Review

1. Write an initial value problem for which $3e^t - e^{-2t} + 5$ is the unique solution.
2. Find a solution to the following initial value problem. How many solutions are there?
$$y'' - y = 0; \quad y(0) = 2$$
3. Find a solution to the following initial value problem. You will probably be able to guess the solution without using row reduction.
$$y'' - y = 0; \quad y(1) = 3e + 5e^{-1}; \quad y'(1) = 3e - 5e^{-1}$$

A Little Bit of Theory

For the questions in this section, let V be the vector space of infinitely differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ (commonly denoted $C^\infty(\mathbb{R})$).

1. Let $T\colon V \to V$ be the linear transformation defined by $T(f) = \frac{df}{dt}$. Are e^{2t} and e^{3t} eigenvectors of T? If so, what are the corresponding eigenvalues?
2. Let $T\colon V \to V$ be the linear transformation defined by $T(f) = \frac{df}{dt} - 2f$. In other words, $T = \frac{d}{dt} - 2I$. What is $T(e^{3t})$? What about $T(e^{3t})$?
3. Let $T_1\colon V \to V$ and $T_2\colon V \to V$ be the linear transformations defined by $T_1(f) = \frac{df}{dt} - 2f$ and $T_2(f) = \frac{df}{dt} - 3f$. Let $T = T_1 \circ T_2$. Is e^{2t} in the kernel of T?
4. Let $T\colon V \to V$ be the linear transformation defined by $T(f) = \frac{df}{dt} - 2f$. What is $T(te^{2t})$? What is $T(T(te^{2t}))$?

Repeated and Complex Roots

1. Find the general solution of the following differential equations.

   (a) $y'' - 6y' + 9y = 0$

   (b) $y''' - 5y'' = 0$
2. Find the general solution of the following differential equations.

   (a) $y'' - 6y' + 10y = 0$

   (b) $y'' + 4y' + 6y = 0.$
3. Write a differential equation such that $t^2e^{3t} + 5$ is a solution.
4. Write a differential equation such that $e^{-2t}\cos(4t) + e^t$ is a solution.

Definitions and Theorems

Definitions:

- Pre-Wronskian
- Wronskian

Main Theorem Today: How to find all solutions of a linear, constant coefficient, homogeneous ODE of any order.

Most important idea today: Factoring the auxiliary equation is equivalent to "factoring" a linear transformation. This (almost) lets us reduce a higher order differential equation to solving several first order equations. (The "almost" is because there may be repeated roots.)