Review

1. Are the following vectors in $\mathbb{R}^3$ linearly independent? Do they span all of $\mathbb{R}^3$?

$$\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 7 \end{bmatrix}$$

Linear Transformations

1. Which of the following are linear transformations?

   (a) $T\colon \mathbb{P}_2 \to \mathbb{P}_2$ defined by $T(p) = \frac{dp}{dx}$.

   (b) $T\colon M_{2\times 3} \to M_{4\times 3}$ defined by $T(B) = AB$ where

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

   (c) $T\colon M_{2\times 2} \to M_{2\times 2}$ defined by $T(A) = A + I$.

   (d) $T\colon C([0,1]) \to C([0,1])$ defined by $T(f) = f^2$.

   (e) $T\colon C(\mathbb{R}) \to \mathbb{R}^2$ defined by $T(f) = \begin{bmatrix} f(0) \\ f(\pi) \end{bmatrix}$.

   (f) $T\colon C([0,1]) \to \mathbb{R}$ defined by $T(f) = \int_0^1 f(x)\,dx$.

2. For each linear transformation in the previous question, find a nonzero element of the range and a nonzero element of the kernel.

Why You Should Love Bases

1. Answer the following questions.

   (a) Is $\{1, x, x^2\}$ a basis for $\mathbb{P}_2$?

   (b) Is $\{1, x, x^2\}$ a basis for $\mathbb{P}_3$?

   (c) Is $\{1, x^2, x, 3x^2 - 2\}$ a basis for $\mathbb{P}_2$?

   (d) What is the dimension of $\mathbb{P}_2$? What about $\mathbb{P}_3$?

2. In each item below, you are given a vector space, a basis for the vector space and a vector in the vector space. Write the vector as a coordinate vector in terms of the basis (you do not have to check that the list of vectors is actually a basis).

   (a) $\mathbb{P}_3$, $\{1, x, x^2, x^3\}$, $3x^2 - 5$

   (b) $M_{2\times 2}$, $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$, $\begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix}$

(c) $\text{span}\{\sin(x), \cos(x)\}$ (this is a subspace of $C(\mathbb{R})$), $\{\sin(x), \cos(x)\}$, $4\cos(x) - \sin(x)$

(d) $\mathbb{P}_2$, $\{1 + x, x + x^2, x^2 + x^3, x^3 - 1\}$, 1

(e) $\mathbb{R}^2$, $\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \end{bmatrix} \right\}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

3. Find a polynomial p such that

$$[p]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

where $\mathcal{B} = \{1, x, x^2 + x\}$.

4. Is $\begin{bmatrix} 1 & 2 \\ 11 & 6 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 10 \\ -5 & 15 \end{bmatrix} \right\}$?

5. Are the following polynomials linearly independent? Do they span all of $\mathbb{P}_2$?

$$3x^2 + x,\ 4x^2 - 3x + 2,\ 7x^2 - 2x + 2$$

6. Do the following functions form a basis for $\text{span}\{\sin(x), \cos(x), e^x\}$? (You may assume without proof that $\sin(x)$, $\cos(x)$, and e^x are linearly independent.)

$$\cos(x) + 3e^x,\ 2\sin(x) - 3\cos(x) + 4e^x,\ 2\sin(x) - 2\cos(x) + 7e^x$$

7. What is the dimension of $\text{span}\{1 + x, x + x^2, x^2 - 1\}$?