

MATH 54 SUMMER 2017, QUIZ 15

Find the change of basis matrix, $P_{\mathcal{C} \leftarrow \mathcal{B}}$, from the basis \mathcal{B} to the basis \mathcal{C} .

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\} \quad \mathcal{C} = \left\{ \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

$$\begin{array}{ccc} \left[\begin{array}{cc|cc} 0 & 1 & 1 & 3 \\ 3 & 2 & 2 & 4 \end{array} \right] & \xrightarrow{\text{Swap } R1 \text{ and } R2} & \left[\begin{array}{cc|cc} 3 & 2 & 2 & 4 \\ 0 & 1 & 1 & 3 \end{array} \right] & \xrightarrow{R1=R1-2R2} & \left[\begin{array}{cc|cc} 3 & 0 & 0 & -2 \\ 0 & 1 & 1 & 3 \end{array} \right] \\ & \xrightarrow{R1=\frac{1}{3}R1} & \left[\begin{array}{cc|cc} 1 & 0 & 0 & -2/3 \\ 0 & 1 & 1 & 3 \end{array} \right] & & \end{array}$$

Therefore

$$P_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{bmatrix} 0 & -2/3 \\ 1 & 3 \end{bmatrix}$$