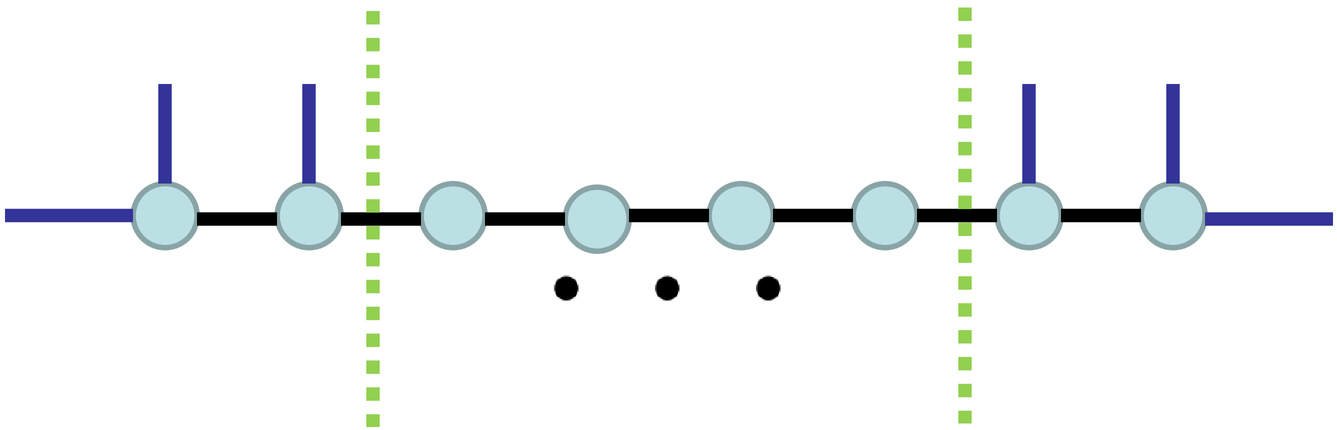



$$(1/2)^{\otimes 4} = 2 \oplus \ldots$$




$$(1/2)^{\otimes 6} = 3 \oplus \ldots$$

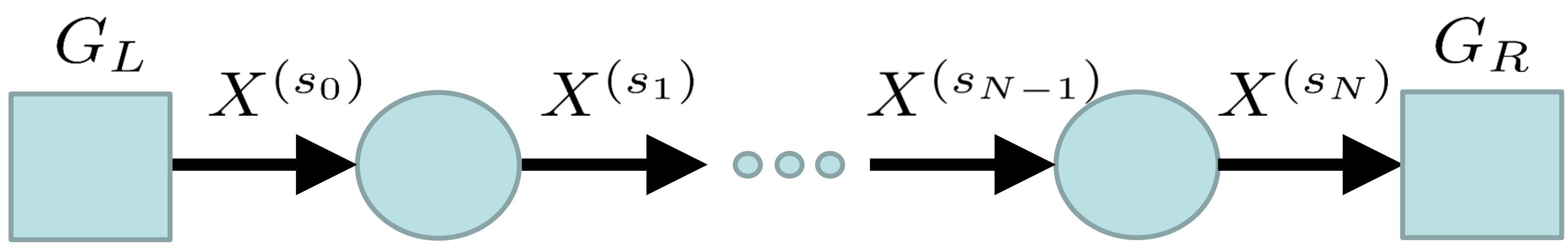
# But Recall...



Looks like a 1D system...

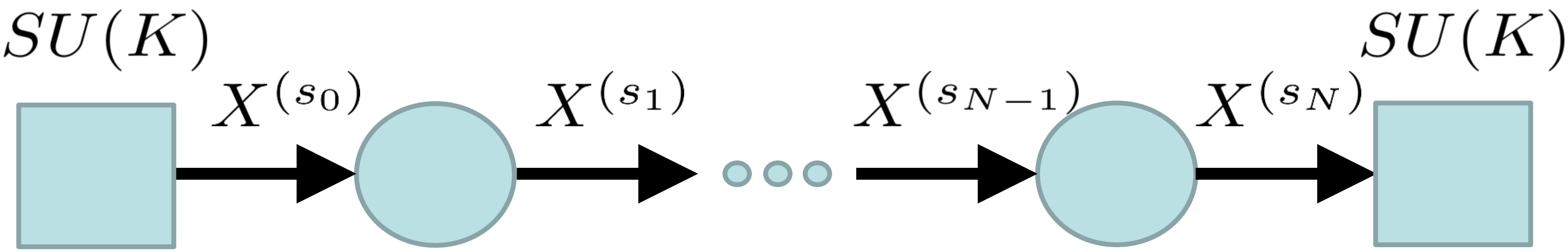
# Spin Chains

# A BPS Operator

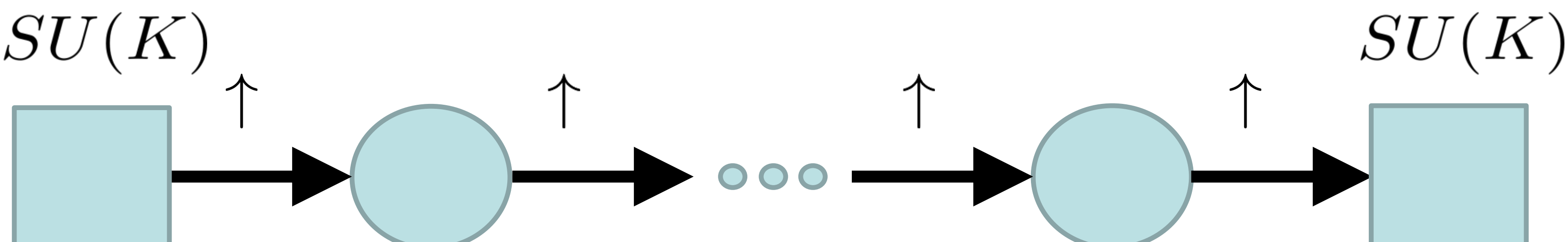



$X_i$ in spin $r$ rep of $SU(2)_{\mathcal{R}}$, $s_i = r$

# For Example...

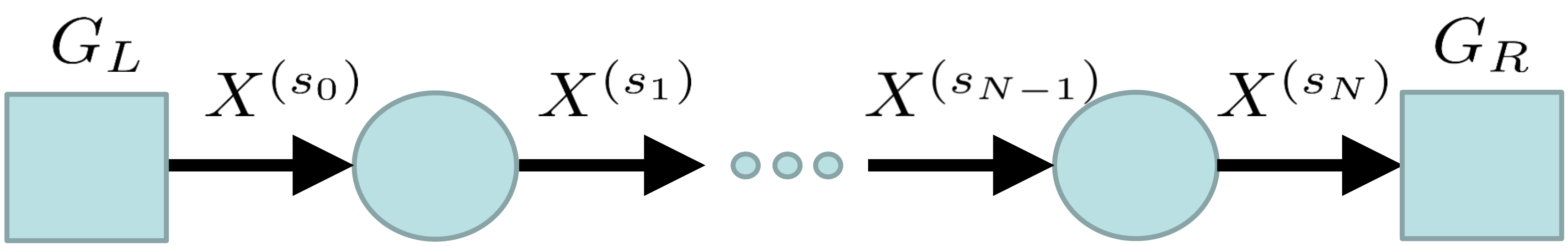



$X_i$ in spin $r$ rep of $SU(2)_{\mathcal{R}}$, $s_i = r$



# A BPS Operator

Bergman Fazzi Rodriguez-Gomez Tomasiello '20; Baume JJH Lawrie '20

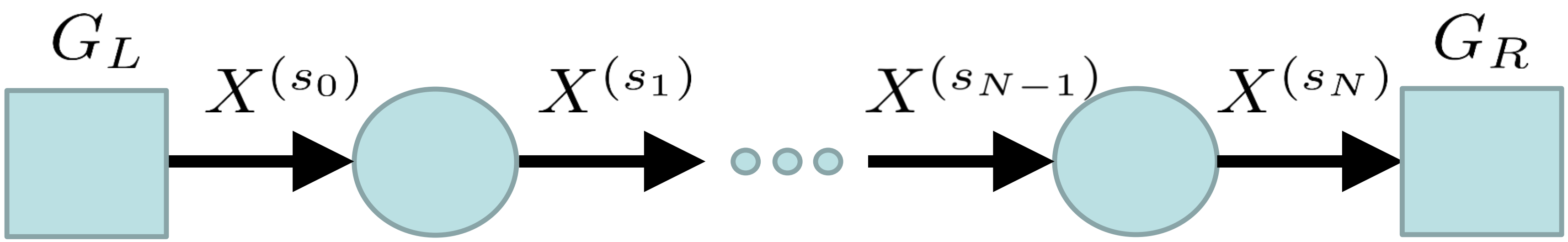



$\mathcal{O} \equiv X^{(s_0)} ... X^{(s_N)}$ in bifundamental $(G_L, G_R)$

$\Delta(\mathcal{O}) = (N+1) \times \Delta(X)$

# Non-BPS Operators?

What if $s_i \neq r$?




$$\mathcal{O} \equiv X^{(s_0)} \ldots X^{(s_N)}$$

$$\Delta(\mathcal{O}) = ???$$

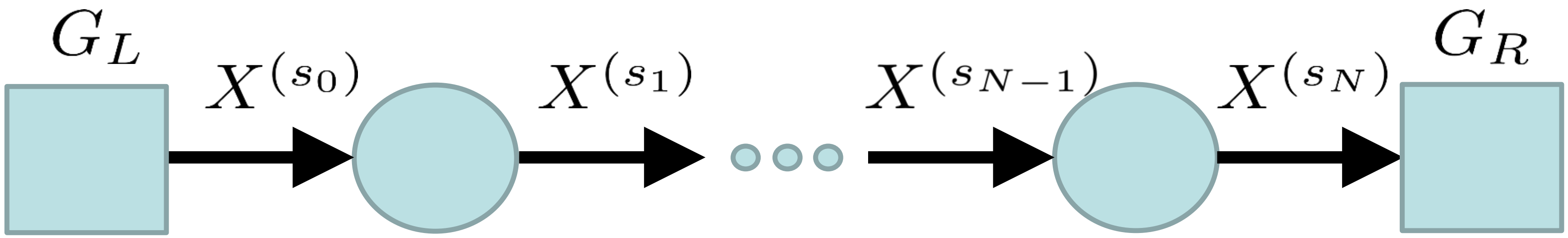
# Large $R$-charge Limit

Baume JJH Lawrie '20

(see also Berenstein Maldacena Nastase '02; Hellerman Orlando Reffert '15)

# Non-BPS Operators?

What if *most* $s_i = r$?

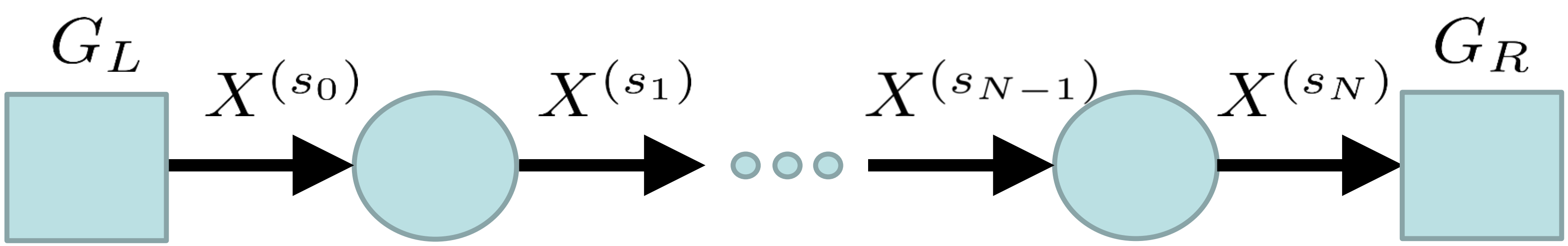



$$\mathcal{O} \equiv X^{(s_0)} \ldots X^{(s_N)}$$

$$\Delta(\mathcal{O}_{\text{eigen}}) = \Delta_{\text{BPS}} + \frac{\gamma}{N^2} + O(N^{-4})$$

# Non-BPS Operators?

What if *most* $s_i = r$?



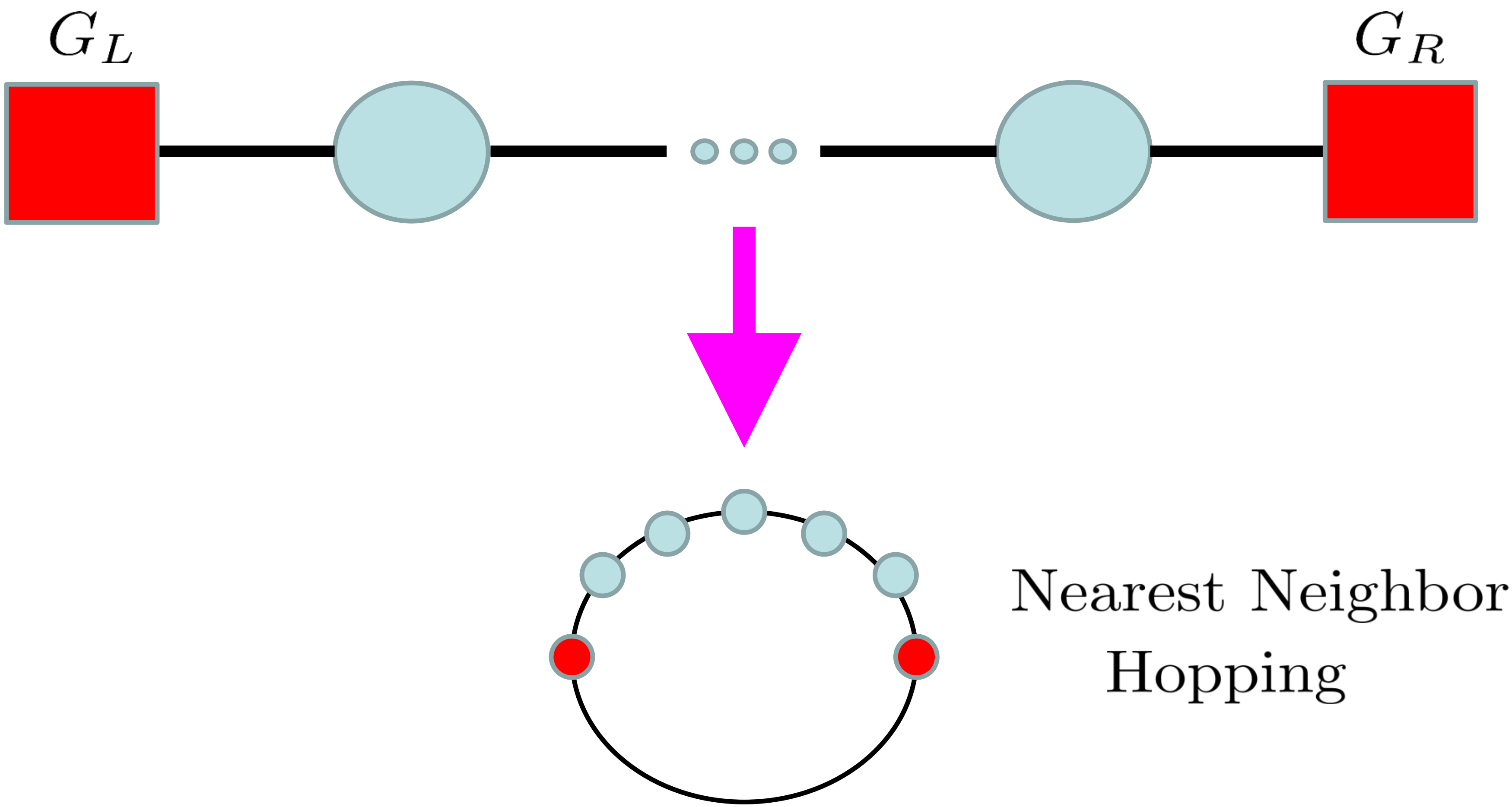

$$\mathcal{O} \equiv X^{(s_0)} \ldots X^{(s_N)}$$

Calculable!

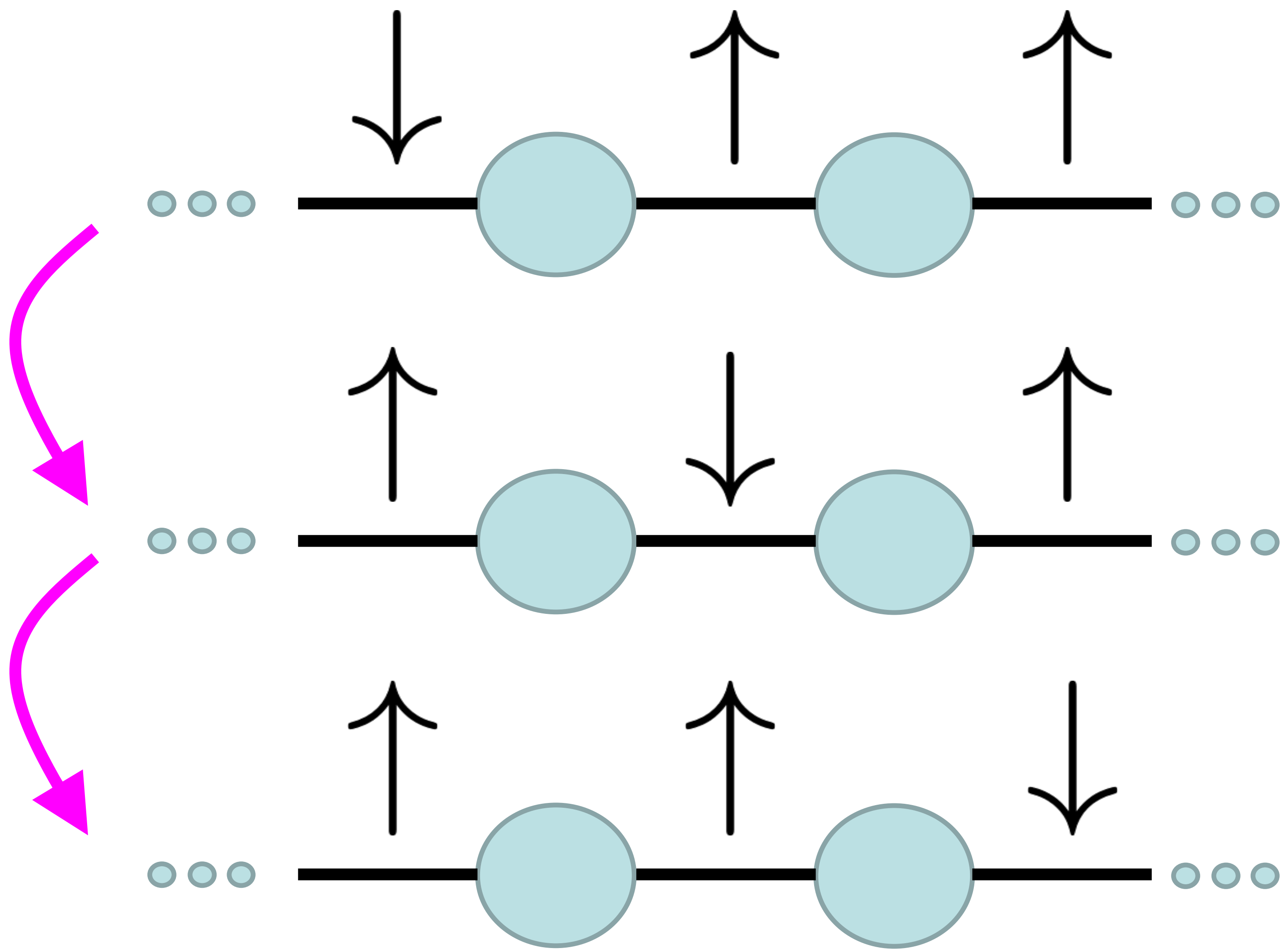
$$\Delta(\mathcal{O}_{\text{eigen}}) = \Delta_{\text{BPS}} + \frac{\gamma}{N^2} + O(N^{-4})$$

# Holographic Hints

$$AdS_7 \times S^4/\Gamma_G$$

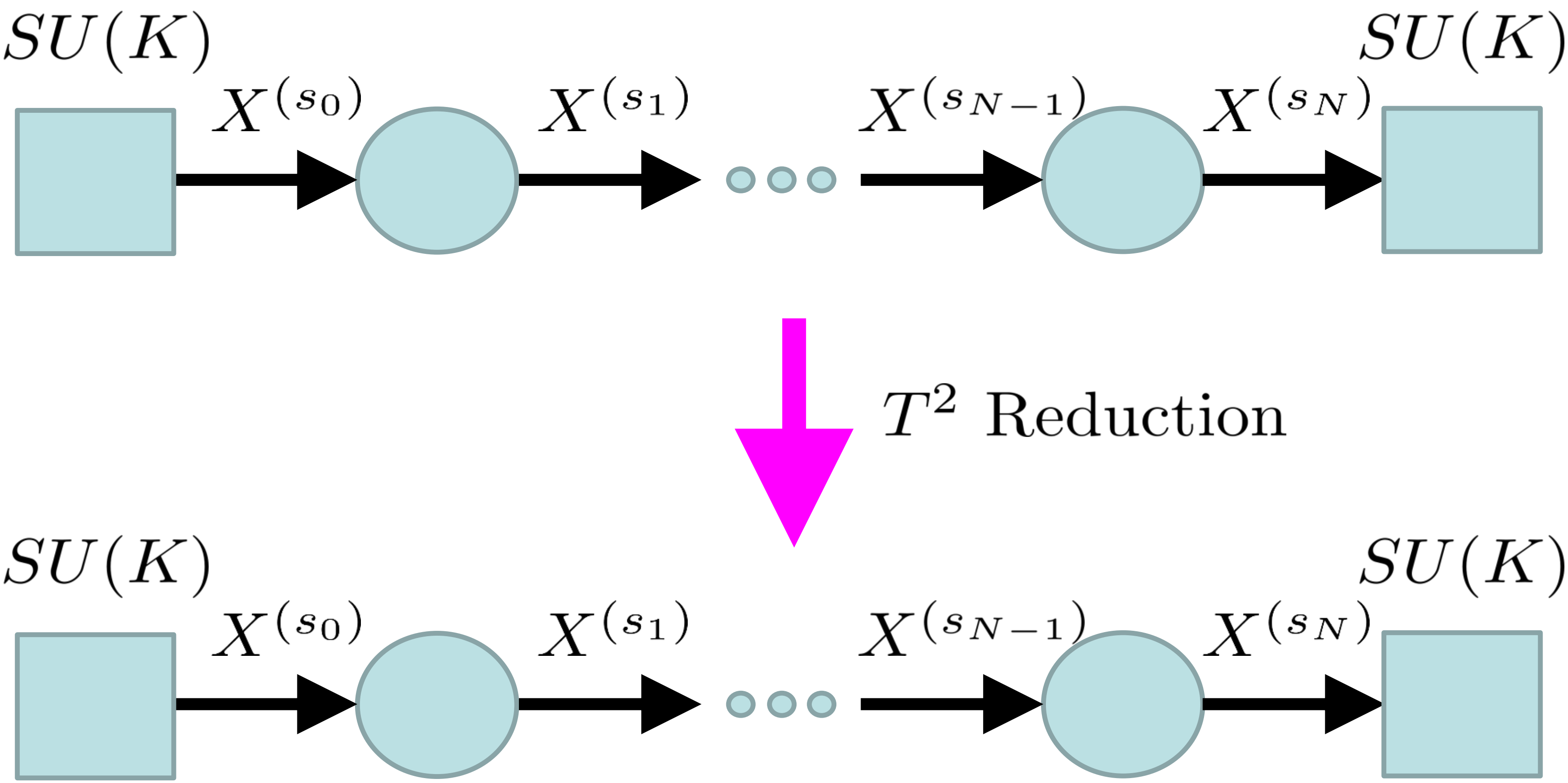


# Impurity Hopping

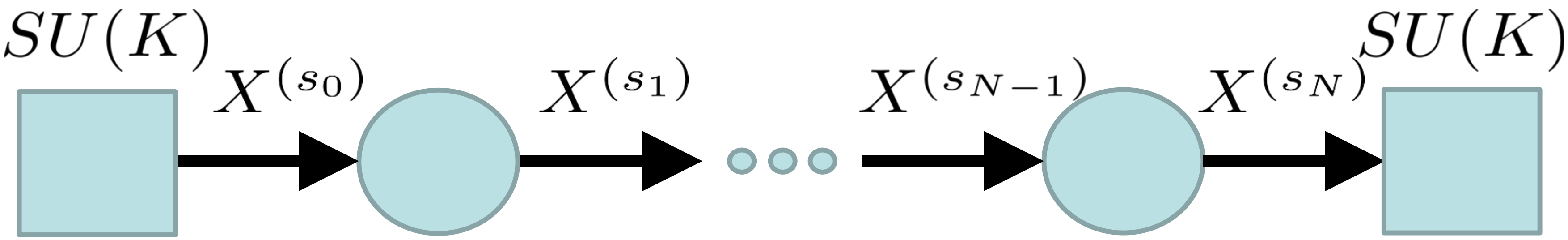


# Direct Calculation
# (4D Case)

# $T^2$ Reduction



# 4D 2-Pt Function




$$\mathcal{O}_j \equiv X_0^{(+1/2)} \ldots X_j^{(-1/2)} \ldots X_N^{(+1/2)}$$

$$\langle \mathcal{O}_i^\dagger(x) \mathcal{O}_j(0) \rangle = \frac{1}{|x|^{2\Delta_{\text{BPS}}}} \left(1 - \gamma_{ij} \log|x|^2\right)$$

Anomalous Dimension Matrix

# The $\gamma_{ij}$ (in 4D)

$$\langle \mathcal{O}_i^\dagger(x)\mathcal{O}_j(0)\rangle = \frac{1}{|x|^{2\Delta_{\rm BPS}}}\left(1-\gamma_{ij}\log|x|^2\right)$$

$$\underline{\gamma} = \lambda_A \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & ... & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix},$$

1D Lattice Laplacian

$$\lambda_A = \frac{g^2 C_{SU(K)}}{8\pi^2} \quad \text{with: } C_{SU(K)} = \frac{K^2-1}{2K}$$

# Eigen($\gamma_{ij}$) (in 4D)

$$\underline{\gamma} = \lambda_G \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & ... & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix},$$

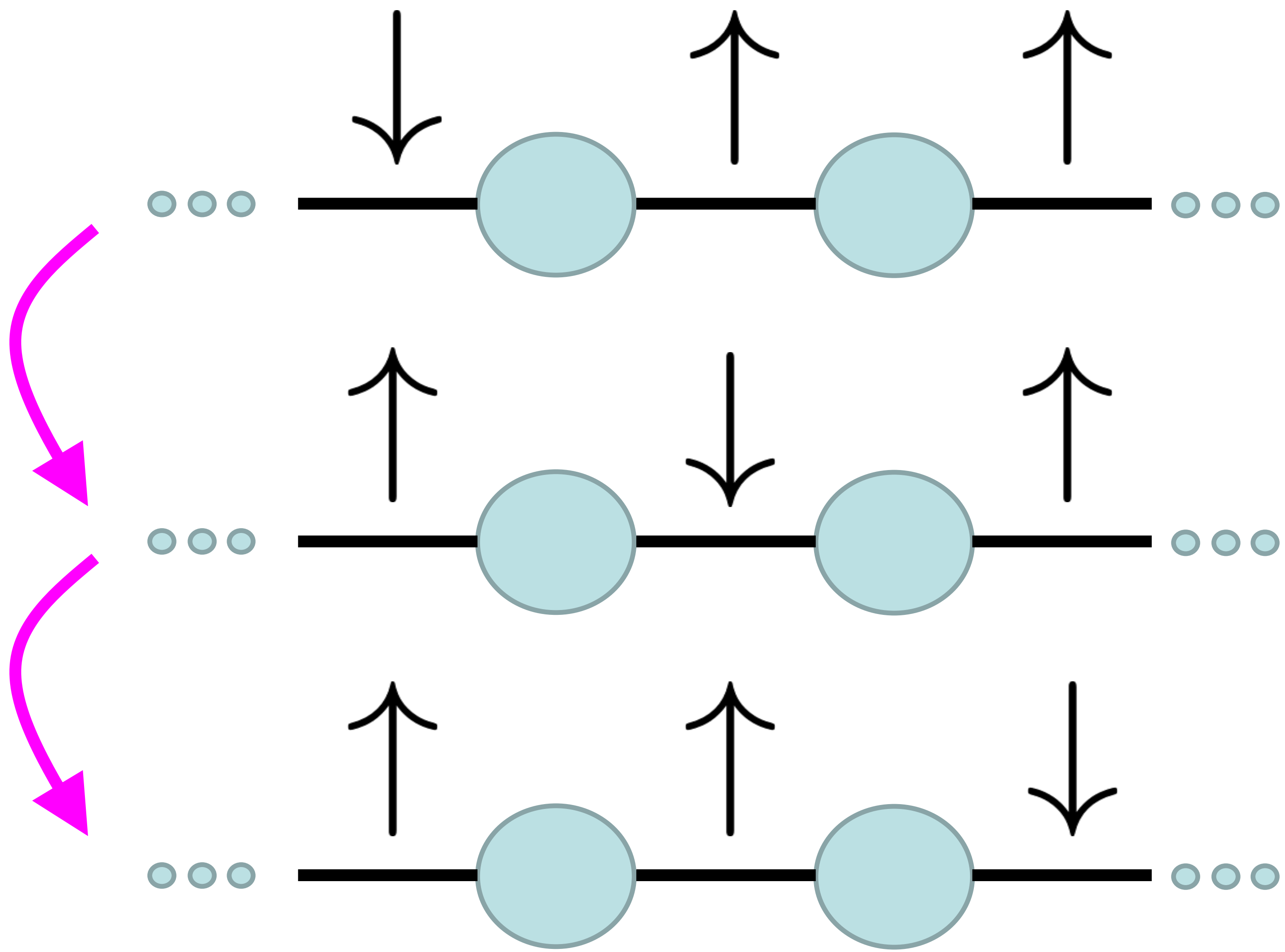
1D Lattice Laplacian

$$\text{Eigen}(\underline{\gamma}) \sim \# \tfrac{g^2 n^2}{N^2} + O(N^{-4})$$

$$\Rightarrow g^2 \text{ can be } O(N^{2-\varepsilon}) \text{ !!!}$$

see BMN '02; Baume JJH Lawrie '20

# Impurity Hopping



# Spin Chain (4D Case)

$$\langle \mathcal{O}^\dagger_{s_0,\ldots,s_N}(x)\mathcal{O}_{s'_0,\ldots,s'_N}(0)\rangle = ???$$

Heisenberg Spin Chain:

$$\widehat{\Delta}_{SU(K)} - \Delta_{\mathrm{BPS}} = -\lambda_{SU(K)} \sum_i \left(2\overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1} - \frac{1}{2}\right)$$

- *Integrable*
- $1/N^2$ spectrum at low impurity #

# Bethe Ansatz

Bethe '31; Leningrad School '70s and '80s

$j = 1, ..., I$ impurities

$\exp(ip_j) = \frac{\mu_j + \frac{i}{2}}{\mu_j - \frac{i}{2}}$, $p_j$ = spin wave momentum

$$\left(\frac{\mu_j + \frac{i}{2}}{\mu_j - \frac{i}{2}}\right)^{2(N+1)} = -\prod_{l \neq j} \frac{(\mu_j - \mu_l + i)(\mu_j + \mu_l + i)}{(\mu_j - \mu_l - i)(\mu_j + \mu_l - i)},$$

Energy: $\Delta - \Delta_{\mathrm{BPS}} = \lambda_A \sum_j \left( \frac{i}{\mu_j + \frac{i}{2}} - \frac{i}{\mu_j - \frac{i}{2}} \right)$

# Assuming Integrability...

Bethe '31; Leningrad School '70s and '80s

$$\widehat{\Delta}_G - \Delta_{\text{BPS}} = -\lambda_G \sum_i Q(\overrightarrow{S}_i \cdot \overrightarrow{S}_{i+1})$$

$$Q_{A_k}(x) = -\frac{1}{2} + 2x$$
$$Q_{D_k}(x) = +\frac{1}{2}x - \frac{1}{2}x^2$$
$$Q_{E_6}(x) = -\frac{3}{4} - \frac{1}{8}x + \frac{1}{27}x^2 + \frac{2}{27}x^3$$
$$Q_{E_7}(x) = -\frac{1}{2} + \frac{13}{24}x + \frac{43}{432}x^2 - \frac{5}{216}x^3 - \frac{1}{144}x^4$$
$$Q_{E_8}(x) = -\frac{148}{125} - \frac{1687}{9000}x + \frac{1297}{18000}x^2 + \frac{593}{20250}x^3 + \frac{79}{97200}x^4 - \frac{77}{243000}x^5 - \frac{1}{48600}x^6,$$

# Bethe Ansatz

Bethe '31; Leningrad School '70s and '80s

$j = 1, ..., I$ impurities

$\exp(ip_j) = \frac{\mu_j + is}{\mu_j - is}$, $p_j =$ spin wave momentum

$$\left(\frac{\mu_j + is}{\mu_j - is}\right)^{2(N+1)} = -\prod_{l \neq j} \frac{(\mu_j - \mu_l + i)(\mu_j + \mu_l + i)}{(\mu_j - \mu_l - i)(\mu_j + \mu_l - i)},$$

Energy: $\Delta - \Delta_{\mathrm{BPS}} = \lambda_G \sum_j \left(\frac{i}{\mu_j + is} - \frac{i}{\mu_j - is}\right)$

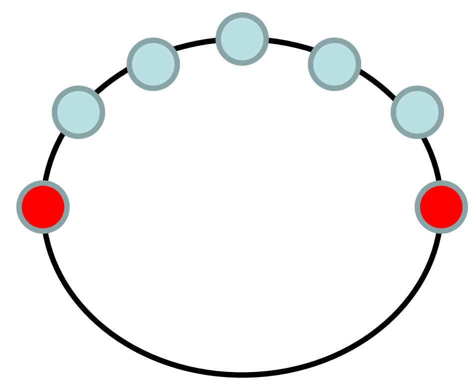
# Direct Calculation
# (6D Case)

# 4D vs 6D

4D Case: marginal coupling $g^{-2} \to g^{-2}_{\text{min}}$

6D Case: $g^{-2} = \langle t \rangle$ dimensionful

Large $N$ / Holographic Regulator:



$$g^2_{6D} \sim \frac{N^{2/3}}{\ell^2_{pl}}$$

$g^2_{\text{eff}} \sim N^{2/3}$ dimensionless

# The $\gamma_{ij}$ (in 6D)

$$\langle \mathcal{O}_i^\dagger(x)\mathcal{O}_j(0)\rangle = \frac{1}{|x|^{2\Delta_{\rm BPS}}}\left(1 - \gamma_{ij}\log|x|^2\right)$$

$$\underline{\gamma} = \lambda_A \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & ... & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix},$$

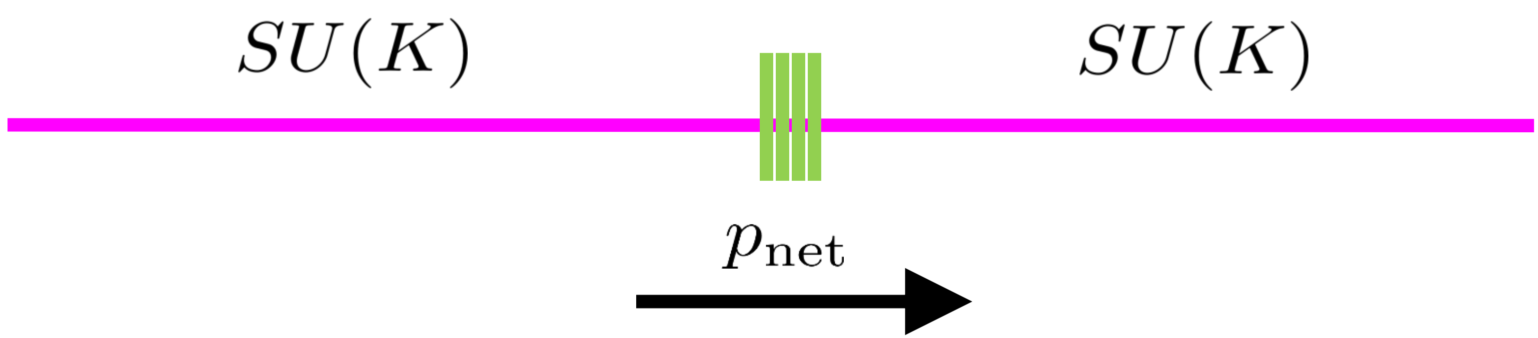
1D Lattice Laplacian

$$\lambda_A = \frac{g^2 C_{SU(K)}}{16\pi^3} \quad \text{with: } C_{SU(K)} = \frac{K^2-1}{2K}$$

# Spectrum

Consists of Spin waves: $\gamma \sim \# N^{2/3} \sum_i \frac{n_i^2}{N^2} + \ldots$

Decoupling: $\sum_i p_i = p_{\text{net}} = 0$ (in 6D but not 4D)



# Bethe Ansatz

Bethe '31; Leningrad School '70s and '80s

$j = 1, ..., I$ impurities

$$\exp(ip_j) = \tfrac{\mu_j + is}{\mu_j - is}, \ \sum_i p_i = p_{\text{net}} = 0$$

$$\left(\tfrac{\mu_j + is}{\mu_j - is}\right)^{2(N+1)} = -\prod_{l \neq j} \tfrac{(\mu_j - \mu_l + i)(\mu_j + \mu_l + i)}{(\mu_j - \mu_l - i)(\mu_j + \mu_l - i)},$$

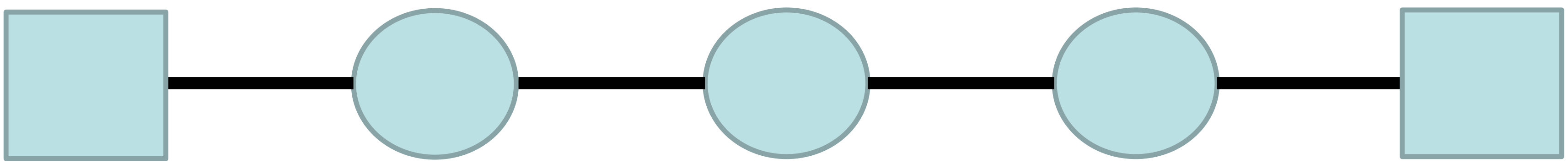
Energy: $\Delta - \Delta_{\text{BPS}} = \lambda_G \sum_j \left( \tfrac{i}{\mu_j + is} - \tfrac{i}{\mu_j - is} \right)$

# Generalizations

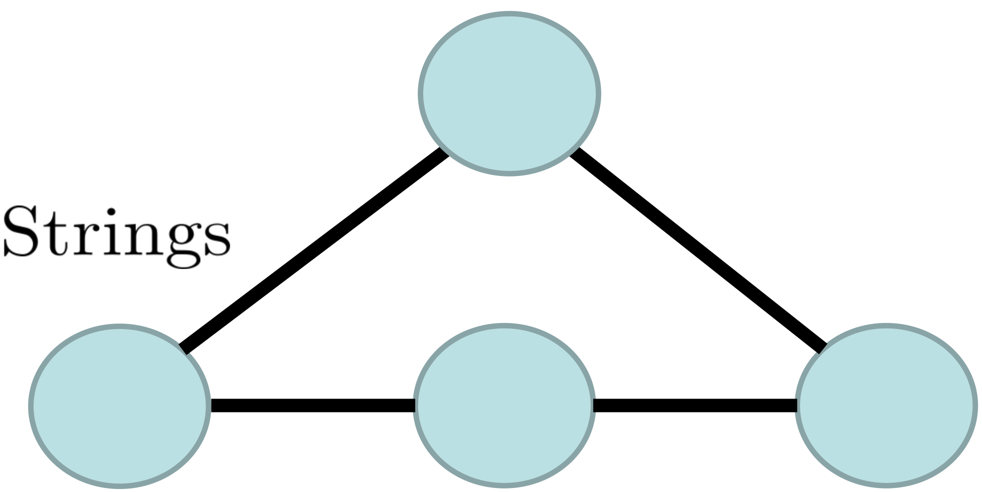
# Periodic Boundary Conditions

Baume JJH Lawrie '20

6D SCFT



6D Little Strings



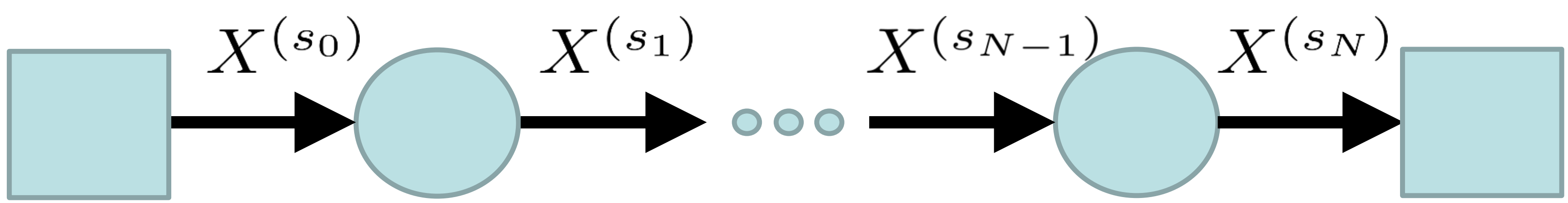
# $\mathfrak{osp}(6,2|1)$ Spins

Baume JJH Lawrie (to appear)

$\mathfrak{su}(2)_{\mathcal{R}} \subset \mathfrak{osp}(6,2|1)$ superconformal algebra

$\Rightarrow$ More general spin chain excitations

(4D versions: see Minahan Zarembo '02; Beisert '03)



# Bethe Ansatz

Arnaudon Crampé Doikou Frappat Ragoucy Avan '04

$$\left(\frac{u_i^I+\frac{i}{2}\mu^I}{u_i^I-\frac{i}{2}\mu^I}\right)^{2(N+1)} = -\prod_J\prod_{j\neq i}\frac{u_i^I-u_j^J+\frac{i}{2}\mathcal{M}_{IJ}}{u_i^I-u_j^J-\frac{i}{2}\mathcal{M}_{IJ}}\;\frac{u_i^I+u_j^J+\frac{i}{2}\mathcal{M}_{IJ}}{u_i^I+u_j^J-\frac{i}{2}\mathcal{M}_{IJ}}$$

$\mu^I$: weight vector for $\mathfrak{osp}(6,2|1)$ state

$\mathcal{M}_{IJ}$ : Cartan for Super-Dynkin (not unique...)

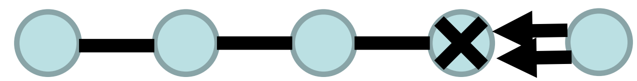
(Choice depends on impurities of interest)

Baum JJH Lawrie *To Appear*

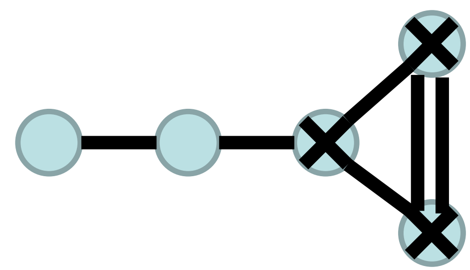
# Subsectors (A-type CM)

Hypermultiplet: $X \to Y^\dagger, \Psi, DX, \ldots$

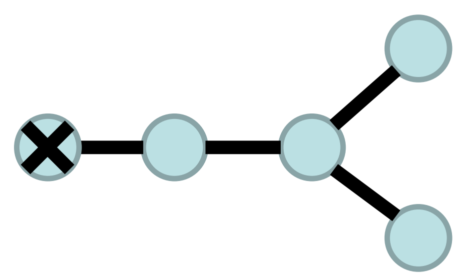
$\mathfrak{su}(2)_R$ $Y^\dagger$



$\mathfrak{su}(1|1)$ $\Psi$



$\mathfrak{sl}(2)$ $DX$



# Beyond $XXX$?

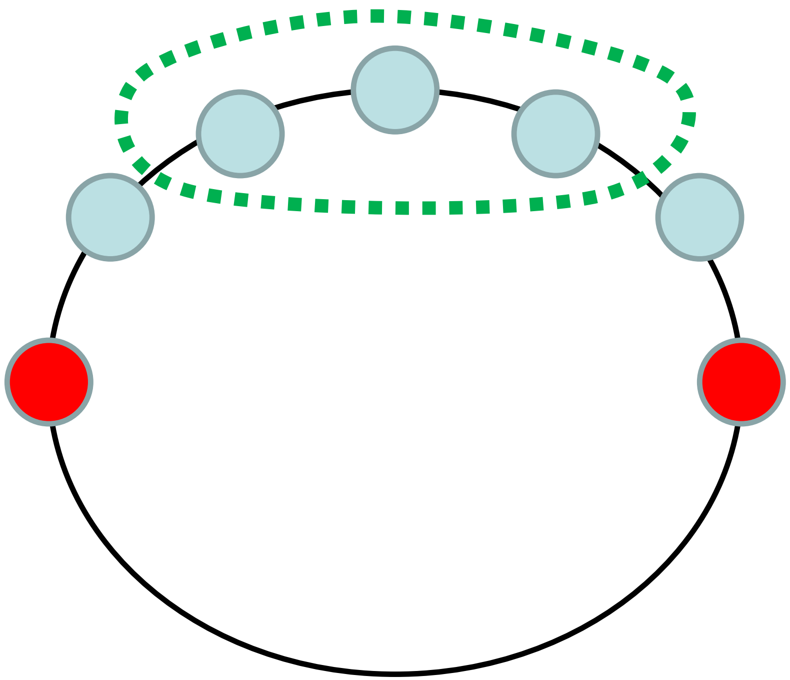
How about $H = -\sum_i \left(2\lambda_{ab} S_i^a \cdot S_{i+1}^b - \frac{1}{2}\right)$?

Yes, $\mathfrak{su}(2)_\mathcal{R}$ breaking, (Higgs branch flows)

But also expect coupling to magnetic field: $\overrightarrow{h} \cdot \overrightarrow{S} \ldots$

# More General Operators?

Closed Loops



# Summary / Future

# Summary / Future

Summary:

- Geometry $\to$ 6D SCFTs and 4D SCFTs
- 1D Structure $\to$ Spin Chains
- Hints of Integrability

Future:

- Integrability at higher order?
- Other semi-protected subsectors?