Math 54 Homework 1 (due Tuesday 6/20)

For the first seven questions, refer to the handout on the course website. For the rest of the questions, refer to Appendix B of the textbook.

1. Answer True or False for each item below. For this question only, you do not need to give any explanation for your answers.

    (a) $1 \in \mathbb{N}$

    (b) $\{1, 2\} \in \mathbb{R}$

    (c) $0 \in \{1, 2, 3\}$

    (d) $\{1, 2, 3\} \subseteq \mathbb{N}$

    (e) "primordial" $\in \{w \mid w$ is an English word that does not contain the letter 'e'$\}$

    (f) $0 \in \varnothing$

2. For each of the following sets, either give an element of the set or explain why the set is the empty set.

    (a) $\mathbb{Z} \setminus \mathbb{N}$

    (b) $\{n \in \mathbb{N} \mid n$ is odd$\} \cap \{n \in \mathbb{N} \mid n$ is a power of 2$\}$

    (c) $\{f \mid f$ is a function from $\mathbb{N}$ to $\mathbb{N}\} \cap \{f \mid f$ is a function from $\mathbb{R}$ to $\mathbb{R}\}$

    (d) $\mathbb{N} \cup \{\sqrt{2}, \sqrt{3}\}$

    (e) $(-3, 5]$

    (f) $\{A \mid A \subseteq \mathbb{Z}\}$

    (g) $\{7, 8, 9, \ldots, 25\} \cap \{2, 4, 6, \ldots\}$

3. For each of the following functions, state whether it is one-to-one and whether it is onto. If it is not one-to-one, explain why not. If it is not onto, explain why not. If it is both one-to-one and onto, find its inverse.

    (a) $f \colon \mathbb{N} \to \{0, 1\}$ defined by

    $$f(n) = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$$

    (b) $g \colon \mathbb{Z} \to \mathbb{R}^2$ defined by

    $$g(n) = \begin{bmatrix} n \\ -n \end{bmatrix}$$

    (c) $h \colon \mathbb{R} \to \{x \in \mathbb{R} \mid x \geq 0\}$ defined by $h(x) = e^x$

    (d) $k \colon \{1, 2, 3, \ldots\} \to \mathbb{N}$ defined by $k(n) =$ the nth digit of π

    (e) $j \colon \{0, 1, 2, 3\} \to \{1, 2, 4, 8\}$ defined by $j(n) = 2^n$.

4. What is the range of each function in question 3?

5. Write down a function from $\mathbb{N}$ to $\mathbb{N}$ that is:

   (a) Both one-to-one and onto.
   (b) One-to-one but not onto.
   (c) Onto but not one-to-one.
   (d) Neither one-to-one nor onto.

   Make sure to explain why the functions you give in parts (c) and (d) are not one-to-one and why the functions you give in parts (b) and (d) are not onto.

6. Let $f\colon \mathbb{R} \to \mathbb{R}^3$ be the function defined by

$$f(x) = \begin{bmatrix} x \\ x \\ x \end{bmatrix}$$

   and let $g\colon \mathbb{R}^3 \to \mathbb{R}$ be the function defined by

$$g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = x + y + z.$$

   (a) What is $g \circ f$?
   (b) What is $f \circ g$?

7. Let $f\colon \mathbb{Z} \to \mathbb{R}$ be the function defined by $f(n) = e^n$.

   (a) What is $f^{-1}(\mathbb{R})$?
   (b) What is $f^{-1}(\{x \in \mathbb{R} \mid x \geq 0\})$?

8. Let $w = 5 + 4i$ and $z = -1 + 2i$. Compute each of the following:

   (a) $\bar{z}$
   (b) $|z|$
   (c) wz
   (d) w/z

9. Let $z = -1 + 2i$. Find real numbers r and θ such that $z = r(\cos(\theta) + \sin(\theta))$.

10. Let $a = 2(\cos(\pi/4) + i\sin(\pi/4))$ and $b = 3(\cos(\pi/3) + i\sin(\pi/3))$. Compute each of the following:

    (a) $|a|$
    (b) a^{-1}
    (c) $Re(a)$
    (d) $Im(a)$
    (e) a^4
    (f) b/a

11. Draw w, z, a, and b (from the previous three problems) in the complex plane.