Review

1. Find at least 3 subspaces of $\mathbb{P}_3$.

Why You Should Love Bases

1. Suppose V is a vector space and $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for V. Let $T\colon V \to \mathbb{R}^n$ be the function defined by $T(\mathbf{v}) = [\mathbf{v}]_{\mathcal{B}}$. Why is T a linear transformation? Talk about this with the people sitting around you.

2. Find a polynomial p such that

$$[p]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

where $\mathcal{B} = \{1, x, x^2 + x\}$.

3. Suppose V is a vector space and $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for V. Can there be a set of more than n linearly independent vectors in V? Why or why not?

Linear Transformations and Bases

1. For each of the following linear transformations, find a basis for the domain and codomain and write the matrix of the linear transformation relative to those bases.

   (a) $T\colon \mathbb{P}_2 \to \mathbb{P}_1$ defined by $T(p) = \frac{dp}{dx}$.

   (b) $T\colon M_{2\times 2} \to M_{2\times 2}$ defined by $T(B) = AB$ where

   $$A = \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix}$$

   (c) $T\colon \operatorname{span}\{\sin(x), \cos(x)\} \to \mathbb{R}^3$ defined by $T(f) = \begin{bmatrix} f(0) \\ f(\pi) \\ f(\pi/2) \end{bmatrix}$.

   (d) $T\colon \operatorname{span}\{\sin(x), \cos(x)\} \to \mathbb{R}$ defined by $T(f) = \int_0^\pi f(x)\,dx$.

2. Which of the linear transformations in the previous questions are one-to-one? Which ones are onto?

3. For each linear transformation from the previous question, find a basis for the kernel and for the range.

4. The following linear transformation is one-to-one and onto (you do not have to check this). Find its inverse.

$$T\colon \mathbb{P}_2 \to \mathbb{R}^3$$

$$T(p) = \begin{bmatrix} p(0) \\ p(1) \\ p(2) \end{bmatrix}$$