Review

1. For A shown below, find:

(a) a vector in Col A
(b) a vector not in Col A

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
(c) a vector in Null A
(d) a vector not in Null A

- 2. If A is an $n \times m$ matrix then how many solutions does $A\mathbf{x} = \mathbf{b}$ have if:
 - (a) Null $A = \{0\}$ and $\mathbf{b} \in \operatorname{Col} A$?
 - (b) Null $A \neq \{0\}$ and $\mathbf{b} \in \operatorname{Col} A$?
 - (c) $\mathbf{b} \notin \operatorname{Col} A$?
- 3. If A is an $n \times n$ invertible matrix, what are Null A and Col A?

Bases, Dimension, and Rank

- 1. Is $\left\{ \begin{bmatrix} 1\\2 \end{bmatrix}, \begin{bmatrix} 3\\4 \end{bmatrix} \right\}$ a basis for \mathbb{R}^2 ?
- 2. Can 2 vectors ever form a basis for \mathbb{R}^3 ? Can 4 vectors ever form a basis for \mathbb{R}^3 ? What are the possible sizes of a basis for \mathbb{R}^3 ?
- 3. Suppose V is a subspace of \mathbb{R}^n . What is the largest possible size of a basis for V? What if you know that $V \neq \mathbb{R}^n$?
- 4. Find a basis for $\operatorname{Col} A$ and a basis for $\operatorname{Null} A$.

$$A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 4 & 5 & -3 \\ 5 & 10 & 0 & 20 \end{bmatrix}$$

- 5. For A as in the previous problem, what is rank A? What is $\dim(\operatorname{Null} A)$?
- 6. Suppose A is a 5×7 matrix of rank 3. What is dim(Null A)? (Hint: think about pivots.)
- 7. If A is an $n \times m$ matrix with linearly independent columns, what is the rank of A?

Vector Spaces

Which of the following are vector spaces?

- 1. The set of convergent sequences of real numbers whose limit is 0.
- 2. The set of functions from \mathbb{Z} to \mathbb{Z} .
- 3. The set of differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that $\frac{d}{dx}f(x) = f(x)$.
- 4. The set of polynomials with real coefficients of degree exactly 3.
- 5. The set of matrices in REF.