Math 54 Homework 32 (Assigned 8/4)

By the end of the day on Saturday I will upload some notes about the things we talked about in class on Friday and which may be helpful on this assignment.

Don't worry if you find some of these problems tricky—especially the last one. Just do your best to answer them and everything will be okay.

1. 10.2: 15

2. 10.2: 16

3. Find all eigenvectors of $\frac{d^2}{dx^2} + \frac{d}{dx}$ with eigenvalue 2.

4. Recall that in class we found that $e^{-(\pi/L)^2\beta t}\sin(\pi x/L)$ and $e^{-(2\pi/L)^2\beta t}\sin(2\pi x/L)$ were two solutions to the heat equation with the usual boundary values. Draw the graphs of both of these on the interval $0 < x < L$ at $t = 0$, $t = 1$ and $t = 10$.

5. (The wave equation is a partial differential equation that can be solved using the same ideas we used in class to find solutions to the heat equation. In this problem you will go through one of the parts of that process.)

   The wave equation is the partial differential equation
   $$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$$
   Suppose that $z(x)$ is an eigenvector of $\frac{d^2}{dx^2}$ with eigenvalue λ. Show that $e^{t\sqrt{\lambda}}z(x)$ and $e^{-t\sqrt{\lambda}}z(x)$ are both solutions to the wave equation.

6. Recall that in class, we found the eigenvectors of $\beta\frac{d^2}{dx^2}$ were
   $$\begin{aligned} &c_1e^{\sqrt{\lambda}x} + c_2e^{-\sqrt{\lambda}x} && \text{with eigenvalue } \beta\lambda \text{ where } \lambda > 0 \\ &c_1 + c_2x && \text{with eigenvalue } \beta\lambda \text{ where } \lambda = 0 \\ &c_1\cos(\sqrt{-\lambda}x) + c_2\sin(\sqrt{-\lambda}x) && \text{with eigenvalue } \beta\lambda \text{ where } \lambda < 0. \end{aligned}$$
   Each of these eigenvectors gave us a solution to the heat equation. We then used the required boundary values to get rid of some of these solutions. If the required boundary values had instead been
   $$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(L,t)}{\partial x} = 0$$
   then what solutions would we have been left with? Try to use the same process we used in class (i.e. separately examine the cases where $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$ and see what happens in each).