Review

1. Find an eigenvector of eigenvalue 5 of the following linear transformation. Recall that $C^1(\mathbb{R})$ is the vector space of differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ and $C(\mathbb{R})$ is the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. (Hint: it is not possible to solve this problem by translating everything to $\mathbb{R}^n$, but if you understand what an eigenvalue is and remember a little calculus, you do have the necessary knowledge to solve it.)

$$T\colon C^1(\mathbb{R}) \to C(\mathbb{R})$$
$$T(f) = \frac{df}{dx}$$

2. What are all the eigenvalues of the linear transformation in the previous problem?

Complex Eigenvalues

1. Find the eigenvalues of
$$A = \begin{bmatrix} 1/2 & -3/5 \\ 3/4 & 11/10 \end{bmatrix}$$

2. With A as in the previous problem, find an eigenvector for each eigenvalue of A.

3. Diagonalize A.

4. With $\mathbf{v}$ given below and A as in the previous three problems, calculate and draw $\mathbf{v}, A\mathbf{v}, A^2\mathbf{v}, \ldots, A^8\mathbf{v}$.
$$\mathbf{v} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

5. If $a_0 = 1$, $a_2 = 2$ and $a_{n+1} = 2a_n - 2a_{n-1}$, find a formula for a_n.