# Review

1. Suppose $A$ is an $n \times n$ matrix. What is $(I + A + A^2 + \ldots + A^{m-1})(I_n - A)$ (i.e. what is the simplest expression that is equal to the given expression)?

2. Find an expression for $(I_n - A^m)(I - A)^{-1}$ not involving inverses.

# Subspaces

1. True or false:

   (a) $\mathbb{Z}$ is a subspace of $\mathbb{R}$

   (b) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

   (c) $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$

   (d) The line defined by $y = x + 3$ is a subspace of $\mathbb{R}^2$. (Hint: try drawing it)

   (e) If $A$ is a $2 \times 2$ matrix then the set of solutions to $A\mathbf{x} = \begin{bmatrix}1\\2\end{bmatrix}$ is always a subspace of $\mathbb{R}^2$.

   (f) If $\mathbf{v}_1, \ldots, \mathbf{v}_m \in \mathbb{R}^n$ then $\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ is a subspace of $\mathbb{R}^n$.

2. If $A$ is an $n \times m$ matrix, is the set of solutions to $A\mathbf{x} = \mathbf{0}$ always a subspace of $\mathbb{R}^m$?

3. Fill in the blanks: If $A$ is an $n \times m$ matrix, $\operatorname{Col} A$ is a subspace of ____ and $\operatorname{Null} A$ is a subspace of ____

4. Is it possible that $\operatorname{Col} A = \operatorname{Null} A$? If so, give an example. If not, explain why not.

5. Suppose $A$ is an $n \times m$ matrix and $B$ is an $m \times p$ matrix such that $\operatorname{Null} A = \operatorname{Col} B$. What can you say about $AB$?

# Bases, Dimension, and Rank

1. Find a basis for
$$\text{span}\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\2\\4\end{bmatrix}\right\}$$

2. Suppose $V$ is a subspace of $\mathbb{R}^n$. What is the largest possible size of a basis for $V$? What if you know that $V \neq \mathbb{R}^n$?

3. Find a basis for $\mathbb{R}^2$ besides $\{\mathbf{e}_1, \mathbf{e}_2\}$.

4. Find a basis for $\operatorname{Col} A$ and a basis for $\operatorname{Null} A$.
$$A = \begin{bmatrix}1 & 2 & 0 & 4\\ 2 & 4 & 5 & -3\\ 5 & 10 & 0 & 20\end{bmatrix}$$

5. Suppose $A$ is a $5 \times 7$ matrix of rank 3. What is $\dim(\operatorname{Null} A)$? (Hint: think about pivots.)