Math 54 Summer 2017, QUIZ 2

Find all solutions of the following system of linear equations.

$$\begin{aligned} 4x_2 + 8x_3 &= 12 \\ x_1 - x_2 + 3x_3 &= -1 \\ 3x_1 - 2x_2 + 5x_3 &= 6 \end{aligned}$$

$$\begin{bmatrix} 0 & 4 & 8 & | & 12 \\ 1 & -1 & 3 & | & -1 \\ 3 & -2 & 5 & | & 6 \end{bmatrix} \xrightarrow{\text{Swap } R1 \& R2} \begin{bmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 4 & 8 & | & 12 \\ 3 & -2 & 5 & | & 6 \end{bmatrix}$$

$$\xrightarrow{R3 = R3 - 3R1} \begin{bmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 4 & 8 & | & 12 \\ 0 & 1 & -4 & | & 9 \end{bmatrix}$$

$$\xrightarrow{R2 = \frac{1}{4}R2} \begin{bmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & 2 & | & 3 \\ 0 & 1 & -4 & | & 9 \end{bmatrix}$$

$$\xrightarrow{R3 = R3 - R2} \begin{bmatrix} 1 & -1 & 3 & | & -1 \\ 0 & 1 & 2 & | & 3 \\ 0 & 0 & -6 & | & 6 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} -6x_3 &= 6 \\ x_2 + 2x_3 &= 3 \\ x_1 - x_2 + 3x_3 &= -1 \end{aligned} \quad \Rightarrow \begin{aligned} x_3 &= -1 \\ x_2 &= 3 - 2x_3 = 5 \\ x_1 &= -1 + x_2 - 3x_3 = 7 \end{aligned}$$

$$\boxed{\begin{aligned} x_1 &= 7 \\ x_2 &= 5 \\ x_3 &= -1 \end{aligned}}$$

Check:

$$4(5) + 8(-1) = 12$$
$$7 - 5 + 3(-1) = -1$$
$$3(7) - 2(5) + 5(-1) = 6$$