# Complex Eigenvalues

For this section, let

$$A = \begin{bmatrix} 3/2 & -5/2 \\ 5/2 & -3/2 \end{bmatrix}$$

1. Find the eigenvalues of $A$ and for each eigenvalue, find a corresponding eigenvector.
2. Find real numbers $a$ and $b$ such that $A$ is similar to
$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$
3. Find $r$ and $\theta$ such that $A$ is similar to
$$r \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$
4. Calculate and draw $\mathbf{e}_1$, $A\mathbf{e}_1$, $A^2\mathbf{e}_1$, $A^3\mathbf{e}_1$, and $A^4\mathbf{e}_1$. How do the vectors you calculated and drew relate to the values for $r$ and $\theta$ you found above?

# Dot Product

1. $$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$
   (a) Find the lengths of $\mathbf{u}$ and $\mathbf{v}$—i.e. find $||\mathbf{u}||$ and $||\mathbf{v}||$.
   (b) Find the distance between $\mathbf{u}$ and $\mathbf{v}$—i.e. find $\text{dist}(\mathbf{u}, \mathbf{v})$.
   (c) Find $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u} \cdot \mathbf{w}$.
   (d) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$ and the cosine of the angle between $\mathbf{u}$ and $\mathbf{w}$.
   (e) Find a unit vector in the same direction as $\mathbf{u}$.
2. If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$ and the angle between them is $\pi/2$ radians ($90°$), what is $\mathbf{u} \cdot \mathbf{v}$?
3. If $\mathbf{u}$ is any vector in $\mathbb{R}^n$, what is $\mathbf{u} \cdot \mathbf{0}$?
4. Is any pair of vectors from problem (1) orthogonal?
5. Show that for all vectors $\mathbf{v}$ in $\mathbb{R}^n$, $\mathbf{v} \cdot \mathbf{v} \geq 0$. When is it equal to 0?
6. Suppose $W$ is a subspace of $\mathbb{R}^n$. Let $H$ be the set of all vectors that are orthogonal to every vector in $W$. Formally,
$$H = \{\mathbf{v} \in \mathbb{R}^n \mid \text{for all } \mathbf{u} \text{ in } \mathbb{R}^n,\ \mathbf{v} \cdot \mathbf{u} = 0\}.$$
   Is $H$ a subspace of $\mathbb{R}^n$?
7. What is the orthogonal complement of $\{\mathbf{0}\}$? (Assume this means the zero vector in $\mathbb{R}^n$.)